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HAND-BOOKS

OF

NATURAL PHILOSOPHY

AND

ASTRONOMY.

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MECHANICS—HYDROSTATICS—HYDRAULICS
PNEUMATICS—SOUND—OPTICS.

ILLUSTRATED BY UPWARDS OF FOUR HUNDRED
ENGRAVINGS ON WOOD.

PHILADELPHIA:
BLANCHARD AND LEA.
1859.

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STEREOTYPED BY J. FAGAN. COLLINS, PRINTERS.

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AMERICAN PUBLISHERS' NOTICE.

In passing this volume through the press, the publishers have availed themselves of the services of a gentleman in every way competent to ensure the accuracy necessary to a scientific text-book. Various errors, which had escaped the author's attention, have thus been corrected, and occasional omissions supplied. A series of questions and examples has also been appended to each subject, for the purpose of drawing the student's attention to the practical applications of the various laws and theories explained in the text.

In order to supply the wants of those who desire to obtain separate manuals on the different subjects embraced in this volume, it has been arranged for binding either in three parts, or as a whole. The First Part embraces Mechanics; the Second, Hydrostatics, Hydraulics, Pneumatics, and Sound; the Third, Optics. The paging at the head of the pages is for the Parts; that at the foot is continuous throughout the volume. It will be seen that the references in the Table of Contents apply to the former, and those in the Index to the latter.

Philadelphia, *August*, 1851.

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AUTHOR'S PREFACE.

In the composition of this work the author has had in view the satisfaction of those who desire to obtain a knowledge of the elements of physics without pursuing them through their mathematical consequences and details. The methods of demonstration and illustration have accordingly been adapted to such readers.

The present volume will, it is hoped, be understood, without difficulty, by all persons of ordinary education, and may, with some aid from the teacher, be with advantage placed in the hands of pupils in the higher classes of the schools for either sex.

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The second course will contain Heat, Electricity, Magnetism, and Astronomy.

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(The numbers refer to paging at the foot of the pages.)

☞ A full special Table of Contents on each subject will be found t the commencement of each Hand-Book.

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A

HAND-BOOK

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MECHANICS.

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HAND-BOOK

OF

NATURAL PHILOSOPHY.

BOOK THE FIRST.

PROPERTIES OF MATTER.

CHAPTER I.

DIVISION OF THE SUBJECT.

1. The material world, the bodies which compose it, and their qualities and properties, form the subjects of contemplation and inquiry to the natural philosopher.

Among these properties, the most important are the following:—

1. Magnitude and Form.
2. States of Aggregation.
3. Impenetrability.
4. Divisibility.
5. Porosity and Density.
6. Compressibility and Contractibility.
7. Elasticity and Dilatability.
8. Inactivity.
9. Specific Properties.

We shall therefore, in the present book, explain and illustrate these qualities.

CHAP. VII.

REGULATION OF FORCE.

CHAP. VIII.

THE PENDULUM.

CHAP. IX.

RESISTING FORCES.

9. *Surfaces* are bounded by *lines.* Thus the surface of this page is bounded by four straight lines, forming its edges; the surface of a circle is bounded by the line called its circumference.

10. *Magnitude is unlimited as well in its increase as in its diminution.*

There is no magnitude so great, that we cannot conceive a greater; and none so small, that we cannot conceive a smaller.

The diameter of the earth measures about 8000 miles; but it is very small compared to the diameter of the sun, which measures nearly 900,000 miles; and this, again, is itself very small compared with the distance between the earth and the sun, which measures little less than 100,000,000 of miles; and even this last space, great as it is, vanishes to nothing, compared with the distance between the sun and the fixed stars.

11. In like manner, there is no limit to the diminution of which magnitude is susceptible.

There is no line so small, that it may not be bisected, or halved, or reduced, in any desired proportion.

Let P 1 (*fig.* 1.) be a line of any proposed length, as for example, the tenth of an inch.

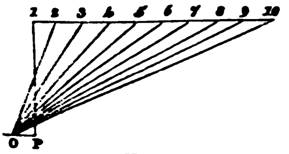

Fig. 1.

Draw P O at right angles to it, and a line 1, 10 parallel to P O. Take upon this line distances 1, 2, 3, 4, &c. successively equal to P O: the line O 2 will cut off half P 1; O 3 will cut off one-third of P 1; O 4 will cut off one-fourth of it; O 5 one-fifth, and so on. Now, as there is no limit to the number of equal parts that may be taken successively on the parallel 1, 10, so there is no fraction so minute, that a more minute one may not be cut from the line P 1.

CHAP. III.

STATES OF AGGREGATION.

12. *Bodies consist of parts similar to the whole.* — All bodies consist of component parts, similar in their qualities to the whole, and into which, as will presently be shown, they are separable or divisible. Thus, a mass of metal may be reduced to powder, by filing, grinding, and a variety of other expedients; each particle of this powder will have the same qualities as the entire mass of metal of which it constituted a part.

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To this expansion there is no practical limit; such a piston may be indefinitely raised, producing an indefinitely-increased space in the cylinder under it, and the air will still continue to expand, so as to fill this space to whatever extent it be increased.

19. *Other gaseous examples.* — Atmospheric air, as will be seen hereafter, is by no means the only example of bodies in the gaseous form. Innumerable varieties of such bodies are found existing in the material world, and still greater varieties result from the experimental operations of the natural philosopher and the chemist.

20. *The same body may exist in either state.* — There are numerous bodies which may exist in any of these three states of solid, liquid, or gas, according to certain physical conditions, which will be explained hereafter; and analogy renders it probable that all bodies whatever may assume, under different conditions, these several states.

21. *Water an example.* — Water affords a familiar example of this: as ice exists in the solid state, as water in the liquid state, and as steam in the gaseous state.

CHAP. IV.

IMPENETRABILITY.

22. *All matter impenetrable.* — Impenetrability is the quality in virtue of which a body occupies a certain space, to the exclusion of all other bodies. This idea is so inseparable from matter, that some writers affirm that it is nothing but matter itself; that is, that when we say that a body is impenetrable, we merely say that it is a body.

However this be, the existence of this quality of impenetrability is so evident as to admit of no other proof than an appeal to the senses and the understanding. No one can conceive two globes of lead, each a foot in diameter, to occupy precisely the same place at the same time. Such a statement would imply an absurdity, manifest to every understanding.

23 *Gaseous bodies impenetrable.* — Even bodies in the gaseous form, the most attenuated state in which matter can exist, possess this quality of impenetrability as positively as the most hard and dense substances.

24. *Air an example.* — If we invert a common drinking glass, and plunge its mouth in water, the water will be excluded from the glass, in spite of the pressure produced by the weight of the external water; because the air which filled the glass at the moment of immersion is still in it, and its presence is incompatible with that of any other body.

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It is true, however, that in this case the water will rise a little above the mouth of the glass; but this effect arises not from the penetrability of the air, but from another quality, viz. its compressibility, which we shall presently explain.

25. *Examples of apparent penetrability explained.* — The numerous examples of apparent penetration which will present themselves to all observers are only cases of displacement.

26. *Walking through the air.* — If we walk through the atmosphere, our bodies may be said in one sense to penetrate it; but they do so only in the same manner as they would penetrate a liquid in passing through it. They displace, as they move, as much air as is equal to their own bulk.

27. *Solid plunged in liquid.* — If a cambric needle be plunged in a glass of water, it might appear to the common mind that penetration took place; but it is evident that the needle displaces a quantity of water precisely equal to its own bulk, and if we had the means of measuring with the necessary precision the position of the surface of the water in the glass, we should find that on plunging the needle in the liquid the surface rises through a space corresponding precisely to a quantity of water equal in volume to the bulk of the needle.

CHAP. V.

DIVISIBILITY.

28. *Divisibility unlimited.* — As all bodies consist of parts which are similar in their qualities to the whole, a question arises whether there is any limit to their subdivision or comminution. Are there, in short, any ultimate particles at which the process of division must cease? or, to speak more correctly, are there any ultimate particles so constituted that any further division would resolve them into parts differing in quality from the entire mass?

29. *Water. Its ultimate atoms compound.* — To make our meaning understood, let us take the example of the most common of all substances, water. The discoveries of modern chemistry have disclosed the fact that water is a compound body, formed by the combination of two gases, called *oxygen* and *hydrogen*. These gases, in their sensible properties and appearance, are similar to atmospheric air, and have never separately assumed the liquid form; but, by certain means which will be explained in a future part of this work, they may be made to combine and coalesce, and when so combined they form water.

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Hence it is certain that the liquid, water, consists of ultimate particles, or molecules, as they are sometimes called; which molecules are composed of particles of the two gases above mentioned combined together.

30. *Water nevertheless divisible without practical limit.* — Now, the question is, can water be practically so minutely divided that its particles admit of no further subdivision, save and except that which would resolve them into their constituent gases, oxygen and hydrogen?

To this it is answered, that, although the process of subdivision may be carried to an extent which has no practical limit, yet that by no process of art or science have we ever even approached to those ultimate constituent atoms which admit of no other division save decomposition.

31. *Other bodies likewise divisible without practical limit.* — Nor is this unlimited divisibility and comminution peculiar to water. It is common to all substances, whether solid, liquid, or gaseous. They may all be reduced to particles of the most unlimited minuteness, and yet each of these minute particles will possess the same qualities as the largest mass of the same substance.

32. *Examples of divisibility.* — As this quality of unlimited divisibility involves conditions of the most profound interest, as well in the sciences as in the arts, we shall offer here several examples in illustration of it.

33. Example I. — *Pulverized marble.* — The most solid bodies are capable of unlimited comminution, by a variety of mechanical processes, such as cutting, filing, pounding, grinding, &c. If a mass of marble be reduced to a fine powder by the process of grinding, and this powder be then purified by careful washing, its particles, if examined by a powerful microscope, will be found to consist of blocks having rhomboidal forms, and angles as perfect and as accurate as the finest specimens of calcareous spars. These rhomboids, minute as they are, may be again broken and pulverized, and the particles into which they are divided will still be rhomboids of the same form and possessing the same character. The particles of such powder being submitted to the most powerful microscopic instruments, and the process of pulverization being pushed to the utmost practical extreme, it is still found that the same forms are reproduced.

34. Example II. — *Polished surfaces covered with asperities. Diamond.* — The polish of which the surfaces of certain bodies, such as steel, the diamond and other precious stones, are susceptible, is an evidence at once of the limited sensibility of our organs, and the unlimited divisibility of matter. This polish is produced, as is well known, by the friction of emery powder or diamond dust, and consequently each individual grain of such powder or dust must leave a little trench or trace upon the surface submitted to such friction. It is evident, therefore, that after this process has been completed, the

surface which presents to the senses such brilliant polish, and apparently infinite smoothness, is in reality covered with protuberances and indentations, the height and depth of which cannot be less than the diameter of the particles of powder by which the polish has been produced.

35. Example III. — *Gold visible on touchstone.* — In the detection of matter in a state of extreme comminution, the sense of sight is infinitely more delicate than that of touch. If we rub a piece of gold upon a touchstone, we plainly see the particles of matter which are left upon the surface of the stone. The touch, however, cannot detect them.

36. Example IV. — *Minuteness of tubular filaments of glass.* — In the preceding examples the comminution, however great, cannot be easily submitted to actual measurement. Certain processes, however, in the arts enable us to obtain exact numerical estimates of a minute divisibility, which without them might appear incredible. If a thin tube of glass, being held before the flame of a blow-pipe until the glass be softened and acquire a white heat, be drawn end from end, a thread of glass may be obtained so fine that its diameter will not exceed the two-thousandth part of an inch. This filament of glass will have all the fineness and almost all the flexibility of silk, and yet a bore proportional to that which passed through the original tube will still pass through its centre. The presence of this bore may be rendered manifest by passing a fluid through it.

37. *Such a filament might penetrate the flesh without producing pain.* — It has been ingeniously conjectured, that if a filament of this degree of fineness could be obtained of a material which would retain sufficient inflexibility, it might be made to penetrate the flesh without producing either pain or injury, inasmuch as its magnitude would be so much less than the pores of the integuments.

38. Example V. — *Wollaston's micrometric wire.* — In the application of the telescope to astronomical purposes, the distance between objects which are present at one and the same time within the field of view of the instrument, is measured by fine threads which are extended parallel to each other across the field of view, and which may be moved towards and from each other until they are made to pass through the objects between which we desire to measure the distance. An experiment, then, which determines the distance between these threads measures the distance between the objects.

But these threads, being placed before the eye-glass of the telescope, and therefore necessarily magnified in the same manner as the objects themselves, would, unless such filaments were of an extreme degree of tenuity, appear in the field of view like great broad bands, and would conceal many of the objects which it might be necessary to observe. It was therefore necessary to resort to the use of filaments of extraordinary minuteness for this purpose. The threads of

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the web of the spider were used with more or less success; but the late Dr. Wollaston invented a beautiful expedient by which metallic threads of any degree of fineness might be obtained.

Let us suppose a piece of platinum wire, the one-hundredth of an inch in diameter, a fineness easily obtainable by the process of wire-drawing, to be extended along the axis of a cylindrical mould, the one-fifth of an inch in diameter, the wire being thus the twentieth part of the diameter of the mould. Let the mould be then filled with silver in a state of fusion. When this is cold we shall have a cylinder of silver, having in its axis a thread of platinum the twentieth part of its diameter.

This compound cylinder is then submitted to the common process of wire-drawing, during which the platinum in its centre is drawn with the silver, the proportion of their diameters being still maintained. When the wire is drawn to the greatest degree of fineness practicable, a piece of it is plunged in nitric acid, by which the surrounding silver is dissolved, and the platinum wire remains uncovered.

39. *Illustrations of its extreme minuteness.* — By this process Dr. Wollaston obtained platinum wire so fine, that thirty thousand pieces, placed side by side in contact, would not cover more than an inch.

It would take one hundred and fifty pieces of this wire bound together to form a thread as thick as a filament of raw silk.

Although platinum is among the heaviest of the known bodies, a mile of this wire would not weigh more than a grain.*

Seven ounces of this wire would extend from London to New York.

40. Example VI. — *Minuteness of organized filaments.* — *Wool.* — *Silk.* — *Fur.* — The natural filaments of wool, silk, and fur afford striking examples of the minute divisibility of organized matter. The following numbers show how many filaments of each of the annexed substances placed in contact, side by side, would be necessary to cover an inch: —

Coarse wool - - - - -	500
Fine Merino wool - - - - -	1250
Silk - - - - - - - -	2500

The hairs of the finest furs, such as beaver and ermine, hold a place between the filaments of Merino and silk, and the wools in general have a fineness between that of Merino and coarse wool.

All these objects are sensible to the touch.

It will be remembered that they are compound textures, having a particular structure, each containing very different elements, which are prepared by the processes of nutrition and secretion.

41. Example VII. — *Thinness of a soap-bubble.* — The optical investigations of Newton disclosed some astonishing examples of the minute divisibility of matter.

A soap-bubble as it floats in the light of the sun reflects to the eye an endless variety of the most gorgeous tints of colour. Newton showed, that to each of these tints corresponds a certain thickness of the substance forming the bubble; in fact, he showed in general, that all transparent substances, when reduced to a certain degree of tenuity, would reflect these colours.

Near the highest point of the bubble, just before it bursts, is always observed a spot which reflects no colour and appears black. Newton showed that the thickness of the bubble at this black point was the 2,500,000th part of an inch! Now, as the bubble at this point possesses the properties of water as essentially as does the Atlantic Ocean, it follows, that the ultimate molecules forming water must have less dimensions than this thickness.

42. Example VIII. — *Thinness of insects' wings.* — The same optical experiments were extended to the organic world, and it was shown, that the wings of insects which reflect beautiful tints resembling mother-of-pearl owe that quality to their extreme tenuity.

Some of these are so thin that 50,000 placed one upon the other would not form a heap of more than a quarter of an inch in height.

43. Example IX. — *Thinness of leaf-gold.* — In the process of gold-beating the metal is reduced to laminæ or leaves of a degree of tenuity which would appear fabulous, if we had not the stubborn evidence of the common experience in the arts as its verification.

A pile of leaf-gold the height of an inch would contain 282,000 distinct leaves of metal! The thickness, therefore, of each leaf is in this case the 282,000th part of an inch. Nevertheless, such a leaf completely conceals the object which it is used to gild; it moreover protects such object from the action of external agents as effectually as though it were plated with gold an inch thick.

44. Example X. — *Wire used in embroidery.* — In the manufacture of embroidery, fine threads of silver gilt are used. To produce these, a bar of silver weighing 180 oz., is gilt with an ounce of gold; this bar is then wire-drawn until it is reduced to a thread so fine that 3,400 feet of it weigh less than an ounce. It is then flattened by being submitted to a severe pressure between rollers, in which process its length is increased to 4,000 feet.

Each foot of the flattened wire weighs, therefore, the 4000th part of an ounce. But as in the process of wire-drawing and rolling the proportion of the two metals is maintained, the gold which covers the surface of the fine thread thus produced consists only of the 180th part of its whole weight. Therefore the gold which covers one foot is only the 720,000th part of an ounce, and consequently the gold which covers an inch will be the 8,640,000th part of an ounce. If this inch be again divided into one hundred equal parts, each part will be distinctly visible without the aid of a microscope, and yet the

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gold which covers such visible part will be only the 864,000,000th part of an ounce.

But we need not stop even here. This portion of the wire may be viewed through a microscope which magnifies 500 times; and by these means, therefore, its 500th part will become visible.

45. In this manner, therefore, an ounce of gold may be divided into 432,000,000,000 parts, and each part will still possess all the characters and qualities found in the largest mass of the metal. It will have the same solidity, texture, and colour, will resist the same chemical agents, and will enter into combination with the same substances. If this gilt wire be exposed to the action of nitric acid, the silver within the coating will be dissolved, but the hollow tube of gold which surrounds it would still cohere and remain suspended.

46. Example XI. — *Composition of blood.* — The organic world affords most interesting and striking examples of the minute divisibility of matter.

None can be selected more remarkable than that presented by the blood of animals, which is not, as it appears to the naked eye, a uniform red liquid, but consists of a transparent colorless fluid called *liquor sanguinis*, or *blood-liquid*, in which innumerable small red particles of solid matter float.

47. *The magnitude and form of its corpuscles.* — In different species these red corpuscles differ both in form and size. They were long considered to be spheroidal, and are even still so stated to be in most works on physics. The observations however of Hewson, Wagner, Gulliver, and others have proved that they are flat or disk-shaped. In the human blood, and in that generally of animals who suckle their young, they are circular, or nearly so, their surfaces being slightly concave, like the spectacle glasses used by short-sighted persons. In birds, reptiles, and fishes, they are generally oval, the oval being more or less elongated in different species. The surfaces of the disks in these species, instead of being concave, are convex, like the spectacle glasses used by weak-sighted persons. The thickness of these disks varies from one third to one quarter of their diameter. Their diameter in human blood is the 3500th part of an inch; they are smallest in the blood of the *Napu* musk deer, where they measure only the 12,000th of an inch.

It would require 50,000 of these disks as they exist in the human lood to cover the head of a small pin, and 800,000 of the disks of the blood of the musk-deer to cover the same surface.

48. *Number of corpuscles in a drop of blood.* — It follows from these dimensions that in a drop of human blood which would remain suspended from the point of a fine needle there must be about 3,000,000 of disks, and in a like drop of the blood of the musk-deer there would be about 120,000,000; yet these corpuscles are rendered not only distinctly visible to the senses by the aid of the microscope,

but their forms and dimensions are rendered apparent. Small as they are, they are divisible, and can be resolved into their elements by chemical agents.*

It is difficult to say which excites more astonishment and admiration, the minuteness of these structures, or the intellectual powers which have submitted them to exact observation and measurement.

49. EXAMPLE XII. — *Minuteness of animalcules; their organization and functions.* — But these globules, small as they are, are exceeded in minuteness by innumerable creatures whose existence the microscope has disclosed, and whose entire bodies are inferior in magnitude to the globules of blood.

Microscopic research has disclosed the existence of animals a million of which do not exceed the bulk of a grain of sand, and yet each of these is composed of members as admirably suited to their mode of life as those of the largest species. Their motions display all the phenomena of vitality, sense, and instinct. In the liquids which they inhabit they are observed to move with the most surprising speed and agility; nor are their motions and actions blind and fortuitous, but evidently governed by choice and directed to an end. They use food and drink, by which they are nourished, and must, therefore, be supplied with a digestive apparatus. They exhibit a muscular power far exceeding in strength and flexibility, relatively speaking, the larger species. They are susceptible of the same appetites, and obnoxious to the same passions, as the superior animals, and, though differing in degree, the satisfaction of these desires is attended with the same results as in our own species.

Spallanzani observes that certain animalcula devour others so voraciously that they fatten and become indolent and sluggish by over-feeding. After a meal of this kind, if they be confined in distilled water so as to be deprived of all food, their condition becomes reduced, they regain their spirit and activity, and once more amuse themselves in pursuit of the more minute animals which are supplied to them. These they swallow without depriving them of life, as, by the aid of the microscope, the smaller, thus devoured, has been observed moving within the body of the greater.

The microscopic researches of Ehrenberg have disclosed most surprising examples of the minuteness of which organized matter is susceptible. He has shown that many species of infusoria exist which are so small that millions of them collected into one mass would not exceed the bulk of a grain of sand, and a thousand might swim side by side through the eye of a needle.

The shells of these creatures are found to exist fossilized in the strata of the earth in quantities so great as almost to exceed the limits of credibility.

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* See Quain's Anatomy, 5th ed. by Dr. Sharpey and Mr. Quain.

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By microscopic measurement it has been ascertained that in the slate found at Bilin, in Bohemia, which consists almost entirely of these shells, a cubic inch contains forty-one thousand millions; and as a cubic inch weighs two hundred and twenty grains, it follows that one hundred and eighty-seven millions of these shells must go to a grain, each of which would consequently weigh the 187,000,000th part of a grain.

All these phenomena lead to the conclusion that these creatures must be supplied with an organization corresponding in beauty with those of the larger species.

50. Example XIII. — *Unlimited divisibility by solution of solids in liquids.* — If a grain of salt be dissolved in 1000 grains of distilled water, each grain of the water will contain the 1000th part of the grain of the salt; and if a grain of this water be mixed with 1000 grains of distilled water, the 1000th part of a grain of salt which it holds in solution will be uniformly diffused through the latter, so that each grain of the latter solution will contain the millionth part of a grain of salt. The presence of the salt in this second solution can be detected by certain chemical tests.

It is evident that this process may be continued to a still greater extent.

51. Example XIV. — *Minute divisibility proved by colour.* — A grain of sulphate of copper, dissolved in a gallon of water, will impart to the whole mass of the liquid a plainly perceptible tinge of blue; and a grain of carmine will give its peculiar red to the same quantity of water. It follows, therefore, that a minute drop of such water will contain such a proportion of either of these substances as the drop bears to the gallon.

52. Example XV. — *Divisibility of musk.* — The sense of smelling, although it does not inform us of the mechanical qualities of minute masses of matter, determines, nevertheless, their presence: thus, it is known that a grain of musk will impregnate the atmosphere of a room with its odour for a quarter of a century, or more, without suffering any considerable loss in its weight.

Every particle of the atmosphere which produces the sense of the odour must contain a certain quantity of the musk.

53. Example XVI. — *Fineness of spider's-web.* — Fine as is the filament produced by the silkworm, that produced by the spider is still more attenuated.

A thread of a spider's web, measuring four miles, will weigh very little more than a single grain!

Every one is familiar with the fact, that the spider spins a thread, or cord, by which his own weight hangs suspended. It has been ascertained that this thread is composed of about six thousand filaments.

54. Example XVII. — *Divisibility shown by the taste.* — The

sense of taste, like that of smelling, may determine the presence of matter, without manifesting, by direct evidence, anything concerning its mechanical qualities.

55. *Effect of strychnine dissolved in water.* — A portion of strychnine so minute as to be scarcely perceptible to the sight, if dissolved in a pint of water, will render every drop of the water bitter. Now, it is evident that in this case, the strychnine being uniformly diffused through the water, the minute portion of it above mentioned is subdivided into as many parts as there are drops of water in a pint.

56. *Effect of salt of silver dissolved in water.* — In like manner, a single grain of the salt of silver, called ammoniacal hyposulphite, will impart a flavour of sweetness to a gallon of water.

Now, a gallon of water will weigh about 46,000 grains; and as the flavour of the salt is perceptible in each grain of the water, it follows that one grain of this salt is thus divided into 46,000 equal parts.

57. *Effect of sugar dissolved.* — A small lump of sugar, dissolved in a cup of tea measuring half a pint, will sweeten the whole perceptibly. In this half-pint of tea there are 31,000 drops. Each drop, therefore, must contain the 31,000th part of the sugar dissolved, and each such drop is perceptibly sweet. But if the point of a needle be inserted in one of these drops, and withdrawn from it, a film of moisture will remain upon it, and the drop will not be visibly diminished. Yet this film of moisture will still be sweet, and will, therefore, contain a fraction of the 31,000th part of the lump of sugar, too minute to admit of numerical estimation.

58. *Is matter, therefore, infinitely divisible?* — It may be asked, whether we are then to conclude, from these various facts, that matter is infinitely divisible, and that there are no original constituent atoms of determinate magnitude and figure, at which all subdivision must cease. Such an inference, however, would be unwarranted, even if we had no other means of deciding the question except those of direct observation, as we should thus impose those limits on the operations of nature which she has imposed upon our powers of observing them.

59. *The existence of ultimate molecules, of determinate figure, may be inferred.* — Although we are unable, by direct observation, to perceive the existence of molecules, or material atoms of determinate figure, yet there are many observable phenomena which render their existence in the highest degree probable, if not positively certain.

60. *Crystallization indicates their existence.* — The most remarkable of such phenomena are observed in the crystallization of salts.

When salt is dissolved in distilled water, as in the preceding example, the mixture presents the appearance of a transparent liquid like water itself, the salt altogether disappearing from sight and touch. The presence of the salt in the water, however, can be established by

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weighing the solution, which will be found to exceed the original weight of the water by the exact amount of the weight of the salt dissolved.

Now, if this solution be heated to a sufficient temperature, the water will gradually evaporate; but this process of evaporation not affecting the salt, the remaining water will still contain the same quantity of salt in solution, and it will consequently become by degrees, a stronger and stronger saline solution, the water bearing, consequently, a less and less proportion to the salt. The water will at length be diminished, by evaporation, to that point, that a sufficient quantity does not remain to hold in solution the entire quantity of salt contained in it. When this has taken place, each particle of water which is evaporated leaving behind it the salt which it held in solution, and this salt not being capable of being dissolved by the water which remains, it will float in such waters in its solid and natural state, undissolved, just as particles of dust, or other matter not soluble in the water, would do. But the saline particles which thus remain floating in the liquid undissolved, will not collect in irregular solid pieces, but will exhibit themselves in regular figures, terminated by plane surfaces, always forming regular angles, these figures being invariably the same for the same species of salt, but different for different species. There are several circumstances attending the formation of these crystals which merit attention.

61. *Process of crystallization.* — If one of these be detached from the others, and the gradual progress of its formation be submitted to observation, it will be found to grow large, always preserving its original figure. Now, since its increase must be produced by the continual accession of saline molecules, disengaged by the water evaporated, it follows that these molecules, or atoms, must have such a shape, that, by attaching themselves successively to the crystal, they will maintain the regularity of its bounding planes, and preserve the angles which these planes form with each other unvaried.

In fact, they must be so shaped, that the structure of the crystal they form may be built up by their regular aggregation into the form which it assumes.

If one of these crystals be taken from the liquid during the process of its formation, and be broken, so as to destroy the regularity of its form, and then restored to the liquid, it will be observed soon to recover its regular form, the atoms of salt, successively dismissed by the evaporating water, filling up the irregular cavities produced by the fracture.

62. *Existence of ultimate molecules inferred from this process.*— Two consequences obviously follow from this phenomenon.

First. That the atoms of the salt dismissed by the water evaporated have such a form, as enables them, by combination, to give to the crystals the shape which they exhibit; and, *Secondly.* That the atoms which are successively attached to the crystals in the process of formation, attach themselves in a particular position, to explain which it is necessary to suppose that corresponding sides of the crystals have attractions for each other, so that the atoms of salt not only attach themselves to the sides of the crystals, but place themselves there in a particular position. In a word, we must suppose that the walls of the crystal are built with these atoms in the same manner, and with the same regularity, as the walls of a building are formed with bricks.

All these, and many similar details of the process of crystallization, are, therefore, very evident indications of a determinate figure in the ultimate atoms of the substances which are crystallized.

63. *Some bodies exist naturally in the crystallized state.* — But besides these substances thus reduced by art to the form of crystals, there are large classes of bodies which naturally exist in this state.

64. *Planes of cleavage.* — There are certain planes called planes of *cleavage*, in the direction of which natural crystals are easily divided. In substances of the same kind, these planes have always the same relative position; but they differ in different substances.

The surfaces of the planes of cleavage are not always observable before the crystals are divided; but when the crystals are divided, these surfaces exhibit an intense polish which no effort of art can equal.

65. *Planes of cleavage indicate the forms of the ultimate molecules.* — We must conclude, therefore, that these planes of cleavage are parallel to the sides of the constituent atoms of the crystals, and their directions therefore form so many conditions for the determination of the shape of these atoms.

This shape being once determined, it is not difficult to assign all the various ways in which they may be arranged, so as to produce regular figures; and we accordingly find that regular figures thus indicated by mathematical reasoning correspond with the forms assumed by the crystals of the same substances.

66. *Inference that all bodies consist of ultimate atoms of determinate figure.* — It follows, therefore, from these effects, and the reasoning established upon them, that the substances which are susceptible of crystallization consist of ultimate atoms of different figure. Now, all solid bodies whatever are included in this class, for they have severally been found in, or are reducible to a crystallized form. Liquids crystallize in freezing: several of the gases have been already reduced to the liquid and solid forms, and analogy justifies the conclusion that all are capable of being reduced to this form.

Hence it appears reasonable to presume that all bodies whatever are composed of ultimate atoms, having determinate shape and magnitude; that the different qualities with which we find different bodies endued, depend upon the shape and magnitude of these atoms; that these atoms cannot be disturbed or changed so long as the body

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ı they belong is not decomposed into other elements, as we : qualities which depend on them unchangeably the same ll the influences to which they have been submitted.

These molecules too minute to be the subjects of direct obser- — We must conclude also that these atoms are so minute in gnitudes that they cannot be observed by any means which rt has yet contrived, but nevertheless that such magnitudes : limits.

'rinciples of mechanical science, however, independent of othesis. — It is necessary, however, to observe that notwith- the strong analogies which support these conclusions as to mate constitution of material substances, the principles of cal science are quite independent of them, and do not rest y hypothesis concerning such atomic constitution, and there- truth of these principles would not be in any wise disturbed ough it should be established that matter is in the most lite- : infinitely divisible, and is not formed of ultimate atoms. oasis of mechanical science is *observed facts*; and since the ıg upon these observed facts is demonstrative, the conclusions, ghtly deduced, have the same degree of certainty as the facts ıich they are inferred.

The ultimate molecules of matter indestructible. — The ex- livision to which bodies are subjected in many natural and processes, and especially when exposed to the application of fire, has naturally suggested to minds not habituated to the 'ocess of scientific reasoning, the idea that bodies are destruc- The ancients, instead of the modern practice of inhumation, l of the bodies of their dead by burning them, upon the sup- that their component parts were by such operation destroyed. more exact reasoning of modern philosophy, however, teaches a power to destroy matter would be as inconceivable in a finite ı a power to create it.

certain that the quantity of matter which exists upon and in :h has never been diminished by the annihilation of a single

er is in fact indestructible by any agency short of divine It may be asked, then, what becomes of the matter com- ı body which, being subjected to the action of fire, gradually ıpletely disappears. The answer is, that in this, as well as in r cases of the apparent destruction of matter, nothing takes cept its subdivision and the change of its form and position.

No matter destroyed in combustion. — When a body is sub- to the action of heat, its elements are decomposed, and its ent particles separated, many of them combine with other s of matter, and form new substances possessing other quali- 'hus, when coal or other fuel is burned, the carbon enters into combination with one of the constituents of the atmosphere called oxygen, and forms a gaseous substance called carbonic acid, which rises into and mixes with the atmosphere. Another element, hydrogen, combines with the same constituent of the atmosphere and forms vapor, which also disperses in the atmosphere.

Sulphur, which is also occasionally present in fuel, combines with the same constituent of the air, forming a gas called sulphurous acid, which also escapes into the atmosphere. Thus the entire matter of the fuel, with the exception of a small portion of incombustible matter which falls into the ash-pit, is dispersed in the air, and no destruction or annihilation takes place.

That no portion of the matter of the fuel is destroyed or annihilated can be established by the incontrovertible experimental proofs of the chemist, for by the expedients of his science all the products of the combustion which have been just mentioned can be preserved, weighed, and decomposed. The oxygen which has entered into combination with each element of the fuel can be reproduced, as well as the constituents of the fuel itself, the latter of which being weighed, as well as the incombustible ash, the weight of the whole is found to be precisely equal to the weight of the fuel which was burned and apparently destroyed.

71. *No matter destroyed in evaporation.*—Liquids when subjected to heat are converted into vapor, and this vapor disperses in the atmosphere, so that the liquid seems to be boiled away; but if the vapor be preserved, as it may be in a separate vessel, and exposed to cold, it will return to the liquid form, and its weight and measure will be found to be precisely the same as that of the liquid evaporated.

72. *Destructive distillation.*—There is a process in chemistry which is called destructive distillation. The term is objectionable, because it implies a destruction where no destruction takes place. If a piece of wood, being previously weighed, be placed in a close retort and submitted to what is called destructive distillation, it will be found that water, a certain acid, and several gases will issue from it, all of which may be preserved, and mere charcoal will remain in the retort at the end of the process. If the water, acid, and gases which thus escape be weighed with the charcoal, the weight of the whole will be found to be precisely equal to that of the wood which was subjected to destructive distillation.

73. *General conclusion.*—Thus various forms of matter may be fused, evaporated, or submitted to combustion; animals and vegetables may die, organized bodies may be dissolved and decomposed, but in all cases their elementary and constituent parts maintain their existence. The remains of our own bodies after death are deposited in the grave, and enter into innumerable combinations with the materials of the soil, with the vegetation which covers it, and the air which circulates above it.

Consequently, these parts enter into an infinite series of other combinations, forming parts of other organized bodies, animal and vegetable, and which, after having discharged their functions, are thrown off again, mixing with the soil, the air, or organized matter, and once more running through the round of physical combinations.

The constituent atoms of matter are thus constantly performing a circle of duties in the economy of nature with infinitely more certainty and regularity than is observed in the most disciplined army or in the best regulated manufactory.

CHAP. VI.

POROSITY AND DENSITY.

74. *The component molecules of a body are not in contact.*—The volume or magnitude of a body is, as has been already explained, the space which is included within its external surfaces. The mass of a body, or the quantity of matter of which it consists, is the collection of atoms or molecules which compose it. If these atoms were in actual contact, the volume would be completely occupied by the mass; but numerous results of observation prove that this is never the case. There is no body so solid or compact that it cannot, by various processes to be explained hereafter, be forced into less dimensions. Now, if its atoms were in absolute contact, and had no unoccupied spaces between them, this compression could not take place. It follows, therefore, that between the atoms or molecules which form the mass of a body there are vacant spaces in the magnitude or volume, which therefore consists partly of the spaces occupied by the atoms, and partly of the spaces which intervene between these atoms.

75. *The spaces which separate them are called pores.*—These intervening spaces which separate the constituent atoms of a mass of matter are called pores; and the quality of bodies in virtue of which their constituent atoms are thus separated by vacant spaces is called *porosity*.

76. *Proportion of mass to pores determines the density.*—In bodies of different species, the pores bear a greater or less proportion to the whole volume; or, in other words, the component atoms of the mass are placed more or less closely together.

This circumstance determines what is called the *density* of bodies. One body is more or less dense than another, according as its constituent atoms are more or less closely packed together, or according as its pores are of less or greater magnitude.

77. *Porosity and density correlative terms.*—*Porosity* and *density*

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are therefore correlative terms. The greater the porosity, the less the density; and the greater the density, the less the porosity.

Gold and platinum are bodies of great density; cork is one of much less density; and air or the gases still less.

When one body is heavier than another under the same bulk, it is concluded that its density is proportionally greater than that of the other, and consequently its porosity proportionally less.

78. *Uniform diffusion of pores constitutes uniform density.*—When the pores are uniformly diffused throughout the dimensions of a body, the body is said to be uniformly dense.

79. *Pores different from cells.*—The porosity of bodies is sometimes illustrated and explained by a sponge, which allows the cavities which pervade it to be filled with water or other fluid; but such an illustration is not strictly apposite. The cavities of a sponge are not its pores, any more than are the cells of a honeycomb the pores of wax. Cellular structures in general present peculiar modifications of matter totally different from what is understood by porosity.

80. *Pores sometimes occupied by more subtle matter.* — The pores of a body are sometimes filled with another material substance of a more subtle nature. Thus, if the pores of a body be greater than the atoms of air, such body being surrounded by the atmosphere, the air will pervade its pores. This is found to be the case, for example, with certain sorts of wood, with chalk, sugar, and many other substances. If a piece of such wood, chalk, or sugar be pressed to the bottom of a vessel filled with water, the air which fills the pores will be observed to escape in bubbles, and to rise to the surface, the water pervading the pores, and taking their place.

81. *Examples of porosity and density.*—The following examples will illustrate the qualities of porosity and density:—

82. EXAMPLE I.—*Porosity of wood.*—If a cylinder of wood be inserted in the bottom of a cup which is attached to the mouth of a glass-receiver placed upon the plate of an air-pump, the cup thus placed being filled with mercury, on withdrawing the air from the interior of the receiver the pressure of the external atmosphere will force the mercury through the pores of the wood, and it will be observed to fall in a shower of silver within the receiver.

83. EXAMPLE II. — *Buoyancy of wood sometimes caused by the air contained in its pores.* — Wood in general is lighter, bulk for bulk, than water, and will therefore float in it; but this comparative lightness is in some cases not a property of the wood, but of the air which fills its pores. To prove this, let a piece of such wood be held beneath the surface of water contained in a vessel placed under the receiver of an air-pump. On exhausting the receiver, the air contained in the piece of wood will force its way out by reason of its elasticity, and will rise in bubbles to the surface of the water. If, when the air has been thus expelled from the pores, the pressure of the

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atmosphere be made to act again upon the surface of the water by opening the cock which admits air to the receiver, the water will be forced into the pores of the wood and will fill the spaces deserted by the air, and the wood will then sink to the bottom.

84. **Example** III.—*The densest substances (gold for example) have pores. Florentine experiment.*—There is no substance so dense as to be divested of pores. The celebrated Florentine experiment, performed at the Academia Del Cimento, in 1661, and often repeated since that time with the same result, showed that gold itself has pores sufficiently large to admit the particles of water to pass through them. A globe of gold, being completely filled with water, was closed by a screw, and submitted to a severe pressure. As a globe is the figure which within the same surface contains the greatest possible volume, any change produced in its figure by external pressure must necessarily diminish its volume. When the globe, therefore, thus filled with water, is submitted to a pressure which changes it to a form slightly elliptical, or turnip-shaped, it would necessarily contain less liquid, and either of two effects must ensue, viz., the globe must burst, or a portion of the liquid must force its passage through the pores of the gold. The latter effect ensued; and as the globe changed its form, the water was seen collecting in a dew on the external surface of the metal. This proved that the particles of water found their way through the pores of the gold without tearing, rupturing, or otherwise doing violence to its general structure.

85. **Example** IV.—*Filtration.*—The process of filtration, so extensively used in the arts and sciences, depends on the quality of porosity. The substance through which a liquid is filtrated has pores large enough to allow the particles of the liquid to pass, but too small to permit the passage of the foreign matter suspended in the liquid, and of which it is intended to purify the liquid by the process of filtration. The most ordinary filters are soft stone, paper, and charcoal.

86. **Example** V.—*Petrifaction.*—Animal and vegetable petrifactions furnish striking examples of porosity, since the stony substance which petrifies them must have been infiltrated through their mass, so as to penetrate all their fibres.

87. **Example** VI.—*Porosity of mineral substances.*—Mineral substances are all more or less porous. Opaque stones are in general more porous than transparent ones. Chalk and marble are formed of the same constituents, with different degrees of porosity. If water be poured on chalk, it is instantly observed passing into its innermost pores; if it be poured on marble, it rests on the surface without penetrating. Stones, however, which resist the admission of water to their pores under ordinary circumstances, will be penetrated by the liquid, provided an intense pressure be used for a sufficient length of time. Thus, stones taken from the bottom of the sea, especially if the depth be considerable, are found penetrated by water to their very centre.

Among siliceous stones, such as agate and flint, there is one called Hydrophane, whose porosity is attended by a singular phenomenon. A piece of this substance, in its common state, is nearly opaque; but if it be plunged in water, it is found, on withdrawing it from the liquid, to be nearly as transparent as glass. In this case the water penetrates the stone exactly as oil penetrates paper; bubbles of air are disengaged from its pores, which are filled with the water absorbed, the presence of which gives the transparency.

88. Example VII.—*Porosity of mineral strata.*—Large mineral masses existing naturally in the strata of the earth present examples of porosity still more striking. Water percolates through the sides and surfaces of caverns and grottos, and, being impregnated with calcareous and other earths, forms stalactites, or pendulous protuberances, presenting curious appearances, with which every one is familiar.

CHAP. VII.

COMPRESSIBILITY AND CONTRACTIBILITY.

89. *Compression diminishes the bulk and augments the density.* — The quality in virtue of which a body allows its volume to be diminished without diminishing its quantity of matter is called compressibility, when the effect is produced by the application of external mechanical force; and contractibility when produced by change of temperature, or any other agency not mechanical.

When the volume of a body is diminished, whether by compression or contraction, its constituent atoms are brought into closer contiguity, its pores are consequently diminished, and its density proportionally increased.

90. *All bodies compressible.*—All known bodies, whatever be their nature, are capable of having their dimensions reduced without diminishing their mass, or quantity of matter; and this is one of the most conclusive proofs that all bodies are porous, or that their constituent atoms are not in contact; for the spaces by which the volume is diminished must, before such diminution, consist of pores.

91. *Compressibility increases with porosity.*—It is evident in general that the more porous a body is the greater is its compressibility. This truth is manifested by innumerable examples derived from organized bodies, especially those of a fibrous texture. All those whose porosity is such as to allow them to be easily penetrated by fluids can be diminished by the application of pressure; and in this case, if they have been previously filled with fluids, these fluids are expelled by the pressure exactly as water is squeezed from a piece of sponge. Innumerable processes in the arts supply examples of this.

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92. *Compression of wood.* — Wood of even the hardest kind, in its natural state, is so porous as to absorb both air and water in considerable quantities. When such wood is used in the arts, in cases where extreme hardness is required, it is previously submitted to severe pressure, by which the fluids absorbed are expelled from the pores, the volume diminished, and the density increased. The wooden wedges used in fastening the rails of the railway in their chairs are prepared in this manner.

93. *Compression of stone.*—Even the most solid stone, when loaded with a considerable weight, is found to be compressed. The foundations of buildings, and the columns which sustain incumbent weights in architecture, supply numerous proofs of this.

94. *Compression of metals.*—Malleable metals are compressed by percussion or hammering; they become thus more compact and dense. In the process of coining, medals and pieces of money are struck by a severe pressure, by which they are made to receive the impression and characters upon them more accurately than softened wax would from the pressure of the hand. Under the blow of the press they not only change their form, accommodating themselves to the characters and figures sunk upon the die, but they are at the same time compressed and rendered more dense, so that the coin or medal has a volume sensibly less than the blank piece had before it was struck.

95. *Compression of liquids.* — Liquids in general are less easily compressed than solids; so much so, that in practical science they are regarded as incompressible.

They are, however, strictly speaking, capable of a slight compression under the operation of considerable mechanical force.

It might be supposed that the Florentine experiment already alluded to, in which water enclosed in a globe of gold, and submitted to mechanical pressure, exuded through the pores of the metal, established the incompressibility of that liquid; and, in fact, the experiment was made with a view of testing that quality in water, but the experiment as executed did not, and could not, establish this conclusion.

It is quite true that if the water had not exuded upon the change of figure of the globe, the compressibility of the liquid would have been established. The mere escape of the water did not, however, prove its incompressibility. To accomplish this it would have been necessary,—*first*, to measure accurately the volume of water which transuded by compression; and, *secondly*, to measure the diminution of volume which the vessel suffered by its change of figure. If this diminution were greater than the volume of water which escaped, it would follow that the water remaining in the globe had been compressed, notwithstanding the escape of the remainder; but this could never have been accomplished *with the necessary* precision in such an *experiment, and, consequently,* so far as the question of compressibility was concerned, nothing was proved by the Florentine experiment.

96. *Compression of water proved.*—A century later, however, in the year 1761, the compressibility of water and other liquids was established. It was found that water, submitted to a mechanical pressure, amounting to fifteen pounds on a square inch, would be diminished in its volume by forty-five parts in a million; that is to say, a million of cubic inches would be reduced to about forty-five cubic inches less.

In more recent experiments, a quantity of water was enclosed in a piece of cannon and submitted to a mechanical pressure amounting to fifteen thousand pounds per square inch. Under this pressure it was diminished by one-twentieth of its volume, and the cannon enclosing it was burst.

97. *Compression of gases.* — Of all forms of matter the gases are the most susceptible of compression. This quality has already been briefly noticed. There appears to be no practical limit to the compression of which this form of matter is susceptible, its volume being diminished in the exact proportion of the compressing force applied to it.

98. *Contractibility of liquids.* — But if liquids are so little compressible, they are, in a very high degree, susceptible of contraction.

If a quantity of water coloured with ink or other colouring matter be included in a glass bulb connected with a tube of small bore, it will be found, that when the bulb is exposed to cold, the level of the coloured water in the tube will descend. This is an effect of the contraction which the liquid undergoes in consequence of its diminution of temperature. This contraction by cold is a universal quality of matter, which will be explained more fully in a subsequent part of this work.

CHAP. VIII.

ELASTICITY AND DILATABILITY.

99. *Elastic and inelastic bodies.* — Elasticity is the quality in virtue of which a body, after having been compressed, recovers its former dimensions, on being relieved from the force which compresses it.

Bodies which retain their compressed state after the force ceases to act, and do not resume their original dimensions, are said to be inelastic.

100. *Elasticity of gases.* — The class of bodies which afford the most striking examples of elasticity are the gases and aeriform bodies.

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If a quantity of air be included in a syringe under a piston, and be compressed by a force applied to the piston, on the removal of that force the air, by virtue of its elasticity, will force the piston upwards until it resumes the position from which it had been driven by the compressing force.

101. *Elasticity of liquids.*— All liquids, when compressed, immediately recover their original dimensions when relieved from the compressing force, and therefore may be said to be perfectly elastic The play of the compressive and elastic principle, however, in the case of liquids, is so extremely limited, that for all practical purposes this form of body is treated as both incompressible and inelastic. All the theorems of those parts of physical science called *Hydrostatics*, *Hydrodynamics*, *Hydraulics*, &c., are based upon the principle that liquids are incompressible and inelastic; for although it be true, as has been stated, that within certain very minute limits they are both compressible and elastic, yet these limits are so small as to produce no appreciable effects under ordinary circumstances.

102. *Expansibility of gases.* — Gaseous bodies are not only compressible and elastic without any practical limit, but also endued with unlimited dilatability. Thus, if a quantity of gas be included in any given volume, and that this volume be augmented in any required proportion, the gas will spontaneously, and without the application of any external agency, dilate itself so as to fill the augmented volume, and this expansion will go on, no matter to what extent the volume be augmented.

103. *Elasticity of solids.* — The quality of elasticity is manifested in solid bodies, but in a less decided manner than in gases. Caoutchouc, or elastic gum, is perhaps of all bodies that which has most elasticity. This quality, combined with the methods recently discovered of varying the form of this substance, has extended considerably the application of it to the useful purposes of life.

104. *Examples of elasticity of solids.* — The following examples will illustrate the quality of elasticity as found in solid bodies.

105. Example I.—*Ivory balls.* — If a flat and hard surface be smeared with a thin coating of oil, and an ivory ball be allowed to drop upon it, the ball will rebound by reason of its elasticity. On examining that part of the surface of the ball which struck the flat surface from which it rebounded, it will be found that a somewhat extensive circular space will have been stained with the oil. If the ball be brought gently into contact with the flat surface, a minute space only would be stained with the oil. Why, then, it may be asked, did a larger space receive a stain when the ball was allowed to drop with a certain force upon the surface? The answer to this is, that the force of the impact *flattens* the surface of the ball to a certain extent; that, in virtue of its elasticity, the ball recovers its spherical figure; and that the force with which it recovers this figure

causes the rebound. The extent of the surface stained by the oil is a little, but not much greater than the extent of the circle flattened by the impact.

If the ball be let fall from several different heights, it will be found that the circular space stained by the oil will be greater, the more elevated the point from which the ball is allowed to depart. This effect is only what might have been anticipated; the greater the height from which the ball falls, the greater will be the force of the impact, and consequently the greater will be the extent over which its surface will be flattened, and the greater, consequently, will be the elastic force which produces the rebound.

106. Example II.—*Caoutchouc balls.*—If such an experiment be made with a ball composed of a substance softer than ivory, and equally elastic, the flattening may be rendered directly perceptible to the senses. This may be made evident by the large caoutchouc balls inflated with air, used in the plays of children. When they strike the ground, they are flattened at the surface over a circle of very considerable magnitude, and which flattening may be exhibited by pressing them on the ground by the force of the hand. This is only an exaggeration of what would actually take place in the case of a ball of ivory or glass.

107. Example III.—*Elasticity of steel springs.*—Elasticity in bodies is sometimes manifested by their disposition to recover their form when disturbed by a force which does not affect their volume. For example, a plate of steel when bent would have the same dimensions which it had before the pressure, yet its elasticity will be rendered apparent by its immediately recovering its original form after the force which bends it had ceased to act. The play of springs of every form affords examples of this. When a straight bar of steel is bent into a curve, both compression and expansion of its molecules take place. The molecules which compose that side which becomes convex are forcibly drawn asunder, and those which form the surface which becomes concave are forcibly compressed. This is evident, inasmuch as the convex side becomes longer, and the concave shorter, by the change of form. The tendency of the molecules, by virtue of their elasticity, to recover their original position, causes those on the convex surface to contract, and those on the concave surface to expand; the combined effects of such contraction and expansion being the restoration of the bar to its original form.

108. *Limits of the elastic force.*—As elasticity results from a derangement of the component molecules of bodies, it will be easily understood that there must be limits, beyond which such derangements cannot be produced without a permanent change in the form of the body; and there are consequently limits to the play of the elastic principle. These limits will be obviously different in different bodies.

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In the case of the most elastic class of bodies, such for example as caoutchouc, these limits are very extensive. In ivory they are more extensive than glass, for ivory will recover its figure after a compression which would cause the fracture of glass. These limits are narrow in the case of such metals as lead; for although considerable compression will not cause the fracture of lead as it would that of glass, yet the derangement which such compression produces amongst the molecules of that metal is greater than their feeble elasticity can resist, and the metal accordingly takes permanently any form given to it.

109. *Elasticity of torsion.*—Elasticity is sometimes manifested by torsion or twisting. Thus, let us suppose a filament of raw silk stretched by a weight attached to it. If this weight be made to revolve several times in the same direction, so as to twist the silk and then be disengaged, the fibre of silk in virtue of its elasticity will untwine itself, causing the weight to revolve in a contrary direction; and this process of untwining will continue until the filament recovers its original position; but the twisting may have been continued to such an extreme, as to exceed the limits of the elasticity of the silk; and in that case a permanent derangement of the molecules of the silk will take place, and it will not recover its original form.

The same effects would ensue if the weight had been suspended to a fine wire of copper, silver, or any other metal, but the limit at which the twisting would produce a permanent derangement of form, or, in other words, the limit of play of the elastic principle, would be different.

110. *Dilatation by elevation of temperature.* — When the extension or augmentation of the volume of a body is produced by any physical agency, such, for example, as heat, not coming under the denomination of mechanical force, it is called *dilatation.* All bodies whatever, when submitted to the action of heat, are susceptible of having their dimensions enlarged; and to this augmentation of magnitude, or dilatation by increase of temperature, there is no practical limit.

Innumerable examples of the operation of this principle in the arts and sciences may be produced.

111. EXAMPLE I.—*Dilatation of liquids in thermometers.*—In the thermometer the dilatation of a liquid is used as the measure of the degree of heat which produces it. This instrument consists of a glass bulb attached to a tube of small bore. The tube and part of the bulb are filled with a liquid. As the temperature to which the instrument is exposed is increased or diminished, the liquid affected by it expands or contracts in a much greater degree than does the glass in which the liquid is contained. The consequence of this is, that in order to find room for its increased volume, a portion of the liquid in the bulb is forced into the tube. The column in the tube consequently becomes longer, and its increase of length, measured by

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a scale attached to the tube, becomes a measure of the increased temperature.

112. EXAMPLE II.—*Useful application of dilatation and contraction of metallic bars.*—The dilatation and contraction of metal consequent upon change of temperature has been applied some time ago in Paris to restore the walls of a tottering building to their proper position. In the *Conservatoire des Arts et Métiers*, the walls of a part of the building were forced out of the perpendicular by the weight of the roof, so that each wall was leaning outwards. M. Molard conceived the notion of applying the irresistible force with which metals contract in cooling to draw the walls together. Bars of iron were placed in parallel directions across the building, and at right angles to the direction of the walls. Being passed through the walls, nuts were screwed on their ends outside the building. Every alternate bar was then heated by lamps, and the nuts screwed close to the walls. The bars were then cooled; and the lengths being diminished by contraction, the nuts on their extremities were drawn together, and with them the walls were drawn through an equal space. The same process was repeated with the intermediate bars, and so on alternately, until the walls were brought into a perpendicular position.

113. *General effects of dilatation and contraction.* — Since there is a continual change of temperature in all bodies on the surface of the globe, it follows that there is also a continual change of magnitude. The substances which surround us are constantly swelling and contracting under the vicissitudes of heat and cold. They grow smaller in winter, and dilate in summer. They swell their bulk on a warm day, and contract it on a cold one. These curious phenomena are not noticed only because our ordinary means of observation are not sufficiently accurate to appreciate them. Nevertheless, in some familiar instances, the effect is very obvious. In warm weather the flesh swells, the vessels appear filled, the hand is plump, and the skin distended. In cold weather, when the body has been exposed to the open air, the flesh appears to contract, the vessels shrink, and the skin shrivels.

CHAP. IX.

INACTIVITY.

114. *All matter inert.*—The quality of matter which stands foremost in importance in all mechanical inquiries, forming the basis of the whole theory of force and motion, is *inactivity* or *inertia*; and important as this quality is, there is perhaps nothing which has given rise to so many erroneous conceptions.

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These errors have chiefly arisen from the adoption of a vicious phraseology on the part of many writers on Natural Philosophy.

Inactivity or inertia, as the form of the words implies, is a negative quality. It consists in the absence of a certain quality, which must be first defined before these terms can be understood.

Activity, as used in mechanics, would signify a power of spontaneous motion; such a power, for example, as accompanies vitality. If a mass of matter, being at rest and uninfluenced by any external agency, could put itself in motion, then it would have activity.

If, however, a mere mass of matter, being at rest, and uninfluenced by any external agency, cannot put itself in motion, then this mass of matter is not endued with activity, or, in other words, it has the quality of *inactivity* or *inertia.*

Inactivity, then, is the quality in virtue of which matter is incapable of spontaneous change. Whatever be its state of rest or motion, in that state it must continue so long as it is not affected by any external agency.

This quality of inactivity is one of the earliest and most universal results of observation. It is equivalent to stating that matter, as mere matter, is deprived of life; for spontaneous action is the only test of the presence of the living principle.

Accordingly, if we observe a mass of *unorganized* matter undergo any change as to motion or rest, we never seek for the cause of the change in the body itself; we look for some external cause producing it.

115. *Inertia is inability to change state of rest or motion.*—At any given moment of time a body, mechanically considered, must be in one or other of two states, rest or motion. Inertia or inactivity is the total absence of all power in the body to change its state. If the body be at rest, it cannot put itself in motion; if the body be in motion, it can neither change that motion nor reduce itself to rest. Any such change must be produced from some external cause independent of the body.

116. *Vis inertiæ, a term leading to erroneous conclusions.* — The phrase *vis inertiæ,* or force of resistance, used in many treatises on Natural Philosophy, has been a fertile source of error. Such a phrase implies a disposition in matter to resist being put in motion when at rest. Now no such disposition is found to exist; and if it did exist, it would be as utterly incompatible with the quality of inactivity as is the power to produce spontaneous motion.

117. *Erroneous supposition that matter is more inclined to rest than to motion.* — Innumerable effects which fall daily under our observation prove to us the inability of mere matter when at rest to put itself in motion, or when in motion to augment its speed; but, on the other hand, we have not the same direct and manifest evidence of its inability to destroy or diminish any motion which it may have

received; and it happens, therefore, that while few will deny to matter the former effect of inertia, many will at first doubt or fail to comprehend the latter.

Philosophers themselves, so late as the epoch signalized by the writings of Bacon, held it as a maxim that matter is more inclined to rest than to motion; and this being so, we cannot be surprised to find those who have not been familiar with physical science still slow to believe that a body once put in motion would continue for ever to move in the same direction and with the same speed, unless stopped by some external cause.

But a careful examination of the circumstances which affect the movement of the bodies around us with which we are most familiar will soon convince us, that in every case in which we observe the motion of those bodies gradually diminished, or entirely destroyed, such effects arise, not, as has been erroneously supposed, from any natural disposition of the bodies themselves to be retarded or brought to rest, but from the operation of causes of which there is no difficulty in rendering an account.

In some of the modern popular works on Natural Philosophy, a trifling experiment is mentioned as an example of the effect of inertia, and explained on principles somewhat erroneous. A card being placed on the top of the finger, and a coin placed on the card, a sudden blow being given with the back of the nail to the edge of the card, it will be projected from its place between the coin and the finger, the coin remaining unmoved on the finger.

This has been explained by stating that the inertia of the coin is comparatively so great, that the friction produced between it and the card is insufficient to move it from its place.

If by these words it be understood that the coin resists the force exerted upon it by means of the friction, it is erroneous, and would be incompatible with that quality of inertia to which the effect is ascribed.

The correct explanation of the experiment is as follows. A part of the momentum given to the card by the blow is communicated to the coin in consequence of the resistance to the motion of the card produced by the friction which takes place between it and the coin. But the coin contains comparatively so much matter, that this moving force, when distributed among its component particles, which it will necessarily be, will give to the whole coin a velocity in the direction of the motion of the card incomparably smaller than that of the card, and so small that the resultant of this force and of that produced by the weight of the coin upon the finger, scarcely deviates from the direction of the weight of the coin: consequently, although the coin remains on the finger, it does not remain precisely in the same position over the finger which it had when it rested on the card; its position will be changed in a slight degree in the direction of the motion of the card.

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118. ***Why the motion of bodies is in general retarded, and ultimately destroyed.*** — When a stone is rolled along the surface of the ground, the inequalities of its form, as well as those of the ground on which it moves, present impediments which gradually retard its movement, and soon bring it to rest. Render the stone round and smooth, and the ground level, and the motion will be considerably prolonged; a much longer interval will elapse, and a much greater space will be traversed, before it will come to rest. But asperities more or less considerable will still remain on the surface of the stone, and on the surface of the ground. Substitute for it a ball of highly polished metal, moving on a highly polished steel plane truly level, and then the motion will continue for a very long time.

But, even in this case, asperities will remain on the surface of the moving body, as well as on the surface on which it moves, which will gradually destroy the motion, and ultimately bring it to rest.

But, independently of the obstructions to the motion of bodies arising from the friction of the surfaces which move in contact with each other, all motions which take place on or near the surface of the earth are necessarily made in the fluid medium of the atmosphere. This fluid, however attenuated, still offers considerable resistance to the motion of bodies through it. An extensive flat surface spread at right angles to the direction of the motion will thus meet a powerful resistance. This resistance arises from the body moved being compelled to push out of its way a volume of air proportional to the extent of the surface which the body presents in the direction of the motion. If on a calm day an open umbrella be carried with its concave surface presented in the direction in which we are moving, a powerful resistance will be encountered, which will increase with every increase of speed.

119. ***Astronomy supplies conclusive proofs of the law of inertia.***— As these causes of resistance to the motion of bodies are everywhere present on and near the surface of the earth, we are unable, by direct experiment, to establish the proposition that a body when once put in motion would continue for ever to move in the same direction, and with the same speed, if undisturbed; but astronomical observations supply an immense mass of evidence to establish this principle. In the heavens we find a vast apparatus, every movement of every part of which establishes incontrovertibly the inertia of matter, inasmuch as the reasoning by which all these motions are explained, and by which all these phenomena are predicted, is based upon the fundamental principle of the complete inertia of matter. The celestial bodies, removed from all the casual obstructions and resistances on the surface of the globe which disturb our reasoning, roll on in their appointed paths with unerring regularity, preserving undiminished all that motion which they received at their creation from the hand which launched them into space. These phenomena alone, unsup-

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ported by other reasoning, would be sufficient to establish the quality of inertia; but, viewed in connection with the other circumstances already mentioned, and with the whole superstructure of mechanical science, leading to innumerable truths verified by daily and hourly experience, no doubt can remain that this important principle is a universal law of nature.

120. *Examples of inertia.*—The following examples will illustrate the qualities of inertia:—

121. Example I.—*Effect of sudden change of speed on horseback or in a carriage.*—If a horse or vehicle of any kind moving with considerable speed be suddenly stopped by any cause which does not at the same time affect the rider or those who are transported by the vehicle, then the body of the rider, or those who are transported, still retaining the progressive motion of which the horse or vehicle is suddenly deprived, will be projected forwards; and unless some means of resistance be adopted, the rider will be thrown over the head of the horse, and the passengers thrown forwards from the vehicle.

In the same manner, if a horse or vehicle being at rest be suddenly started forwards with considerable speed, the rider, or the persons placed upon the vehicle, not being as suddenly affected by the same forward motion, will be thrown backwards.

In both these cases the effects are the consequence of the quality of inertia. In the one case they manifest the tendency of the bodies to continue the motion they have already received, and in the other they manifest the disposition of the same bodies to continue at rest.

122. Example II.—*Leaping from a carriage in motion.*—If a passenger in a carriage which moves with considerable speed leap to the ground, he will fall in the direction in which the carriage is moving; for in descending to the ground his entire body will still retain all the progressive motion which it had in common with the carriage. When his feet touch the ground, they and they alone will be suddenly deprived of this progressive motion, which being retained by the remainder of his body, he will fall as if he were tripped up by some object impeding his motion, in the direction of the carriage.

123. Example III.—*Coursing.*—The sport of coursing presents many amusing and instructive examples of the force of inertia. From the movements of the hare, one might suppose that he, indeed, is an expert mechanical philosopher. The hound which pursues it being a comparatively heavy body, and moving at the same or a greater speed, cannot suddenly arrest its course, because, in virtue of its inertia, it has a tendency to proceed forward in the same straight line.

The hare, a comparatively light body, and moreover being prepared for the evolution, first gradually retards its motion so as to

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diminish the effects of inertia, and at the moment when the hound, urged to its extreme speed, is in the act of seizing the game, the hare dexterously turns at an acute angle to its former course, leaving the hound propelled forwards in the direction in which it was previously moving.

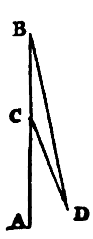

Fig. 2.

Thus, if the line A B (*fig.* 2.) represent the direction in which the hound was pursuing the hare, the hare, having arrived at the point C, suddenly turns in the direction C D; while the hound, unprepared for the trick, and hurried forward by the inertia of its motion, is carried on in the direction A B to the point B, while the hare has passed along the line C D to the point D. The distance now between the hound and the hare is the line B D, the base of the obtuse angle formed by two lines, C B and C D, simultaneously moved over by the hound and the hare.

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CHAP. X.

SPECIFIC PROPERTIES.

124. *Properties general and specific.* — The qualities of matter which have been illustrated and explained in the preceding chapters, are those which are common, in a greater or less degree, to all bodies, in whatever form or under whatever circumstances they may exist. There remains to be noticed another group of properties which may be denominated for distinction, specific properties, being found in some species of matter and not in others, or at least varying in degree so extremely in different sorts of bodies as to give them specific characters.

125. *Elasticity and hardness.* — Although the property of elasticity in its general sense may be considered as one which, in various degrees, is common to all bodies, yet it is manifested in so peculiar a manner in bodies of different forms, that it may be not incorrectly considered as giving them a specific character. It is intimately connected with another mechanical quality which may be called *hardness.*

This quality consists in a certain degree of coherence, by which the constituent molecules of a body keep their relative position so as to resist any force which tends to change the figure of the body.

126. *Relative hardness of metals.* — *Hardness* is distinct from *density,* as we frequently find the most dense bodies possess this quality in a much less degree than lighter substances. Glass, for

example, is harder than gold, or even than platinum, which is still harder and denser than gold. A piece of glass will scratch the surface of gold or platinum, an effect which shows that the particles of gold or platinum yield and are displaced more easily than those of glass.

Again, in comparing different species of metals one with another, their hardness is evidently independent of their densities. Gold and platinum, the most dense of metals, are softer than iron or zinc, which are much lighter. Among the hardest of the metals are iron, zinc, copper, manganese, nickel, titanium, and pelagium. The softest of the common metals is lead, but the new metals developed by chemical enquiries, such as potassium and sodium, are so soft as to yield under the finger like putty.

127. *Hardness of a metal may be modified.* — Some metals are capable of having their structure modified without the combination of any other substance with them, so as to render them harder or softer within certain limits at pleasure. Thus steel, when heated, and then suddenly cooled by being plunged in cold water, becomes harder than glass; but if it be cooled more gradually, then it becomes soft and flexible.

128. *Effects of elasticity.* — Elasticity manifests itself in various ways according to the form and character of the body to which it belongs. The elasticity of a flat and thin bar of steel is manifested by the force with which it will recover its figure when bent by lateral pressure; the elasticity of an ivory ball is manifested by the force with which it will recover its figure when flattened by impact against some hard surface. In the case of a steel spring, the body yields to a slight pressure, readily changing its form; in the case of an ivory ball, the body does not yield to mere pressure, and requires the force of impact to produce change of form.

When the force with which the form of a body is recovered is equal to the force by which its form has been changed, the elasticity is said to be perfect. Thus, if a bent spring recover its position when relieved from the force which bent it with an energy equal to such bending force, then the spring is said to be perfectly elastic; but when the restoring force is less than the bending force, the elasticity is imperfect. In the same manner, if an ivory ball flattened by a blow recover its form with a force equal to that of the blow which flattens it, the elasticity is perfect, but otherwise imperfect.

129. *No body either perfectly elastic or perfectly inelastic.* — It has been erroneously said in some popular treatises, that while some bodies are highly elastic, others are utterly deprived of this quality.

It is more exact to say that no body whatever is, in an absolute sense, either perfectly elastic or perfectly inelastic. All bodies possess some degree of elasticity, however small, and no known body is absolutely and perfectly elastic. But some bodies such as the gases,

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for example, possess elasticity in so high a degree, and others, such as the liquids, in a degree comparatively so small, that not only, in popular language, is the one considered elastic and the other inelastic, but it has been found convenient to assume hypothetically these two qualities of perfect elasticity and perfect inelasticity as the bases of those divisions of physical science, in which the laws which regulate the phenomena of liquids and gases are developed.

It has been already shown, however, that liquids themselves admit of some compression, and it may be added that they recover their volume with a force sensibly equal to the compressing force; and to this extent, and in this sense, they are therefore almost perfectly elastic.

130. *Elasticity not proportional to hardness.* — The quality of elasticity is intimately connected with that of hardness; so much so, that it has sometimes been said that one quality is proportional to the other. This is, however, erroneous. Many of the gums, and eminently that called caoutchouc, are highly elastic, and yet these substances are among the softest of the solids. The elasticity of caoutchouc is nearly perfect, and yet this substance, especially when it is warm, has great softness. On the other hand, glass, flint, marble, and ivory, afford examples of solids in which hardness is combined with great elasticity.

Putty, wet paste, moist clay, and similar bodies, afford examples of substances nearly deprived of elasticity. The figure of any of these may be changed by pressure or by impact, and no tendency to recover the figure so changed is perceptible.

131. *Vibratory metals.* — Sound, as will be explained hereafter, is produced by vibration imparted to the air by some solid body which is itself in a state of sympathetic vibration. It is obvious, therefore, that the metals best suited for bells, and other forms of matter, intended to produce sound, must be those which are most elastic.

132. *Hardness and elasticity of metals affected by their combination.*—The hardness and elasticity of metals are affected in a striking manner by their combination.

It often happens that two metals, neither of which is eminently hard or elastic, produce by their combination in certain proportions, one which possesses these qualities in a high degree. Thus, bells formed of pure copper, or of pure tin, will have little sonorous quality; but if these two metals be combined in a certain proportion, the combined metal will give a beautiful musical sound. The compounds of different metals which have this quality are accordingly known as *bell metal.*

133. *Flexibility and brittleness.*—When a body easily yields, and changes its form in obedience to a force exerted at right angles to its length, as, for example, when a bar being supported at the middle is pressed upon the ends, it is said to be flexible; but if upon the ap-

tion of such a force, instead of yielding and changing its form, it breaks, it is said to be brittle.

Flexibility and brittleness are specific qualities which bodies possess in an infinite variety of degrees.

In general, brittleness is connected with hardness; nor is it, as might at first appear, at all inconsistent with certain forms of elasticity. Glass, for example, which is highly elastic, is also the most brittle of known substances.

Brittleness, like hardness and elasticity, is a quality which the same body may acquire, or be deprived of, according to certain conditions to which it may be subjected.

Thus, the metals, iron, steel, brass, and copper, if they be heated and suddenly cooled, by being plunged in cold water, will become brittle; but if, when heated, they are buried in a hot sand-bath, and allowed to cool very gradually, then they will lose their brittleness, and acquire the contrary quality of flexibility.

134. *Malleability.*—Malleability is a quality by which the metals in general are eminently distinguished, but which they possess in extremely different degrees. This property is one in virtue of which a substance admits of being reduced to thin plates or leaves under the blow of a hammer, or the intense pressure of rollers. No process is of more extensive use in the arts. In large iron works, great lumps of metal at a white heat, but still solid, are taken from the furnace, stuck upon the end of a long bar of iron, and placed under a sledge hammer of enormous weight, which rapidly strikes them, and reduces them to an elongated form approaching to that of an iron bar. The metal, being still red hot, is then passed between rollers, which are formed to the shape of the transverse section of the rails used on our railways.

When pressed between and drawn through these rollers, the rail has acquired its proper form, but is still red and soft; and when received from the rollers is so flexible, that it bends by its own weight like a rod of wax. It is then laid on a flat surface, where it cools and hardens, and assumes the condition of the rails on which we travel.

The malleability of bodies depends on the combination in them of the qualities of tenacity and softness. Without softness, they could not yield to the impact of the hammer or the pressure of the roller; without tenacity, they would be fractured by the severe process of their fabrication.

The most malleable of the metals are gold, silver, iron, and copper.

135. *Malleability varies with temperature.* — The malleability of a metal varies in degree according to its temperature. There are certain temperatures in which this quality exists in the highest degree. Iron is most malleable when it first attains the white heat which follows the red; zinc becomes malleable at a much lower temperature, possessing this quality in the greatest degree between 300°

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100°. Some metals possess the quality of malleability in so
t a degree as to be in this respect specifically different from
ls in general: among them may be mentioned antimony, arsenic,
th, and cobalt, all of which are brittle.

e *metals* may be rendered brittle as they are rendered hard, by
heated and then suddenly cooled, in which case they lose
malleable quality. This quality, however, may always be re-
l by again heating them and cooling them gradually, as before
bed.

5. *Process of annealing.*—This process of gradually cooling,
is of great importance in the arts, is called *annealing*.

tals are also rendered brittle, and deprived of their malleability,
nstant hammering. Thus, a bar of iron may be hammered
it entirely loses its flexibility. In this case, as before, the mal-
ity may be restored by heating and annealing.

7. *Welding.*—Metals which are highly malleable admit of be-
mited, piece to piece, by the process called *welding*. In this
ss, the two pieces of metal are raised to that heat at which they
ost malleable, and the ends being laid one upon the other, are
ly beaten by a welding-hammer. The particles are thus driven
such intimate contact, that they cohere, and form one uniform
. Different metals may in some cases be thus welded together.

8. *Ductility.*—The property in virtue of which a metal admits
ing drawn into wire, is called *ductility*. This quality is also
ently specific, being possessed by some sorts of metal in a very
degree, while others are entirely destitute of it.

9. *Ductility different from malleability.*—Ductility is a quality
h must not be confounded with malleability; for the same
ls are not always ductile and malleable, or, at least, do not pos-
these qualities in the same degree.

on possesses ductility in a much greater degree than it possesses
eability, for it admits of being drawn into extremely fine wire,
gh it cannot be beaten into extremely thin plates. Tin and
, on the other hand, are highly malleable, being capable of being
ced to extremely attenuated leaves; but they are not ductile,
they cannot be drawn into small wire.

old and platinum possess both ductility and malleability in a high
ee. Gold has been drawn into wire so fine, that 180 yards'
th of it did not weigh more than one grain, and an ounce weight
ld consequently extend over fifty miles.

10. *Tenacity.*—The property in virtue of which a body resists
separation of its parts, by extension in the direction of its length,
lled *tenacity*. This manifestation of strength must be carefully
nguished from that of which the absence or feebleness is ex-
sed by brittleness. The one form of strength may exist in the
est degree in *a body in which the other is* in the lowest degree.

A thin rod of glass, if laid at its middle point on any support, will be broken by the slightest force pressing on its ends; but the same rod, if suspended by one end in a vertical position, will sustain an immense weight attached to the lower end without being broken. It has at once great brittleness and great tenacity; while its longitudinal strength is considerable, its lateral strength is almost nothing.

Different bodies vary extremely in their tenacity. Experiments have been made on an extensive scale for determining the tenacity of those bodies most used in the arts. The tenacity of metals has been tested by suspending a weight from the end of a wire.

141. *Table showing the relative tenacities of metals.*—In the following table, the greatest weights are given which were found to be supported by wires of the different metals, having a diameter of $\frac{787}{10000}$ths of an inch:

	Weights supported.	
Iron	549·250	lbs.
Copper	302·278	"
Platinum	274·320	"
Silver	187·137	"
Gold	150·178	"
Zinc	109·540	"
Tin	34·630	"
Lead	27·621	"

The process of annealing, which improves the malleability and ductility of metals, is found in some cases, as, for example, in iron, copper, and the combinations of zinc and copper, to diminish their tenacity.

In organized substances, those which possess a fibrous texture have greater tenacity than those of cellular tissue. Hence, we find that cotton has much less tenacity than thread, rope, or silk.

142. *Tenacity of fibrous textures.*—L'Abbé Labillardière found that threads of the following substances, having the same diameter, were capable of supporting weights in the proportion of the annexed numbers:—

Silk	3400
New Zealand Flax	2380
Hemp	1633
Flax (common)	1175
Ditto (Pita) (*Agave Americana*)	700

143. *Chemical properties.*—There is an endless variety of specific properties of bodies, the exposition and investigation of which belong properly to chemistry. It will be sufficient here to notice briefly the distinctions between these qualities and those which form the proper subjects of Natural Philosophy, commonly so denominated.

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If two substances, being mixed together, retain respectively their separate qualities, the combination is said to be a mechanical mixture. Thus, if blue and yellow powders be mingled together, the mixture will appear green; but on examining it with a microscope, it will appear to consist of large blocks of matter, of the colours blue and yellow, these being the particles of the separate powders retaining their distinctive qualities, which are mechanically mingled. The combination produces upon the eye a green colour, the effect of the separate particles being too minute to be separated by unassisted vision.

There are two gases, called oxygen and hydrogen, which have the common mechanical properties of atmospheric air. If one ounce weight of hydrogen be mingled with eight ounces of oxygen, the gases will be interfused and mingled; but the entire mass will retain the same mechanical qualities as before, and the separate particles will remain side by side in the mixture, exactly as did the particles of blue and yellow powder in the preceding example.

But if an electric spark be imparted to this mixture, a striking change will take place. The mixture will in an instant be reduced to water, or rather to vapour, which, being cooled, will be soon converted into water. In fact, the oxygen has in this case united with the hydrogen, and the mass has lost the mechanical qualities which it possessed, and has acquired those of the liquid water. This is a chemical phenomenon.

There is a metal called *sodium*, and a gas called *chlorine*, each of which, separately, is poisonous, and destructive of life, if taken into the stomach or lungs. If these two substances be brought together, they immediately explode, and burst into flame. If the substance resulting from this phenomenon be preserved and cooled, it will be found to be common kitchen-salt, one of the most wholesome condiments, and highly antiputrescent. Thus, two ingredients possessing the most noxious properties, when combined chemically, lose those properties, and produce a substance wholly different in form and quantity.

The investigation of this and all similar phenomena belongs to the province of chemistry.

BOOK THE SECOND.

OF FORCE AND MOTION.

CHAPTER I.

THE COMPOSITION AND RESOLUTION OF FORCES.

144. *Force produces, destroys, or changes motion.* — Any agency which, applied to a body, imparts motion to it, or produces pressure upon it, or causes both of these effects together, is called in mechanics, *a force.*

To determine a force, therefore, with precision, three things are necessary:

First, the point of the body to which it is applied, technically called its *point of application ;*

Secondly, its *intensity* or *quantity ;* and

Thirdly, its *direction.*

145. *Force expressed by weight.* — It is in general convenient and customary to express the intensity or quantity of forces by equivalent weights. Weight is the sort of force with which we are most familiar. Every one is acquainted with the effect produced by the pressure of a given weight; and whatever be the force whose intensity or quantity it is required to express, a weight may be named which would produce the same effect. Thus, if a piece of iron, attracted by a magnet, be resisted by any surface, it will press against this surface with a certain force. A weight may in this case be assumed, which, being placed in the dish of a balance, would press upon the surface of the dish with the same force. The intensity of the attraction of the magnet on the iron would then be expressed by the amount of such an equivalent weight.

146. *Direction of a force.* — When a force applied to any point of any body causes that point to move, the direction of its motion is the direction of the force. If the force do not produce motion, but mere pressure, then the direction of the force is that in which the pressure is directed, and in which the point would move in obedience to the force, if it were free.

147. *Effect of forces acting in the same direction.* — If two or more forces act upon the same point, and in the same direction,

their effect will be equivalent to a single force which is equal to their sum. This is so self-evident, that it scarcely needs demonstration.

If a vehicle be drawn by three horses, one placed before the other, one horse pulling with a force of 80, another with a force of 60, and the third with a force of 40 lbs., then the combined action of the three horses upon the vehicle will be equal to the action of a single horse which should pull with a force of 180 lbs., which is equal to 80 lbs. + 60 lbs. + 40 lbs.

148. *Resultant of forces in the same direction.* — A single force acting on a body which would thus produce the same motion or pressure as several forces acting together, is called technically the *resultant* of these forces. Thus, in this preceding example, the force of 180 lbs. acting on the vehicle in the same direction as the three independent forces of 80 lbs., 60 lbs., and 40 lbs., is the resultant of those three.

149. *Resultant of opposite forces.* — If two forces act upon a body in opposite directions, then the lesser of these forces will neutralize so much of the greater as is equal to its own quantity, and an effective force will remain in the direction of the greater, equal to their difference. This is also self-evident. If, for example, a vehicle be pulled backwards by a weight of 100 lbs. acting over a pulley, and that it be drawn forwards by a horse acting with a force of 150 lbs., then 100 lbs. of the horse's force will be neutralized by the weight which draws the vehicle backwards, and an effective force of 50 lbs. will remain in the direction of the horse's traction.

This principle is stated generally by saying that the resultant of two forces applied to the same point in opposite directions, is equal to their difference, and in the direction of the greater.

If any number of forces act upon the same point, some in one direction, and the others in the direction immediately opposed to it, then the resultant of such a combination of forces will be found by taking the difference between the sum of all the forces which act in the one direction, and the sum of all the forces which act in the other direction; the direction of such resultant being that of the forces whose sum is the greater.

150. *Resultant and components correlative terms.* — The several forces whose combined effect is equivalent to that of a single force are called the *components* of that single force.

Thus *resultant* and *components*, as applied to forces, are correlative terms. The resultant is mechanically equal to the combination of its components, and the components are mechanically equal to the resultant.

In all mechanical investigations, one of these can be substituted for the other, the components for the resultant or the resultant for the components, without in any wise changing the condition of the body on which such forces act.

151. *Resultant of forces in different directions.* — When two forces applied to the same point act in the direction of different and diverging straight lines, such as A X and A Y (*fig.* 3.), then the direction and quantity of their resultant is not so evident as in the case just mentioned. It is indeed apparent that the combined effect of two such forces on the point A must be in some direction, such as A Z, intermediate between A X and A Y; but how this direction A Z divides the angle formed by the two components is not apparent.

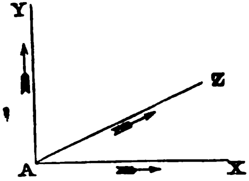

Fig. 3.

The following example, in which, as usual, weights are used to represent the forces in question, will, however, elucidate this.

Let two weights A and B (*fig.* 4.) be attached to the extremities of a flexible cord which passes over two pulleys, M and N. Let another cord be knotted to this at any intermediate point, such as P; and let a third weight O be suspended from it. The weight C will then draw the cord which unites A and B into an angle M P N. The system, after some oscillations, will come to rest, and when it is at rest, it will be evident that the point P is solicited by three forces; 1st, by the weight A acting in the direction of the line P M; 2dly, by the weight B acting in the direction P N; and 3dly, by the weight C acting in the direction of the line P C.

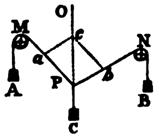

Fig. 4.

Now, it is evident that the weight C acting in the direction P C would equilibrate with an equal force acting in the opposite direction P *c*. Since, then, the weight O would precisely counterpoise an equal weight in the direction of P *c*, and that it is also in equilibrium with the weights A and B, which act in the directions P M and P N respectively, it follows that the resultant of the forces A and B acting in the directions P M and P N will be a single force equal to O acting in the direction P *c*.

It now remains to show in what manner this direction of the resultant of the two diverging forces M and N is connected with their quantities.

Let us suppose, for example, that the weight A is 6 oz., the weight B 4 oz., and the weight O 6½ oz. If then we take upon the line P O a distance of 6½ inches, and if we draw two lines, one *c a* parallel to P N, and the other *c b* parallel to P M, so as to form a parallelogram P *a c b*, we shall find, on measuring the side P *a*, that it is 6 inches, and on measuring the side P *b*, that it is 4 inches.

152. *The composition of forces.* — Hence it appears, that while the diagonal P *c* consists of as many inches as there are ounces in the resultant of the two forces, the sides of the parallelogram which are

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in the direction of these two forces respectively consist of as many inches as there are ounces in these two forces. This result may be ennunciated in general terms as follows:

If two forces acting upon the same point be represented in quantity and direction by two lines drawn through that point, then the resultant of such forces will be represented in quantity and direction by the diagonal of the parallelogram of which these lines are the sides.

But it may be objected that the result we have obtained by the use of three particular weights may be accidental, and that it may not always happen that the third weight C, which balances the other two, will throw the conducting cords into such directions as would give the remarkable result here obtained.

The validity of such an objection may be easily tested by varying the weights at pleasure, and by submitting the position of the string to the same process of measurement as we have given above. It will then be found that in whatever manner the three weights may be varied, the knot which unites the three strings P M, P N, and P O will invariably establish itself in such a position, that while the diagonal P *c* will measure as many inches as there are ounces in the resultant C, the sides P *a* and P *b* will measure as many inches as there are ounces in the components A and B.

The proposition which we have here established is of the utmost importance in all mechanical investigations, and is known as the principle of the *composition of forces.*

153. *Resultant and components mechanically interchangeable.* — In virtue of this principle, whenever two forces in different directions act upon the same point of a body, a single force determined as above by the diagonal can be substituted for them without changing the mechanical state of the body; or, on the other hand, if a single force act upon any point of a body, two forces acting on the same point may be substituted for them, provided such forces can be represented, in quantity and direction, by the sides of a parallelogram whose diagonal represents in quantity and direction the single force for which they are substituted.

154. *Resolution of force.* — As the expedient of substituting a single force for two others is called the *composition of forces*, the reverse process, of substituting for a single force two others, is called the *resolution of forces.*

155. *Resultant of any number of forces acting in any directions.* — If any number of forces whatever act upon the same point of a body, and in any directions whatever, a single force can always be assigned which will be mechanically equal to them, and will therefore be their resultant.

After what has been established, nothing is more easy than the solution of this question.

Let the several single forces supposed to act upon the point in question be expressed by A, B, C, D, E, &c.

1st, let the resultant of A and B be found by the principle of the parallelogram of forces explained above, and let this resultant be A'.

2d, let the resultant of A' and C be found by the same principle, and let this resultant be B'.

3d, let the resultant of B' and D be found, and let this resultant be C'; and so on.

In this way we shall finally arrive at the determination of a single force, which will be equivalent to, and will therefore be the resultant of the entire system.

156. *Composition of forces applied to different points.* — In what precedes we have supposed the forces whose combined effects are to be determined as applied to the same point of the body on which they act. It often happens, however, that the forces are applied to different points. We shall therefore now proceed to consider this case; and, first, we shall take the more simple condition under which the forces act in parallel directions.

157. *Resultant of parallel forces.* — As before, we shall consider the forces represented by weights.

Let P and P' (*fig.* 5.) be the points to which the two forces in question are applied, and let these forces be represented in direction by parallel cords P M and P' M' passing over pulleys, and let them be represented in quantity by two weights A and A' suspended from these cords. Now the resultant of these two weights, A and A', or the single force which would be equal to them, may be determined by means precisely similar to those which we have adopted in the case of diverging forces, by ascertaining where a single force may be applied and what will be its amount, so as to balance the two forces A and A'.

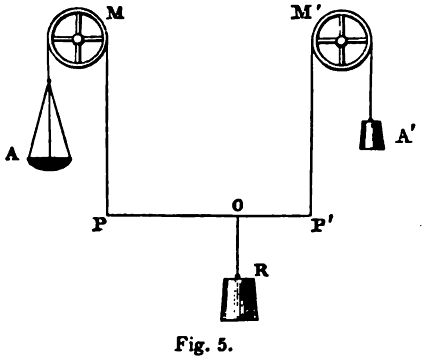

Fig. 5.

For this purpose, let us suppose a weight, R, to be suspended from a point, O, between P and P'.

Instead of suspending a determinate weight from the string carried over the pulley M, let us suppose the dish of a balance to be suspended there, capable of receiving any heavy matter which may be placed in it. Things being thus arranged, let sand be poured into the dish A, until it is found that the three weights A, A′, and R are in equilibrium. Let us suppose, for example, that the weight A′ is 6oz., and the weight R 10 oz. If the weight of the sand and of the scale which bears it at A be ascertained, it will be found to be 4 oz.; and it therefore follows that the sum of the two weights at A and A′ being 10 oz is equal to the weight R. Hence it follows that the resultant of the two parallel forces A and A′ is in this case a force equal to their sum.

Now if the experiment be varied in any manner, the same result will still be obtained. Thus, if the weight A′ be 8 oz. and the weight R be 20 oz., then the weight of the sand in the dish A will be found to be 12 oz.; the sum of A and A′, 12+8, being still equal to R, which is 20. In a word, in whatever manner the weights A and R may be varied, so long as R is greater than A, the weight A′ will invariably be their difference; and we therefore conclude in general that *the resultant of two parallel forces acting in the same direction upon two different points of the same body, is a force parallel to their direction, and equal to their sum acting at some intermediate point.*

Now, it remains to determine what is the intermediate point between P and P′ at which this resultant will act.

After having established an equilibrium by pouring the quantity of sand into the dish A, if we measure the distances P O and P′ O, we shall find that they are invariably in the *inverse proportion* of the two weights A and A′; that is to say, if the weight A′ be 8 oz. and the weight A 12 oz., then the proportion of P O to P′ O will be 8 to 12, and this will be found to be invariably the case. If the position of the string supporting the weight R be varied, as it may be, it will always be found that the ratio of the two weights A and A′, which establish an equilibrium in the system, will be inversely as the distance of the points P and P′ from the point O, where the resultant is applied; while the distance P O represents in quantity the component A′, the distance P′ O will represent in quantity the component A.

158. *Composition of parallel forces acting in the same direction* —This general principle, which is of great importance in mechanics, may be enunciated as follows:—

The resultant of two forces which act on different points of the same body in parallel lines, and in the same direction, is a single force equal to their sum acting parallel to them, and in the same direction, at an intermediate point which divides the line joining the two points of application of the components in the inverse proportion of the quantities of those components.

If the forces A′ and R be considered as components, the force A may be considered as the opposite of their resultant. It consequently follows that the resultant of A′ and R is a force equal in quantity to their difference, and applied at P, acting in a line parallel to them, and in the direction *of the greater force* R.

159. *Composition of parallel forces acting in opposite directions* — This general principle may be enunciated as follows: —

The resultant of two forces which act on different points of the same body in parallel lines in opposite directions, will be a single force equal to their difference, and acting at a point beyond the greater of the two forces, and so situated that the point of application of the greater of the two forces will divide the distance between the lesser and the resultant in the inverse proportion of the quantities of the lesser and of the resultant.

Having the means of determining the resultant of two parallel forces, we can determine the resultant of any number of such forces by taking them respectively in pairs, as we have done in the case of diverging forces.

Thus, let any two forces of such a system be taken, and the resultant found. Then, considering such resultant as a component, let it be combined with a third component, and their resultant found; and so on.

160. *Case of two equal opposite and parallel forces, called a couple.* — There is a case of parallel forces which does not admit of a single resultant, and which is of considerable importance in mechanical inquiries.

This case is that in which two equal forces act upon two points of a body in parallel and opposite directions. The effect of such forces cannot be represented by any single force. In fact, such a combination of forces has no tendency to produce in a body any progressive motion, but has a tendency to cause it to revolve round a point intermediate between the direction of the two forces.

Such a system of forces is called a *couple*.

161. *Mechanical effect of a couple.* — The mechanical effect of such a system depends, consequently, on the intensity of the forces, the perpendicular distance between their lines of direction, and on the direction of the plane which passes through their lines of direction.

If P and P′ (*fig.* 6.) be the points of application, and one of the forces act in the direction of P M, while the other acts in the direc-

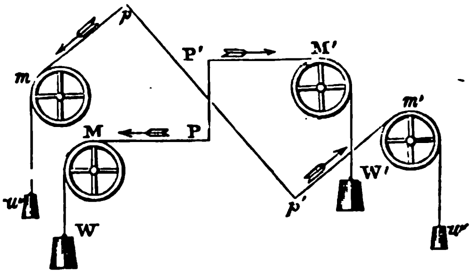

Fig. 6.

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tion of $P'\,M'$, then their effect will depend on their intensity, on the length of the perpendicular distance $P\,P'$ between their directions, and on the direction of the plane in which the lines $P\,M$ and $P'\,M'$ lie.

Representing such forces by weights, as before, let us suppose strings attached to the points P and P' carried over the pulleys M and M', and supporting the two equal weights, $W\ W'$.

The obvious tendency of these weights is to turn the line $P\,P'$ round in the direction in which the hands of a clock would move. Now this tendency cannot be counteracted by any single force, but it may be resisted by another *couple* applied to two other points of the body. Let us suppose, for example, that $p\ p'$ be two other points of the same body, either situate in the same plane as the lines $P\,M$ and $P'\,M'$, or in any parallel plane, and that strings be applied to them extended by weights, in the same manner as in the former case, the strings lying in each parallel plane.

Let $p\,m$ and $p'\,m'$, carried over pulleys, support weights w, but let them be so applied to the line $p\ p'$ that they shall have a tendency to turn the body round *contrary* to the motion of the hands of a clock, and therefore contrary to the effect of the former couple. Now let the weights w be so adjusted by trial, that this second couple shall exactly balance the first couple, and keep the body at rest, which may be done by using for the purpose the dish of a balance and sand, as in the former experiment. When the equilibrium is thus established, it will always be found that the weights W and w will bear to each other *the inverse proportion of the distance between the parallel cords*, that is to say, the weight W will be greater than the weight w in the exact proportion of the distance $p\ p'$ to the distance $P\,P'$; or, to express this in the usual manner by arithmetical symbols, we should have

$$W : w :: p\,p' : P\,P';$$

from which it follows that

$$W \times P\,P = w \times p\,p'.$$

162. *Equilibrium of couples.*—This conclusion involves the entire mechanical *theory of couples*, and may be enunciated as follows:—

Two equal and parallel forces acting in contrary directions on a body, have a tendency to make that body revolve round an axis perpendicular to a plane passing through the direction of such two parallel and opposite forces; and such tendency is proportional to the product obtained by multiplying the intensity of the forces by the distance between their directions; and, consequently, all couples in which such products are equal and have their planes parallel are mechanically equivalent, provided that their tendency is to turn the body round in the same direction; but if two such couples have a tendency to turn the body in contrary directions, then two such *couples have equal and contrary mechanical effects, and would, if simultaneously applied to the same body, keep it in equilibrium.*

163. *Condition under which two forces admit a single resultant.*— Two forces not being parallel in their directions which are applied to different points of the same body, present two different cases, in one of which only they admit of a resultant.

1st. If the two forces applied at P and P′ (*fig.* 7.), not being parallel, are nevertheless in the same plane, their directions P M and P′ M′, if prolonged, will necessarily meet at some point such as O. In this case we may imagine the two forces to be applied at O, and their resultant will be represented in quantity and direction by the diagonal O C of a parallelogram whose sides represent, in quantity and direction, the two forces, according to the principle already explained.

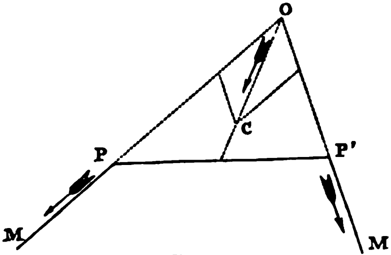

Fig. 7.

2d. But if the forces applied at the points P and P′ (*fig.* 8.), not being parallel, are at the same time in different planes, then the directions, though indefinitely prolonged, will never intersect, and they will not have any single resultant; in other words, their mechanical effect cannot be represented by that of any single force.

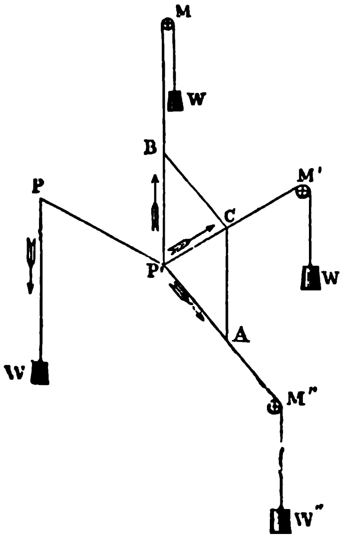

Fig. 8.

164. *Mechanical effect of two forces in different planes.*— It can be demonstrated, however, that the mechanical effect of such a system of two forces as here described, whose directions lie in different planes, and which, though not parallel, can never intersect, will be mechanically equal to the combined action of *a couple* such as already described, and a *single force;* in other words, such a system will have a double effect on the body to which it is applied: 1st,

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a tendency to produce revolution; and, 2dly, a tendency to produce a progressive motion; and if it were not held in equilibrio by some equal antagonist forces, the body would at the same time move forward in some determinate direction, and revolve round some determinate axis.

To render this intelligible, let us imagine the forces to be represented by weights acting on strings, and passing over pulleys.

In *fig.* 8., let P and P′ be the two points of application of the two forces; let the force acting at P be vertical, and be represented by the weight W, suspended by a string at the point P.

Let the other force applied at P′ be horizontal, and in a direction P′ M′ perpendicular to P′ P, and let it be represented by the weight W′ suspended by a cord which passes over the pulley M′. Attached to the point P′, let another string be carried vertically upwards to M, and then passed over a pulley, and let a weight be suspended to it equal to the weight W. Now take upon the line P′ M′ a distance P′ C, consisting of as many inches as there are ounces in the weight W′, or, which is the same, in the force which stretches the cord P′ C. Take also upon the vertical line P′ M as many inches P′ B as there are ounces in the weight W, and draw the line B C. From C draw C A parallel to P′ B, and from the point P′ draw P′ A, parallel to B C. Carry a string from P′ along the line P′ A, and let it pass over a pulley M″, and suspend from it the weight W″, consisting of as many ounces as there are inches in the line P′ A.

Now, according to this statement, the weight W will consist of as many ounces as there are inches in P′ B, and the weight W″ will consist of as many ounces as there are inches in the line P′ A. It follows, therefore, from the principle of the composition of forces already established, that the combined effects of these two forces W and W″ acting in the lines P′ B and P′ A upon the point P′, will be the same as the single action of the weight W′ acting in the direction P′ C upon the same point P′, and that they may be consequently substituted for the latter without changing the effects upon the body. Let us then detach the weight W′ and relieve the point P′ from its action, leaving the weights W and W″ acting in its place. The point P′ and the body to which it belongs will then be affected in the same manner by the three weights W, W, and W″, as it was by the two original weights W and W′. It follows, therefore, that the effect of the two original weights W and W′ is mechanically equivalent to the effect of the weight W″ and the two equal weights W.

This is equivalent to stating that the two forces acting at the points P and P′, in the directions P W and P′ C, are equivalent in their effect to three forces, viz. a single force, represented in intensity by the line P′ A, and acting along that line, and a *couple* acting in a vertical plane passing through the line P P′; the distance between the two forces being equal to P P′ and their intensities being equal to W.

The total effect, therefore, would be equivalent to a single force acting in the direction P′ A, and a *couple* producing rotation round an axis perpendicular to a vertical plane through P P′.

CHAP. II.

COMPOSITION AND RESOLUTION OF MOTION.

165. *Direction and velocity of motion.* — Motion has two qualities, direction and velocity. If we would define, in a precise and intelligible manner, the state of a body which is in motion, we must therefore state first the direction in which it moves, and next, the rate at which it moves, or the speed which it has in such direction.

If the motion of a body be rectilinear, that is to say, if it move continually in the same straight line, then such straight line is its direction. But it is evident that a body may move in two opposite directions in the same straight line: thus, if the line of the motion be east and west, the body may move either from east to west, or from west to east, without departing from the line in question.

A term is wanting in our language for the convenient expression of this condition attending the motion of a body. In French, the word *direction* expresses the line in which the body moves, and the word *sens* expresses the direction of its motion in such line.

166. *Direction of motion in a curve.* — If a body move in a curved path, such as A B (*fig.* 9.), the direction of its motion is continually changed; but at any point of the curve, such as P, it is considered to have the direction of a tangent P T at that point.

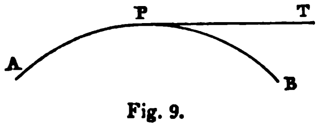

Fig. 9.

167. *Velocity or speed.* — The velocity of a moving body is expressed by stating the relation between any space through which the body moves, and the time in which such motion is performed. Thus, we say that the speed with which a man walks is four miles an hour,

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speed with which a stage-coach travels is ten miles an hour, and speed of a railway train is thirty miles an hour.

t is evident that the same speed may be differently expressed, rding to the different units of time or distance which may dopted. We may express the velocity by stating the space d over in a given time, or the time taken to move over a given

.

e given time may be an hour, a minute, or a second, and the space a mile, a foot, or an inch. If we say that a railway moves at thirty miles an hour, or that it moves at eight hundred and eighty yards a minute, or forty-four feet a second, we ss exactly the same speed, the only difference being that different of time and distance are adopted.

e selection of the units of time and distance for the expression locity is of course arbitrary.

is usual, however, to adopt such units that the velocity may be ssed by a number which is neither inconveniently great nor nveniently small. If the motion of the body be extremely rapid, xpress its velocity by adopting a large unit of space or a small of time; and if, on the other hand, the motion be very slow, xpress the velocity by a small unit of space or a large unit of

spaces or times which are extremely great or extremely small more difficult to conceive than those which are of the order of itude that most commonly falls under the observation of the s, we shall convey a more clear idea of velocity, by selecting for xpression spaces and times of moderate rather than those of exe length. The truth of this observation will be proved, if we der how much more clear a notion we have of the velocity of a ay train, when we are told that it moves over fifteen yards per d, or between two beats of a common clock, than when we are that it moves over thirty miles an hour. We have a vivid and ct idea of the length of fifteen yards, but a comparatively obscure of the length of thirty miles; and, in like manner, we have a h more clear and definite idea of the duration of a second than ave of the duration of an hour.

68. *Table showing the velocity of certain moving bodies.*—In following table are collected examples of the velocities of various cts, which may be found useful for reference, and which will e as standards in the memory for various classes of motion.

TABLE SHOWING THE VELOCITIES OF CERTAIN MOVING BODIES.

Objects moving.	Miles per hour.	Feet per second.
Man walking	3	*4⅖
Horse trotting	7	*10¼
Swiftest race-horse	60	88
Railway train (English)	32	*47
" (American)	18	26⅖
" (Belgian)	25	36⅔
" (French)	27	40¼
" (German)	24	35⅕
Swift English steamboats navigating the channels	14	*20½
Swift steamers on Hudson	18	26⅖
Fast sailing vessels	10	14⅔
Current of slow rivers	3	*4⅖
" rapid rivers	7	*10¼
Moderate wind	7	*10¼
A storm	36	52⅘
A hurricane	80	117⅓
Air rushing into vacuum at 32° F., and bar. 30 in	884	1296⅔
Common musket-ball	850	1246⅔
Rifle-ball	1000	1466⅔
24 lb. cannon-ball	1600	2346⅔
Bullet discharged from air-gun, air being compressed into hundredth part of its volume	466	683 7/15
Sound when atmosphere at 32° Fahr	743	1090⅔
Do. at 60° Fahr	762	1118⅗
Earth moving round sun	67,374	98,815⅕
A point on earth's surface at equator by diurnal motion	1037	1520 14/15

169. *Uniform velocity.* — The speed of a body in motion may be *uniform* or *varied.*

The speed is uniform when all equal spaces, great or small, are moved over in equal times.

It is possible to conceive a body in motion, moving over a mile per minute regularly, and yet the speed not to be uniform; for though each successive mile may be performed in a minute, the subdivisions of such mile may be performed in unequal times: thus, the first half of each successive mile might occupy forty seconds, and the other half twenty seconds. To constitute a uniform motion, therefore, it is necessary that all equal portions of space, no matter how small they are, shall be passed over in equal times.

170. *Principles of composition and resolution of force, equally applicable to composition and resolution of motion.*—As forces which produce pressure would, if the bodies on which they act were free to

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* Nearly.

[redacted]

move, produce motion in the direction of such pressure, whose velocity would be proportional to the pressure, all the principles which have been established in the last chapter respecting the components and resultant of forces will be equally applicable to motion. Hence, without further demonstration, we may consider as established the following principles, called the composition and resolution of motion.

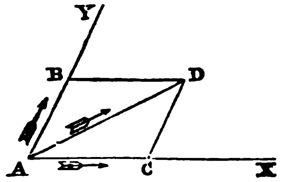

Fig. 10.

If a body A (*fig.* 10.) receive at the same time two impulses, in virtue of one of which it would move in the direction A Y, over the space A B, in one second; and in virtue of the other, it would move in the direction A X, over space A C, in one second; then the two impulses, acting upon it simultaneously, will cause it to move over the diagonal A D of the parallelogram, whose sides are A B and A C, in one second.

Conversely, also, a single motion A D may be considered as equal o the combination of two motions, A B and A C, along the sides of ny parallelogram of which A D is a diagonal.

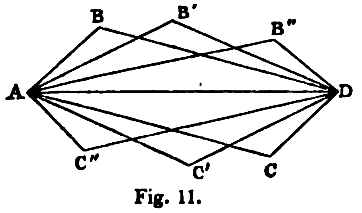

Fig. 11.

Now, as an infinite variety of parallelograms may have the same diagonal, it follows that any single force may be considered as equal to an infinite variety of combined forces or motions. In *fig.* 11., the line A D is the diagonal of the parallelograms A B D C, A B' D C', A B'' D C'', &c.

171. *Resultant of two motions in same direction.*—The effect of a force which acts upon a body in motion, must be either to change its velocity, or to change its direction, or to produce both these effects together.

If a body being in uniform motion in a certain straight line, such as A B (*fig.* 12.), receive at P the impulse of a force in the direction in which it is moving, the effect of such impulse will be to augment

Fig. 12.

its velocity, and such increase of velocity will be exactly equal to the velocity which the same force would impart to the body, if the body had been at rest.

Thus, if the body had been moving towards B at the rate of ten feet per second, and that it received an impulse at P, in the direction of B, which would have imparted to it, being at rest, a velocity of five feet per second, then the velocity of the body after the impact will be fifteen feet per second.

This is evident, because the previous motion which the body is supposed to have had in the direction of B cannot in any way impair the effect of a force tending to make it move in the same direction.

172. *Resultant of two motions in opposite directions.*—But if the impulse which it has received at P had been given in the direction P A, contrary to that of its motion, then such impulse would deprive the body of just so much velocity in the direction P B as it would have imparted to it in the direction P A, had it been at rest. In this case, therefore, the velocity after the impact will be diminished from ten feet per second to five feet per second.

These consequences are obviously analogous to the corresponding conclusions, which were explained in the last chapter, respecting the combined effects of forces acting upon a body in the same or any parallel directions.

173. *Resultant of two motions in different directions.*—If a body, being in motion from A to B (*fig.* 13.), receive at the point P an impact which, had it been at rest, would have caused it to move in the direction P C, then the body, commencing from the point P will have a motion compounded of the motion which it had before receiving the impact at P, and the motion which that impact would have given to it had it been at rest. The motion, therefore, which it would have on leaving P in this case, will be determined by the parallelogram of forces, according to the principles already established.

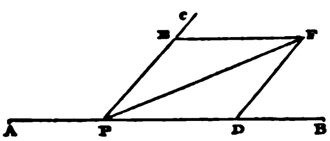

Fig. 13.

Thus, if we suppose that, by virtue of the motion which the body had along the line A B, it would have moved from P to D in a second, and that the motion which it would, being at rest at P, have received from the impact would have carried it from P to E in one second, then the motion which the body will actually have in leaving P after receiving the impact, will be along the diagonal P F of the parallelogram whose sides are P D and P E.

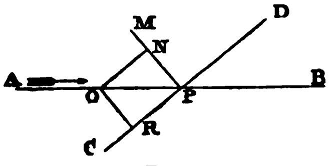

Fig. 14.

If a body moving with a certain determinate velocity in the direction A B (*fig.* 14.) encounter at P a fixed object that has a flat surface O D, its motion will not be destroyed, but will be modified by the resistance of this surface.

The effect of the surface, supposing it, as well as the body, to be perfectly hard and inelastic, will be to destroy so much of the motion of the body as is in a direction perpendicular to it.

To determine this, let us draw the line P M perpendicular to the surface C D, and taking P O as the space which the moving body moves over in a second, from O draw O N parallel to P C, and O R parallel to P N, so that P N O R shall be a parallelogram, of which P O is the diagonal. The force, therefore, with which the body will strike the surface at P will be equal to the two forces, one in the direction of R P, and the other in the direction of N P, the sides of this parallelogram, inasmuch as these two forces are, by what has been already explained, equal to the single force represented by the diagonal O P. But the reaction of the surface C D will destroy the perpendicular force N P, and therefore the force R P alone will remain in the direction R P D. The body, therefore, after it strikes the surface, will move from P toward D with a velocity, in virtue of which it will describe spaces equal to R P in one second.

174. ***Examples of composition and resolution of motion.*** — The following examples will illustrate the principle of the composition and resolution of forces which have been explained in the present and the last chapter.

175. EXAMPLE. — *Swimming across a stream.* — If a swimmer direct his course across a river in which there is a current, his body will be at the same time subject to two motions: first, that which he receives from his action in swimming, which, if there were no current, would carry him directly across the river in a direction perpendicular to its course in a certain time, as, for example, fifteen minutes; and the other in virtue of the current, by which, if he were to float on the water without swimming, he would be carried down the stream with a velocity equal to that of the stream, at a rate, for example, of five thousand feet in fifteen minutes.

Now, in actually passing over the river, the swimmer is affected at the same time by both these motions, and consequently his body will be carried in fifteen minutes along the diagonal of the parallelogram, of which these motions are sides.

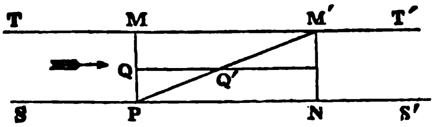

Fig. 15.

If S S and T T′ (*fig.* 15.) be the banks of the river, the direction of the *stream being expressed by the arrow*, and P be the point of departure of the swimmer, let P N be the distance down which the stream would carry the swimmer in the time which he would take, if there were no current, to cross the river from P to the opposite point M of the other bank. Then, as he crosses, he will be impelled by the two motions. By swimming, he will continually approach the bank T T′, at which he will arrive as soon as he would do if there were no current; because the current which crosses him laterally will not interfere with his course in swimming, which is perpendicular to it. Therefore, at the time that he would arrive at the middle point Q of the stream, if there were no current, he will still have arrived at the middle point Q′ of the stream; but in virtue of the current, that point will lie, not opposite the point from which he started, but lower down on the stream at Q′, and, in fine, he will arrive at the opposite bank at the point M′, just so far below the point M as is equal to the space through which the current moves in the time which the swimmer takes to cross the river. The actual course followed by the swimmer, therefore, by the combined effects of his action in swimming and of the effect of the stream, will be the diagonal P M′ of the parallelogram, one side of which, P N, represents the space through which the current moves in the time the swimmer takes to cross the river, and the other, P M, the space through which the swimmer would move in the same time if there were no current.

By a due attention to the principles of composition of motion, the swimmer may be enabled, notwithstanding the current, to cross the river in a direction perpendicular to its banks.

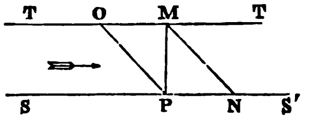

Fig. 16.

As before, let P (*fig.* 16.) be the point of the bank from which he starts, the current of the river being in the direction of the arrow. Let P N be the distance through which the current would run in the time which the swimmer would take to cross the river. From N draw the line N M to the point of the bank directly opposite to P, and from P draw the line P O parallel to M N. Now, by the principles of the composition of motion which have been already explained, a force in the direction of P M will be equal to two forces simultaneously acting, one in the direction of P N, and the other in the direction of P O; consequently, if the swimmer, leaving the point P, direct his course to the point O, and use such action in swimming as would enable him to pass in still water from P to O in the same time which the current requires to run from P to N, then his actual motion, in consequence of the combined effect of his action in swimming and the force of the current, will be in the direction P M, directly across the stream; as this direction will be the diagonal of the parallelogram, whose sides represent the two forces to which his body is exposed.

[redacted]

under these circumstances, although he will pass directly
the stream from P to M, instead of being carried diagonally
t, as in the last example, along the line P M′ (*fig.* 15.), he
e a longer time and greater exertion to cross the river. In
ner example, as his motion was perpendicular to the stream,
e of the current did not in any wise retard his course in
ng, but merely changed the point of the opposite bank at
e arrived; and although he passed over a greater space in
er, yet the increased distance over which he moved was due,
is own exertion, but to the current. In the latter instance,
, a part of his exertion was expended in stemming the cur-
as to prevent his descending the stream, and another part in
the river.

Example II.—*Effect of wind and tide on a ship.*—A ves-
elled at the same time by wind and tide in different direc-
esents another example of the composition of motion. If
ose the wind to impel the vessel in the direction of the keel
e stem to the stern, and at the same time the tide to act at
ngles to the keel, giving the vessel a lateral motion, the real
f the vessel will be intermediate between the direction of the
d the direction of the tide at right angles to the keel. The
irection of this course may be determined by the composition
on, provided the force of the tide and the effect of the wind
n.

us suppose, for example, that the force of the tide is four
n hour, and that the effect of the wind is such that if there
tide the vessel would be carried in the direction of its keel
ate of seven miles an hour. The actual course of the vessel
n be the diagonal of a parallelogram, one side of which is in
ection of the vessel measuring seven miles, and the other at
ngles to the keel, and in the direction of the tide measuring
iles.

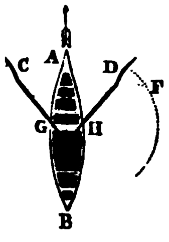

Fig. 17.

177. Example III.—*Rowing a boat.*—The action of the oars in impelling a boat is an example of the composition of forces. Let A (*fig.* 17.) be the head, and B the stern of the boat. The boatman presents his face towards B, and places the oars so that their blades press against the water in the directions C E, D F. The resistance of the water produces forces on the sides of the boat in the directions G L and H L, which by the composition of force are equivalent to the diagonal force K L, in the direction of the keel.

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178. Example IV. — *Motion of fishes, birds, &c.* — Similar observations will apply to almost every body impelled by instruments projecting from its sides, and acting against a fluid. The motion of fishes, the act of swimming, the flight of birds, are all instances of the same kind.

179. Example V. — *Effect of wind on sailing vessels.* — Numerous other examples may be derived from navigation, illustrative of the composition and resolution of forces and motion. The action of the wind on the sails of a vessel, and the effects produced by the reaction of the water, is one of these.

Let A B (*fig.* 18.) represent the position of the sail, and let the wind be supposed to blow in the direction C D oblique to the sail, its force being represented by the line C D. Let C D be considered as the diagonal of a parallelogram whose sides are E D and F D, the former perpendicular to the sail, and the latter in the direction of its surface. We may then consider the actual wind to be represented by two different winds, one of which will blow perpendicular to the surface of the sail, and the other will strike the sail edgewise, and will therefore produce no effect.

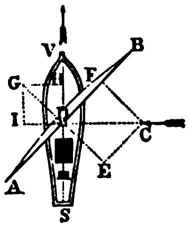

Fig. 18.

The effective part, therefore, of the wind C D will be represented by E D.

Now, this force E D, thus acting perpendicularly to the sail, will still act obliquely to the keel.

Let the force therefore of the wind upon the sail be represented by D G, the diagonal of a parallelogram, one side of which D H is in the direction of the keel, and the other D I perpendicular to it. The component of the wind therefore expressed by D E may be imagined to be replaced by two distinct winds, one D H in the direction of the keel, and the other D I at right angles to the keel.

The original force of the wind C D will thus be expressed by three distinct winds, one D F striking the sail edgewise, and therefore inefficient; another D I acting at right angles to the vessel, and therefore producing lee way; and the third D H acting in the direction of the keel from stern to stem, and therefore propelling the vessel.

In this case, it appears that a wind blowing at right angles to the course of the vessel, and having therefore in itself no tendency to propel the vessel, is nevertheless by the resolution of forces rendered efficient for propulsion.

It is easy to show that this principle may be pushed further, and that a wind which may blow in a direction nearly opposite to the course of the vessel, may, by the proper application of the same principle of the resolution of force, be made to propel a vessel. In *fig.* 19.

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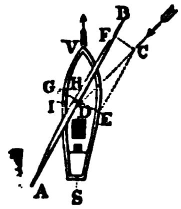

Fig. 19.

the wind is represented as blowing in the direction C D, forming an acute angle with the course of the vessel, and therefore to a proportional extent opposed to it. Let us suppose that the rigging admits of placing the sail so as to form a still more acute angle with the course of the vessel than does the wind.

The sail will thus lie between the line C D, representing the direction of the wind, and the line D V, representing the direction of the keel. As before, C D, by two successive resolutions of forces, is shown to be equal to three different winds: 1st, a wind represented by F D, blowing edgewise on the sail, and therefore inefficient; 2dly, a wind D I blowing at right angles to the course of the vessel, and therefore producing lee way; and 3dly, a wind D H in the direction of the keel, from stern to stem, and therefore efficient for propulsion.

It is evident, however, that the more oblique the wind may be to the course of the vessel, the smaller will be the component of its force represented in the diagram by D H, which is efficient for propulsion; and the greater will be the component F D, acting edgewise on the sail, and therefore inefficient. This will be apparent from the mere inspection of the two diagrams.

The limit of the practical application of this principle in sailing is determined by the play given by the rigging to the sails. There is in each species of rigging a practical limit beyond which it is impossible to place the sails obliquely to the keel. A wind which blows upon the quarter near this limit cannot be rendered useful. It will be apparent, therefore, that different kinds of rigging supply different limits to the application of this principle, and we accordingly find that one form of vessel is capable of sailing nearer to the wind than another.

180. Example VI. — *Tacking an application of the composition of motion.* — But even though the wind should blow in a direction immediately opposed to the course of the vessel, we are enabled, by another application of the principle of the resolution of force, to press such an adverse wind into the service, and to render it available in carrying the vessel directly against its own force.

This object is attained by the expedient called *tacking*, which is nothing but a practical application of the resolution of force. Supposing the course which the vessel has to pursue to be due west, while the wind is blowing due east, then the course of the vessel due west is to be regarded as the diagonal of a parallelogram whose sides are directed alternately to some points north and south of west.

Let A W (*fig.* 20.) be the course of the vessel departing from A; it sails from A to D, some points north of west; from D it sails to

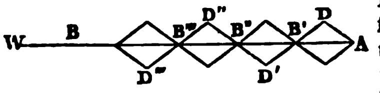

Fig. 20.

D′, some points south of west; from D′ it sails to D″, parallel to A D; and from D″ to D‴, parallel to D D′; and so on. Thus, at first, instead of sailing direct from A to W, it reaches B′ by sailing over two sides A D and D B′ of a parallelogram whose diagonal is A B′. Again, instead of sailing directly from B′ to B″ it sails from B′ to D′, and from D′ to B″, over two sides of a parallelogram whose diagonal is B′ B″; and so on. The vessel is thus conducted over the sides of a series of parallelograms, the succession of whose diagonals forms its right course.

181. EXAMPLE VII. — *Ball dropped from the topmast of a vessel.* — If a ball of lead be taken to the topmast of a vessel which is advancing under sail or steam, and be let drop downwards, it might at first be supposed that this ball would fall at a point vertically under that from which it was discharged, in which case it would necessarily strike the deck just so far behind the mast as would be equal to the space through which the vessel had advanced in its course during the time of the fall.

But we find, on the contrary, that the ball strikes the deck at the foot of the mast precisely in the place where it would have struck it had the vessel been at rest.

This fact is explained by the effect of the composition of motion.

Let A B (*fig.* 21.) be the course of the ship. Let B T be the position of its mast at the moment the ball is discharged; and let B′ T′ be the position of the mast at the moment the ball falls on the deck, the ship having advanced through the space B B′ during the interval of the fall. The ball by being discharged from the top of the mast has in common with the ship its progressive motion; and if it had not been discharged, it would have moved through the space T T′, precisely equal to B B′, in the time of the fall.

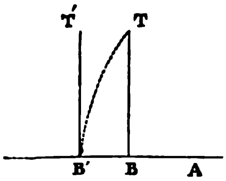

Fig. 21.

This progressive motion during the fall is combined with the vertical motion, and if the vertical descent were made with a uniform velocity, the ball would, by reason of the combined effect of the two motions, move along the diagonal of the parallelogram T B B′ T′. But the vertical descent of the ball not being uniform, but first moving more slowly, and then moving more rapidly, it will move, not along the diagonal of the parallelogram, but along a curve represented by the dotted line in the figure, which curve will be explained hereafter; but at the termination of the fall the ball will be found at the foot of the mast.

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182. **Example VIII.**—*Object let fall from a railway carriage.*—e same illustration is presented in a still more striking form by the vement of a carriage on a railway. Let us suppose that a carriage moved along a line of railway, at the rate of 60 miles an hour, and all is dropped from it at the height of 16 feet as it moves. If this l fall vertically it would strike the ground at a point 29⅓ yards ind the point of the carriage from which it was dropped; for the e of falling 16 feet is one second, and in one second a carriage ring 60 miles an hour would move over 29⅓ yards. The carriage ild, therefore, be 29⅓ yards in advance of the point at which the l was let fall. But it is found that, as in the former case, the ball l meet the ground at a point vertically under the part of the carge from which it was let fall, which renders it evident, that during fall the ball advances with the same progressive motion as the riage.

Let T (*fig.* 22.) represent the point from which the ball is disenged, and B the point of the rails vertically under it; the height T B ing 16 feet. Let B B′ be 29⅓ yards. In one second after the ball s been disengaged, the point T will have been carried forwards to vertically above B′. At the moment the ball was disengaged from it participated in the progressive motion of the carriage, and conquently having a motion in the direction T T′, which, if it had not en disengaged, would have carried it to T′ in one second. During e fall this motion is combined with that of the vertical descent, and e combination of the two motions will cause the ball to move over e curve represented by the dotted line, and to arrive at B′, the int vertically under T′, at the end of a second.

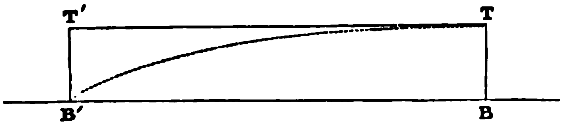

Fig. 22.

These effects may be illustrated in a still more striking manner by pposing a vertical tube 16 feet high carried with the train, like the oke-funnel of the engine. If a ball were held over the centre of e top of the mouth of this funnel, and let fall, it would, if the train ere at rest, strike the bottom of its centre point, falling directly ong the centre or axis of the tube. If the tube be carried forward ith any velocity, such as sixty miles an hour, the side of the tube ill not be carried against the ball by such progressive motion as ight be imagined, but the ball during its descent will still keep exactly the centre of the tube, as it would have done had the tube been t rest.

183. EXAMPLE IX. — *Billiard playing an application of the composition and resolution of motion.* — The skill of the billiard player depends on his dextrous application of the composition and resolution of force. All the movements of the balls, in obedience to the cue, and whether reflected from each other, or from the cushions, are determined by this principle.

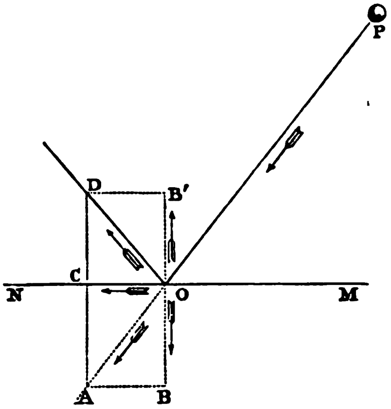

Fig. 23.

Let P (*fig.* 23.) be a billiard-ball driven in the direction P O by the cue. At O let it strike the cushion M N. The force of the impact may be represented by the dotted line O A, which is the diagonal of a parallelogram, one side of which is O B perpendicular to the cushion, and the other O C in the direction of the cushion. The effect, therefore, is the same as if, at the moment the ball struck the cushion, it were influenced by two independent forces represented by O C and O B. The force O B being perpendicular to the cushion is destroyed by its reaction; but the ball, being elastic, receives a rebound in the contrary direction O B′. The force O C being in the direction of the cushion is not destroyed, and being combined with that of the rebound O B′ will cause the ball to move along the diagonal O D of the parallelogram, of which these two lines O C and O B′ are the sides. If O, instead of being a point upon the cushion, had been a point upon the surface of another ball, the effects would be the same, only in this case the line M N would represent, not the cushion, but a tangent plane to the ball at the point of impact.

The skill of the billiard player consists in a knowledge of the combination of such effects of the composition and resolution of force; but instead of deriving it from the physical principles, he obtains his knowledge by long-continued experience. He knows the effects, but cannot explain them.

In *fig.* 24. a stroke is represented in which a *cannon* is made, after successive reflections from each of the four cushions, at the points marked O. The ball P is first directed in the line P O, upon the ball P′, so that being reflected from it, it strikes the four cushions successively at the points marked O, and is finally reflected so as to strike the third ball P″. At each of the reflections from the ball P′ and the four cushions, the same composition and resolution of force

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lace as is represented in *fig.* 23., and the diograms showing mposition and resolution are given in *fig.* 24.

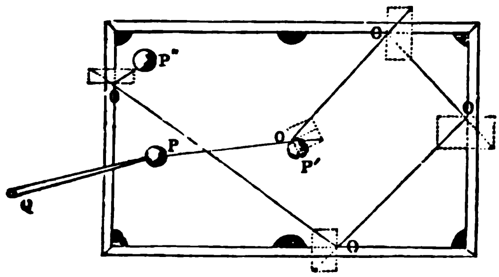

Fig. 24.

EXAMPLE X. — *Example proving the diurnal rotation of* s. — The principle of the composition and resolution of force n ingeniously applied for the purpose of obtaining a direct ration of the diurnal motion of the earth.

high tower or steeple be erected on the surface of the earth, lent that, in consequence of the revolution of the globe upon the top of the tower will be moved in a greater diurnal circle e base, being more distant from the common centre round he entire world is moved. The top of the tower, therefore, thing placed upon it, has a greater velocity from west to east, s the direction of the earth's rotation, than has the bottom.

if we imagine a heavy ball to be let fall from the top of the wards the base, this ball will be affected by two motions: 1st, ich it has in common with the top of the tower from west to virtue of the earth's diurnal motion; and 2dly, that vertical which it has in falling. The course it will follow will there-pend on the combination of these two motions, and it will he ground at a point east of that which it occupied at the com-nent of its fall, by a space equal to that through which the the tower is carried during the time of the fall. But during ne interval, the base of the tower is also moving eastward, but, been explained, through a less space.

as the ball is carried eastward through the space through the top of the tower is carried, while the base of the tower is eastward through a less space, the ball, instead of falling at e of the tower, which it would do, if there were no diurnal ro-of the earth, will fall just so much east of the base as is equal difference between the motion of the top and the motion of the of the tower.

the *difference between its two motions* must be an extremely minute quantity, it might be supposed that such an experiment, though beautiful in theory, would be impracticable; the quantity which would indicate the effect of rotation being smaller than could be accurately measured.

The experiment, nevertheless, has been made, and the result has been, within the limits of error, such as would be produced by a diurnal revolution of the earth in twenty-four hours.

185. *Motion absolute and relative.* — *Motion* is distinguished into *absolute* and *relative*.

186. EXAMPLE XI. — *A person walking on the deck of a ship.* — If a man walk upon the deck of a ship from stem to stern, he has a motion relative to the deck, measured by the space upon it over which he walks in a given time; but while he thus walks from stem to stern, the ship and its contents, including himself, are carried in an opposite direction.

If it should so happen that his own progressive motion from stem to stern is exactly equal to the progressive motion of the ship, then he will be at rest with regard to the surface of the sea, and will be vertically above the same point of the bottom; for his own motion on the surface of the deck in one direction is, by this supposition, exactly equal to the motion of the ship in the other direction.

If he walk upon the deck at a slower rate than the progress of the ship, then he will have a motion relatively to the sea, in the direction of the ship's motion, equal to the difference between the rate of the ship and the rate of his own motion on the deck; but if he move from stem to stern on the deck at a more rapid rate than the ship advances, then he will have a motion relative to the sea contrary to that of the ship, and equal to the difference between the rate of the ship and the rate of his own motion on the deck.

But these motions are again compounded with that of the earth, in which the man who walks, the ship which sails, and the sea itself participate; and if the absolute motion of the man who walks upon deck is required to be determined, it would be necessary to combine, by the principles of the composition of motion, 1st, the motion of the man upon the deck; 2dly, the motion of the ship through the water; 3dly, the motion of the earth on its axis; and 4thly, the motion of the earth in its orbit.

187. EXAMPLE XII. — *Gymnastic and equestrian feats.* — Many of the feats exhibited in gymnastic and equestrian exhibitions are explained by the principles of the composition and resolution of motion.

As an example, let us take the case in which, the exhibitor standing on the saddle, a table or other elevated object is held before him, above the height of the horse, but below that of the rider, so that the horse may pass under the table, which would obstruct the progress of the rider. In this case the rider leaps over the table, and

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returns to the same point of the saddle which he left when the horse had passed under the table.

Now, this feat demands from the rider a muscular exertion, extremely different from that which might be expected.

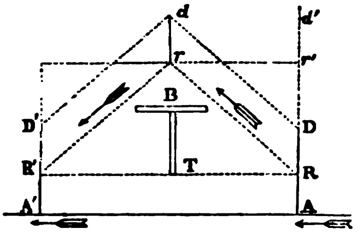

Fig. 25.

Let us suppose the course of the horse to be represented by A A' (*fig.* 25.), the table by T B, and the position of the rider upon the saddle when the horse approaches the table, and when the leap is about to be effected, by R D. At this point the rider leaps, not with a force which would project him over the table, but with one which would project him *vertically upwards* to the position represented by *r' d'*. This motion, combined with the progressive motion which the rider has in common with the horse, and which is represented by R T, will cause the rider to move, not directly upwards, in immediate obedience to his exertion, but in the diagonal direction R *r*, so that at the moment the horse comes directly under the table, the rider is directly over it at *r d*. The upward force of the leap being here expended, the body of the rider begins to fall, and, if not urged by a progressive motion, would fall on the table. But retaining the progressive motion, the descending tendency of the fall is combined with such motion, and the rider accordingly descends in the diagonal direction *r* R', and arrives at the point R' precisely at the moment that the saddle borne by the horse arrives at the same point, so that the rider returns to his position on the saddle at R' which he left at R.

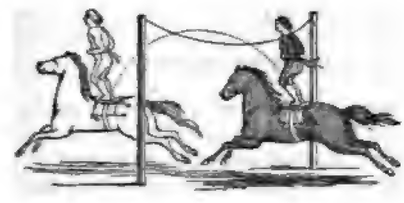

Fig. 26.

Strictly speaking, in this example, the motion of the rider does not take place in the right lines represented by the diagonals in the figure, but in lines slightly curved (*fig.* 26.). This, however, makes no difference in the principle involved in the case.

188. Example XIII. — *Flying a kite.* — The flying of a kite presents an example of the principle of the composition of forces. Let K (*fig.* 27.) be the point upon which the force of the wind may

be considered as concentrated, and let the direction and quantity of this force be represented by the horizontal line K W. This line K W may be taken as the diagonal of a parallelogram, one side of which K L is in the direction of the surface of the kite, and the other K M perpendicular to it. The former component K L, being in the direction of the surface of the kite, glides off without effect; the latter K M, being perpendicular to the kite, is effective. If the string of the kite be in the direction K A, directly opposed to K M, then the tension of the string will balance the force K M, and the kite will remain suspended in the air, without rising higher; but if the direction of the string be K A′, making an angle with K M, then the tension of the string acting in the direction K A′, and the component of the wind acting in the direction of K M, will produce a resultant action in some intermediate direction such as K N, and this resultant will cause the kite to rise to the point N, describing a circle round the centre A′, and the kite will ascend in this circle until the string K A′ takes the direction of the component of the wind to which it is opposed.

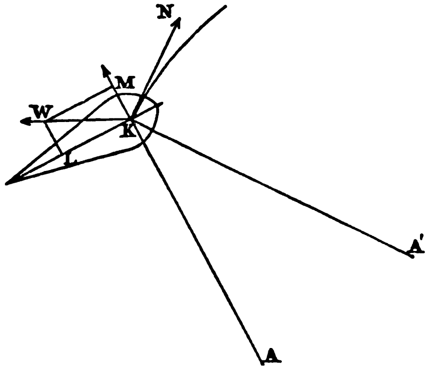

Fig. 27.

* * *

CHAP. III.

THE FORCE OF MATTER IN MOTION.

189. *Force of a moving mass.*—That a mass of matter moving in any manner exercises a certain force against any object which may lie in its way, is the physical law which, of all others, is the most early, the most frequent, and the most universal result of observation and experience. The child has hardly emerged from the nurse's arms, before it becomes conscious of the force with which its body would strike the ground if it fell.

190. *Momentum of solid masses.*—The moving mass of a hammer-head will exercise a force upon a nail sufficient to make it penetrate wood, an effect which no common pressure could produce. The force

[redacted]

f the hammer-head of the sledge-hammer will compress and vary he form of a mass of metal under it.

By the force of a descending mass of matter, the die impresses ipon a piece of metal the image and characters of the coin with more idelity and effect than the pressure of the hand could produce them, f similarly applied to wax.

The force of a moving mass will cause a punch to penetrate a ick plate of iron, or a shears to cut the same plate, with as much cility as a needle would penetrate, or a common scissors cut this aper.

The moving masses of the spear, the javelin, and the arrow, and f the bullet and the cannon-ball, have been used as destructive proectiles in war, ancient and modern.

191. *Momentum of liquids.*—This quality equally appertains to atter in the liquid form.

The force of the torrent, when a river overflows its banks, has cared away buildings and levelled towns. The force of the stream is sed to turn the mill and discharge various mechanical functions.

They who have stood at the foot of Niagara, have been conscious f the frightful energy of the mechanical power developed by the otion of the mass of waters forming that cataract.

192. *Momentum of air.*—The same quality belongs to matter even i its most attenuated form of air. The force of air in motion caries the ship over the sea, and, acting upon the diverging arms, imels the mill. The tempest agitates the deep, and flings the largest essel, with destructive force, upon the rock. The force of the movng atmosphere in the hurricane devastates countries, overturning uildings, and tearing up by the roots the largest trees.

It will therefore be understood how important it is to investigate the aws which govern a quality of matter which developes such effects.

193. *Conditions which determine the force of a moving mass.*—It s not enough to know that matter in motion will exercise this force n any object which it encounters; we must be able to express, with rithmetical precision, the conditions on which the energy of this force n each case depends; we must be able to determine, when two different masses of matter are moved under known conditions, what is he proportion between the forces which they would respectively exert n any object which they might strike.

194. *The moving force augments with the velocity.*—If a leaden ball, of a certain magnitude, move with a certain velocity, it will strike an object with a certain determinate force.

If another leaden ball, of the same magnitude, moving beside it with the same velocity, strike the same object at the same time, it will evidently strike it with the same force, so that the force exerted by the two balls will be precisely double the force which would be exerted by either of them.

195. *When the velocity is the same, the moving force augments with the mass moved.* — But if we suppose the two balls moulded into one, so as to make a ball of double magnitude, the force of the impact will still be the same. It results, therefore, in general, that if two bodies be moved with the same velocity, one having twice the quantity of matter of the other, the force of the latter will be twice that of the former.

By the same mode of reasoning, it may be shown generally that, in whatever proportion the mass of the body in motion be increased, the force with which it would strike an object in its way will be increased in the same proportion.

This force, exerted by a mass of matter in motion, is called in mechanics by the term *momentum*, and sometimes by the phrase *moving force* or *quantity of motion*.

When the velocity is the same, therefore, the momentum or moving force of bodies is directly proportional to their mass or quantity of matter.

It is found that any force which would impel a ball with a given velocity must be doubled, if the ball require to be impelled with double the velocity, and increased in a threefold proportion if the ball be required to be impelled with three times the velocity, and so on. It is evident, therefore, that the moving force of a body will be augmented in the exact proportion in which its velocity is increased, its mass or quantity of matter remaining the same.

Thus the force with which a ball weighing an ounce and moving at ten feet per second will strike any object, will be exactly ten times the force with which the same ball, moving at one foot per second, would strike such an object.

These fundamental principles are so obviously consistent with universal experience, that they can scarcely be said to require proof.

196. EXAMPLE I. — *Force necessary to project a stone by the hand.* — If we project a stone from the hand, we give it but a slight impulse, if our purpose be to impart to it a slow motion; but if we desire to project it with greater speed, we exercise a greater force of projection. Now, as the stone receives all the force communicated by the hand, it is evident that the increase of force in this case is exhibited by the increase of velocity of the motion of the projectile, the body projected being the same.

If we find a certain muscular exertion sufficient to project a ball 10 lbs. weight with a certain velocity, we shall find it necessary to use double the force if we desire to project a ball 20 lbs. weight with the same velocity.

197. EXAMPLE II. — *Force necessary to row a skiff.* — A man's force exerted upon oars can propel a skiff weighing 100 lbs. through the water at the rate of ten feet per second; but the same force applied in the same manner to a vessel weighing a hundred tons would

not move it faster than a sixteenth of an inch per second; for since the same force would be then applied to a body more than two thousand times heavier, the speed of the motion would be more than two thousand times less.

198. *Arithmetical expression for momentum.* — To express momentum arithmetically, let M express the mass of the body, and V its velocity; then

$$M \times V$$

will express its moving force: and if M′ express the mass of another body, and V′ express its velocity,

$$M' \times V'$$

will express its moving force.

Nothing can be more simple or easy than the use of such symbols, provided it be observed that the same units are used in each case for matter, and space, and time. Thus, for example, if in the first case the mass M be expressed by its weight in pounds, and the velocity V by the number of feet moved over per second, then it is necessary, in the second case, that the mass M′ should also be expressed in pounds, and the velocity V′ also in feet per second.

For example, if the mass M be 10 lbs., and the velocity V 5 feet per second, then the product

$$M \times V = 50;$$

the meaning of which is, that the moving force of the body M having the velocity V, will be the same as the moving force of a body weighing 50 lbs., moved at one foot per second.

In like manner, if the mass M′ be 5 lbs. and its velocity V′ 10 feet per second, then its momentum will be

$$M' \times V' = 50;$$

showing that in this case also the momentum is the same as that of a body weighing 50 lbs. moving through one foot per second.

In these two cases the momenta are equal, although the mass of one body be double that of the other; but it will be observed that the lighter mass obtains as much force by its superior velocity as the other has by its superior quantity.

199. *General condition of the equality of moving forces.* — These examples being generalized, we obtain the the following theorem, which is of considerable use in mechanics: —

When the momenta of two bodies are equal, their velocities will be in the inverse proportion of their quantities of matter.

The meaning of which is, that the velocity of the lesser body will be just so much greater than the velocity of the greater, as the quantity of matter in the latter is greater than the quantity of matter in the former.

A ball of cork which strikes upon a plank of wood, even with the greatest velocity, will scarcely produce an indentation of its surface. A ball of lead striking on the same plank with the same velocity, will penetrate it.

The force of the latter will be greater than the former in the same proportion as lead is heavier than cork.

CHAP. IV.

THE COMMUNICATION OF MOMENTUM BETWEEN BODY AND BODY.

200. *Effects of Collision.*—When a body in motion encounters another body, certain changes ensue in the motion and in the moving force of both bodies.

These changes are in general of a complicated kind, depending on the degree of elasticity of the bodies, their form, weight, and other physical circumstances.

201. *Case of inelastic masses.*—To simplify the question, however, at present, we shall consider the bodies completely devoid of elasticity, and so constituted that after the one impinges on the other they shall coalesce and move as one body in some determinate direction, and with some determinate speed.

Although these conditions be not strictly fulfilled in practice, they lead to conclusions of the greatest practical utility, and which approximate more or less to the results of experience, which results, however, are modified by various physical and mechanical conditions, the consideration of which is here omitted.

202. *Effect of a moving body striking another at rest.*—If we suppose a mass of matter M (*fig.* 28.) moving in a certain direction A B, with a velocity V, to encounter another mass of matter M' at rest, and that after the impact the two masses shall coalesce and move together; let it be required to ascertain what will be the direction, velocity, and force of the united masses after such impact.

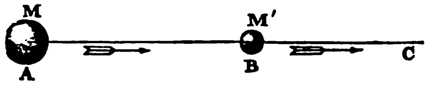

Fig. 28.

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The mass M', being supposed to be previously at rest, can have no motion save what it may receive from the mass M; and consequently, after the impact, it must move in the same direction A B as the mass M moved in before the impact.

Whatever moving force M' may obtain by its coalition with M must be lost by the mass M.

This is an immediate and very important consequence of the quality of inactivity or inertia, which has been already fully explained.

Bodies cannot generate force in themselves, nor can they destroy force which they have independent of any external agency. Whatever force, therefore, the mass M' may acquire after the coalition of the two bodies, must be lost by the mass M; and consequently the total moving force of the united masses, after the impact, must be exactly equal to the moving force of the mass M before impact. Now, according to what has been already explained, the moving force of the mass M before impact was

$$M \times V.$$

If v be taken to express the velocity of the united masses after such coalition, then since the quantity of matter in the combined masses is $M + M'$, its moving force after coalition will be expressed by

$$(M + M') \times v.$$

But this moving force cannot be greater or less than the moving force of M before impact; for to suppose it greater would be to assume that the bodies have a power of creating force in themselves, with which they were not endued by any external agency; and to suppose it less would be to suppose them capable of destroying a force which they had independently of any external agency.

Both of these suppositions, however, are equally incompatible with the quality of inactivity or inertia, which implies the absence of all power to create or to destroy any moving force.

It follows, therefore, that the momentum of the mass M, before impact, is equal to the momentum of the united masses after impact, which is thus expressed in arithmetical symbols:—

$$M \times V = (M + M') \times v.$$

That is to say, the mass M, multiplied by its previous velocity, is equal to the united masses multiplied by their subsequent velocity.

To find, therefore, the velocity with which the united masses will move, it is only necessary to divide the moving force of the mass M, before impact, by the sum of the united masses M and M'.

203. Example I. — *Boat towing ship.* — As an example of this, let us suppose a boat connected with a ship by a tow-rope, the latter being slack, to be propelled by rowers, at the rate of ten miles an hour, the boat weighing one hundred-weight, and the ship weighing fifty tons, being one thousand times the weight of the boat. The

moment the rope becomes tight, or, as seamen call it, *taut*, the moving force of the boat is divided between the mass of itself and of the ship, and the common velocity of the two will be determined by the principles above explained.

To find it, therefore, it will be necessary to diminish the previous velocity of the boat in the proportion of the weight of the united masses of the ship and boat to that of the boat; that is, in the ratio of 1001 to 1.

The velocity of the boat having been ten miles an hour, or $14\frac{2}{3}$ feet per second, the velocity with which the ship would be towed will be, therefore, the $\frac{44}{3003}$ part of a foot per second.

Thus, the force which was sufficient to propel the towing boat alone through $146\frac{2}{3}$ feet in ten seconds, will only tow the ship through little more than one-sixth of a foot in the same time.

204. *Collision of two bodies moving in same direction.*—We have here taken the case in which the body M′ is at rest. Let us now suppose that it has a motion in the same direction A B as that of the mass M, but with a velocity V′, less than V, the velocity of M, so that the body M shall overtake the body M′, and that after coalition they shall move together with a common speed; what will this common speed be? It follows, as a consequence of the quality of inactivity or inertia, that, after their coalition, the united masses cannot have a moving force either greater or less than they had before their union; consequently, the moving force of the coalesced bodies will be found by adding together the two moving forces which they had before their coalition. These two forces, according to what has been explained, being expressed by $M \times V$ and $M' \times V'$, the moving forces of the coalesced mass will be

$$M \times V + M' \times V'.$$

But if v express the velocity of the united masses, its moving force must also be expressed by this velocity v, multiplied by the total mass, that is, by $M + M'$; and, consequently, we have

$$M \times V + M' \times V' = (M + M') \times v.$$

It follows, therefore, from this, that if we divide the sum of the moving forces of the two bodies, before their union, by the sum of their masses, the quotient will be the velocity of the united masses after their union, the moving force of the united masses being equivalent to the sum of their moving forces before their union.

It is evident that this common velocity, which the combined bodies will have after their union, will be less than the speed of the body M which overtakes the other, and greater than that of the body M′ which is overtaken; the one gains and the other loses velocity, and, consequently, it is evident that the one must gain and the other lose momentum. Now, this gain and loss, on the one side and on the other, are always precisely equal. Whatever momentum the body struck

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receives from the body striking, the latter loses, and neither more nor less. There is, therefore, a precisely equal change of momentum.

205. *Equality of action and reaction.* — This general principle, which, as we have seen, is a direct consequence of the quality of inertia, and which, if not established, would lead to the conclusion that mere matter has a power in itself to produce or destroy its motion, is usually announced under the form of the following mechanical dogma or maxim:—

ACTION AND REACTION ARE EQUAL AND CONTRARY.

The term *action*, applied to the case above explained, is the power which the striking body has to give increased momentum to the body struck; and *reaction* expresses the correlative power which the body struck has of depriving the striking body of an exactly equal quantity of momentum.

This celebrated law of the equality of action and reaction, therefore, means nothing more than the equal interchange of momentum, in contrary directions, between two bodies which come into collision, and, as such, is an immediate consequence of the quality of inertia.

206. Example I. — *Mutual action of two boats.* — If a boat weighing a hundred-weight, rowed at the rate of fifteen feet per second, be suddenly connected by a towing-line with another boat which is rowed in the same direction at the rate of ten feet per second, and which weighs two hundred-weight, then the momentum of the first being expressed by 15, and that of the second by 20, their combined momenta will be 35, which, being divided by their united weights 3, gives the quotient $11\frac{2}{3}$, or 11 ft. 8 in., the space per second through which they will be rowed together.

207. *Effect of collision of two bodies moving in contrary directions.* — Finally, let us consider the case in which the two bodies M and M′ (*fig.* 29.) are moving, not in the same, but in contrary directions. Let the body M be supposed to be moving from A towards C, and the body M′ from B towards C; and let C be the point at which they will coalesce, the momentum of M being supposed to be greater than that of M′. On their coalition, the momentum of M′ will destroy just so much of the momentum of M as is equal to its own amount; for it is evident that equal and contrary forces must destroy

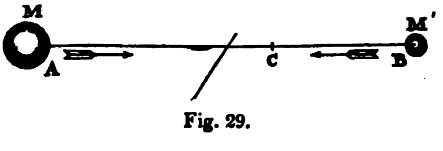

Fig. 29.

each other. After the union of the two bodies, the momentum, therefore, that will remain undestroyed, will be the excess of the moving force of M over the moving force of M′, and this excess will be in the direction C B of the progressive motion of the body M. But, as we have seen, the moving force of the body M is expressed by $M \times V$, and the moving force of the body M′ is expressed by $M' \times V'$. It therefore follows that the moving force which the united masses will have in the direction A B will be

$$M \times V - M' \times V'.$$

But it is also apparent that the moving force of the united masses will be expressed by the quantity of the united masses multiplied by their velocity. If we express, as before, this velocity by v, then we shall have

$$M \times V - M' \times V' = (M + M') \times v.$$

This statement implies nothing more than that the momentum of the masses, after their coalition in the direction of the greater force, will be equal to the difference between their momenta in contrary directions before coalition.

It follows, also, from this, that the velocity of the united masses in the direction of that which has the greater force before their coalition, will be found by dividing the difference between their momenta by the sum of their masses.

208. *This effect verifies the law of the equality of action and reaction.* — This case affords another example of the application of the maxim, that action and reaction are equal and contrary.

The effect which takes place when the bodies coalesce may be thus explained.

When the two bodies M and M′ meet at C (*fig.* 29.), the action of the body M upon the body M′ destroys the entire momentum of the latter in the direction B C, and, in addition to that, imparts to it a certain momentum in the contrary direction C B, which will be expressed by the mass M′, multiplied by the velocity which that mass has in common with M, in the direction C B, after their coalition. This velocity is expressed by v, and, consequently, the momentum imparted to M′, in the direction C B, after impact by the action of the body M, will be $M' \times v$. The total action, therefore, of the body M upon the body M′ will be made up of the momentum of M′, which is destroyed in the direction B C, and which is expressed by $M' V'$, and the momentum which is imparted to it in the contrary direction C B, and which is expressed by $M' v$. Therefore, the entire action of M upon M′ will be expressed by

$$M' \times V' + M' \times v.$$

Such, then, being the effect of the action of M upon M′, let us now consider what will be the effect of the reaction of M′ upon M. Upon

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the collision at C, when M′ loses its entire momentum in the direction C A, it will destroy in the body M an exactly equal amount of momentum; that is to say, the body M will lose, in the direction A C, a momentum equal to $M' \times V'$, which is lost by the body M′. But M also imparts, by its action to M′, a momentum in the direction of C B, which is expressed by $M' \times v$. Now, the reaction of M′ upon M, in receiving this momentum in the direction of C B, must deprive M of exactly the same momentum in that direction; that is, it must deprive it of a momentum in the direction of C B, expressed by $M' \times v$. Hence it follows that the total amount of momentum lost by reason of the reaction of M′ upon M is expressed by

$$M' \times V' + M' \times v,$$

which is precisely equal to the momentum gained by M′.

It appears, then, that in this case the law of equal action and reaction is still fulfilled, the action of M upon M′ being precisely equal to the reaction of M′ upon M.

209. *Collision of equal masses with equal and opposite velocities.* — When two equal bodies meet, moving with equal velocities in opposite directions, their shock will immediately destroy each other's momentum; for in this case, the momenta being equal and contrary, it will be mutually destroyed. The force of the shock produced by the two bodies in this case will be equal to the force which either, being at rest, would sustain, if struck by the other moving with double the velocity; for the action and reaction being equal, each of the two will sustain as much shock from reaction as from action.

210. *Examples of railway trains and steamboats.* — If two railway trains, moving in contrary directions at twenty miles an hour, sustain a collision, the shock will be the same as if one of them, being at rest, were struck by the other moving at forty miles an hour.

If two steamboats, of equal weight, approach each other, one moving at twelve miles an hour, and the other fifteen miles an hour, each will suffer a shock from the collision, the same as if it were struck by the other moving at twenty-seven miles an hour.

211. *Pugilism.* — In the combats of pugilists, the most severe blows are those struck by fist against fist; for the force suffered by each in such case is equal to the sum of the forces exerted by either arm. Skilful pugilists, therefore, avoid such collisions, since both suffer equally and more severely.

212. *Collision of masses moving in different directions.* — In what precedes we have limited our observations to the cases in which the bodies which coalesce are moving either in the same or opposite directions in the same straight line. Let us consider now the case in which they move in different straight lines before their coalition.

Let a body M (*fig.* 30.) be supposed to move in the line A B, and from A towards B. At some intermediate point C suppose it to be

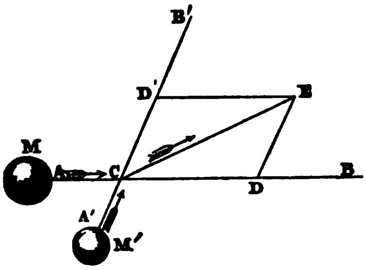

Fig. 30.

struck by another body M', moving in the line A' B' from A' towards B', and suppose that at the moment M arrives at C, M' also arrives at that point and coalesces with it, and that after their union the bodies move together. The question is, in what direction, with what force, and with what velocity they will be moved. This question is easily solved by the application of the principle of the parallelogram of forces already explained.

Let the velocity with which M moves in the direction A B be expressed by V, and let the velocity with which M' moves in the direction A' B' be expressed by V'; then M × V will be the moving force of the body M directed from C towards B, and M' × V' will be the moving force of the body M' directed from C towards B'. Let the distance C D represent the force M × V, and let C D' represent the force M' × V' and complete the parallelogram C D E D'. Draw its diagonal C E. This diagonal will then represent the direction and the quantity of the momentum which the combined masses M and M' will have after their coalition. If we would find the velocity, it will only be necessary to divide the number expressed by this diagonal C E by the number expressing the sum of the masses M and M'; the quotient will be the velocity with which the combined masses will move from C to E.

213. *How action and reaction are modified by elasticity.*—When a body which strikes a hard surface is elastic, the effects of action and reaction are modified in a manner which it will be necessary to explain.

Let us suppose, for the simplicity of the explanation, that the form of the body is that of a sphere or globe. When it strikes a hard surface with any force, it will be momentarily flattened at the point of impact, and will take an oval form; the force of the impact will compress it in the direction of the blow, and it will be elongated in a direction at right angles to the body.

Thus, if its spherical form before the blow be represented in *fig.* 31., and if D C be the diameter which is in the direction of the impact, C being the point at which the impact takes place, and A B be the diameter at right angles to D C, then the body at the moment of receiving the force of impact will take an oval form represented in *fig.* 32., the diameter A B will be elongated, and D C contracted by the force of the impact. But the body, by force of its elasticity, will make an effort

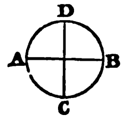

Fig. 31.

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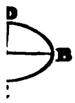
er its figure, and the point C will react upon the surface
strikes, and by that effort the body will recoil; because the
diameter D C, in its effort to recover its original length, will press the matter of the body against the hard surface at C, and this pressure being resisted will cause the body to rebound.



32.

214. *Perfect and imperfect elasticity.*—If the elasticity of the body be perfect, the force with which the spherical figure is restored will be equal
ith which it has been compressed into an ellipse, and this
ng resisted by the surface, the body will rebound with the
ce as that with which it struck it. But if the elasticity of
r be imperfect, then the restoring force will have less in-
han the compressing force, and the body will rebound with
than that with which it struck the surface.

Rebound of an ivory ball.—If an ivory ball, a substance
ossesses elasticity in a high degree, be dropped upon any
l smooth surface which is level, it will rise very nearly to
ht from which it was dropped. It would rise exactly to that
ut for two causes: first, the want of perfect elasticity; and,
, the resistance of the air.

Oblique impact of an elastic body.—When an elastic body
surface in a direction not perpendicular to that surface, it
reflected in another direction, which will depend partly on the direction in which it strikes the surface, and partly on the degree of elasticity of the striking body. Let A B (*fig.* 33.) be the surface, and let the body be supposed to strike it at C, having moved in the direction D C; and let us first suppose that the body is perfectly elastic. The force of the impact at C being represented by C E, may be resolved into two forces, C F perpendicular to the surface A B, and C G parallel to
may therefore suppose the ball at C to be affected by two
rces. Now, since the ball is supposed perfectly elastic, the
ent C F will cause a rebound in the direction of C F′ equal to
e C F. If, therefore, we take C F′ equal to C F, the body will,
e impact, be affected by two forces, C G in the direction of the
and C F′ perpendicular to it, and it will accordingly move in
gonal C E′ of the parallelogram of which C F′ and C G are
but this parallelogram being in every respect equal to the

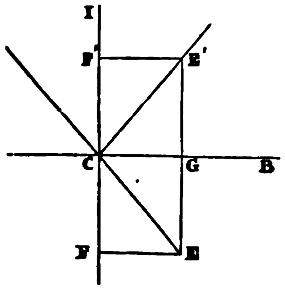


Fig. 33.

parallelogram C F E G, the angle formed by C E' with the surface C B will be equal to the angle formed by C E and C D with the same surface. Hence it appears, that in this case the body will be reflected from the surface which it strikes at the same angle as that with which it strikes it; that is to say, the angle D C A will be equal to the angle E' C B.

217. *When the striking body is perfectly elastic, the angles of incidence and reflection are equal.* — This principle is usually announced thus: When a perfectly elastic body strikes a hard surface and rebounds from it, the angle of incidence will be equal to the angle of reflection, and these angles will be in the same plane.

By the angle of incidence is understood the angle which the direction D C of the original motion of the ball forms with the perpendicular C I to the surface struck; and by the angle of reflection is understood that which the direction C E', in which the body recoils, forms with the same perpendicular.

218. *When the striking body is imperfectly elastic, the angle of incidence is less than the angle of reflection.* — But if the body which struck the hard surface be imperfectly elastic, then the recoil produced by the component C F (*fig.* 34.), perpendicular to the surface C B, will be less than the force with which the body strikes the surface at C. The line C F', therefore, which represents this force of recoil, will be less than C F, and the parallelogram C F' E' G, while it has the side C G in common with the parallelogram C F E G, has the side C F' less than C F. Consequently it is obvious that the angle which C E' makes with the perpendicular C I will be greater than the angle which D C makes with the same perpendicular.

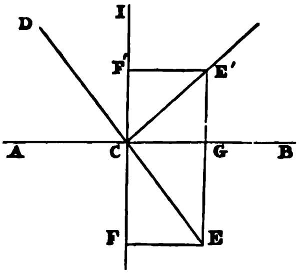


Fig. 34.

This principle is announced thus: When a body imperfectly elastic strikes a hard surface, the angle of incidence will be less than the angle of reflection, and the difference between these angles will be greater, the more imperfect is the elasticity of the body.

219. *The laws of motion.* — Before the true principles of inductive science were so well understood and so generally admitted as they are at present, the exposition of the property of inertia, and of its most important consequences, was embodied by Newton in three formularies, called by him *the laws of motion*, which have attained great celebrity in the history of mechanical science, although these mechanical maxims have lost much of their importance by the more general diffusion of correct principles of physical science; they are, *nevertheless*, entitled to notice, and ought to be registered in the me-

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mory of all students, were it only for their illustrious origin, and to commemorate the difficulty which the true principles of induction had to struggle against in the extermination of the errors of the old philosophy. These laws of motion are announced as follows: —

FIRST LAW.

Every body must persevere in its state of rest, or of uniform motion in a straight line, unless it be compelled to change that state by force impressed upon it.

SECOND LAW.

Every change of motion must be proportional to the impressed force, and must be in the direction of that straight line in which the force is impressed.

THIRD LAW.

Action must always be equal and contrary to reaction; or the actions of two bodies upon each other must be equal, and their directions must be opposite.

220. *Meaning of these laws.* — The first of these propositions is little more than a definition of the quality of inertia.

The terms in which the second is expressed require qualification and explanation, without which they might be subject to erroneous interpretation.

If a body be at rest, it is true that every motion it receives must be proportional to and in the direction of the force impressed, that is, of the force which produces the motion; but if a body be already in motion in one direction, and receive a force in another direction, then the new direction which the body takes will not be in the direction of the force impressed, nor proportional to it, but will be in the direction of the diagonal of a parallelogram, one side of which represents the force with which the body previously moved, and the other the new force which is impressed upon it. This has been already fully explained.

In the third law, the word *action* means the moving force which one body receives when another acts upon it, and the word *reaction* means the moving force which the latter loses in consequence of communicating force to the former.

The equality of action and reaction is, therefore, subject to the same qualification as must be given to the terms of the second law of motion.

Both propositions are literally true only when the two motions in question are directed in the same straight line. When they are directed in different straight lines, then the proposition must be interpreted by the principles of the composition of forces, as already explained.

221. *Maxim, that there is always the same quantity of motion in the world, explained.*—The consequence of the equality of action and reaction, combined with what has been explained respecting the indestructibility of matter in a former chapter, have led some philosophers to the adoption of the startling physical dogma, that there has been, and must be always, the same quantity of matter and the same quantity of motion in the world; in other words, that nothing short of divine agency can create or destroy either the smallest portion of matter or the smallest moving force. So far as this applies to matter, it has been already explained; but a few words may be useful to elucidate the import of this maxim as applied to moving force or momentum, as it may naturally be objected, that as all creatures endowed with animal life have the power of spontaneous motion, how can it with truth be said that there is always the same quantity of moving force in the world?

222. *Cannot a living agent produce new motions?*—Must not an animal, who by the act of its will puts its body in progressive motion from one place to another, and after a time by another act of its will causes this motion to cease, first create a new motion, and then destroy it? Is there not in such a power manifest contradiction to the maxim which states that there is always the same quantity of moving force in the world; for while the animal is in motion, is there not the momentum of the mass of its body in existence, which did not exist before it began to move? and is not this momentum destroyed when it ceases to move?

223. *Spontaneous motion of an animal explained.*—Let us examine what takes place in such cases. If an animal commence to move its own body on the surface of the earth in any given direction, it obtains its progressive motion by the action of its feet upon the ground.

Between the mass of the body of the animal and the earth on which it treads, there is therefore an action and reaction, which are equal and opposite. Whatever moving force the body of the animal acquires in one direction, the earth loses in the other; and therefore the animal may be considered as robbing the earth, so to speak, of the moving force which its body gains.

But when the animal, after moving to the desired point, brings his body to rest, there is another action upon the earth. His body is deprived of the momentum which it had acquired by the action of the feet upon the ground, so that the momentum with which the mass of its body moved is now imparted to the earth, gradually, as his motion is retarded, and altogether when he comes to rest. It therefore appears, that the animal takes from the earth his progressive momentum when he begins to move, and returns it to the earth when he comes to rest.

224. *Case of the motion of a railway train explained.*—If a rail-

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ray train, weighing 100 tons, be started from a state of rest by a comotive engine, and attain a velocity of 50 miles an hour, it will ave acquired a certain moving force of 100 tons, moving at 73⅓ feet er second, which is equivalent to a mass of 7,333⅓ tons, moving at he rate of one foot per second. Now this momentum is obtained by he action of the driving-wheels of the engine on the rails, produced y the force of the steam.

This action is attended with an equal and opposite reaction upon he rail, and through the rail upon the earth. The earth, therefore, y this reaction, loses as much momentum in the direction in which he train moves as the train gains; therefore the earth loses, in this ase, a momentum equal to 7,333⅓ tons moved one foot per second, hich momentum the train has acquired.

Now, when the same train is about to stop, the moving force which t possesses is imparted to the rails, as it must be by the resistance of he rails on the wheels that the train is brought to rest. The rails herefore in this case, and with them the earth, receive back, gradually, its moving force, as the train is gradually stopped; and when he train has been brought to actual rest upon the rails, its entire moving force, equal to 7,333⅓ tons moving one foot per second, is restored.

225. *The earth a great reservoir of momentum.*—It appears, therefore, that the earth may be regarded as a great reservoir of momentum as it is a great reservoir of matter, and that a moving force, which by any action whatever, whether mechanical or vital, which is produced upon it, must be borrowed from, and will be restored to it when such moving force ceases. The analogy, therefore, of matter and momentum is complete, and the maxim above mentioned must be accepted. As all apparently new bodies must be composed of materials derived from the earth, and as all bodies apparently destroyed are merely decomposed, and their atoms return to the common stock of matter which constitutes the globe, so all momenta must be obtained from the common reservoir of forces in the earth and restored to it.

226. *Would spontaneous progressive motion be possible in the absence of a mass like the earth to react upon?* — But perhaps another objection may be raised in the minds of some to this reasoning. It may be asked, whether, if the body of a man or animal, endued with life, could be imagined to be suspended in space, out of contact with the earth, could not such man or animal, by the act of its will, put its body in progressive motion? We answer at once in the negative

Any attempt to move the limbs would produce in the body of such animal an equal action and reaction. If any limb were projected with any given force, a reaction would take place in other parts of the body, which would *be projected backwards* with the same force, *and the general mass of the body would* have no progressive motion.

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fore has a tendency to make all these particles move with equal velocities, and in parallel lines perpendicular to the surface.

It is easily conceived that if two leaden balls of equal magnitude be placed side by side at the same height, they will fall together with the same velocity to the surface, and strike the earth at the same moment side by side. Now, if the matter of these two balls be moulded into a single ball, the effect will remain the same, since their form cannot affect the operation of gravity upon them.

In the same manner, if ten or a hundred leaden balls of equal magnitude be disengaged together, they will fall together; and if they be moulded into one ball of great magnitude, it will still fall in the same manner. Hence it follows that masses of matter, however they may vary in magnitude and weight, will descend to the surface of the earth with the same velocity, and if they fall from the same height will arrive at the surface of the earth in the same time, provided they be affected by no other force but that of gravity.

235. *Effects apparently incompatible with this explained.* — There are some circumstances developed in the fall of bodies, and the effects of the resistance of the air upon them, which are apparently incompatible with what has been just stated. If a feather and a leaden ball suspended at the same height be disengaged, it is evident that they will not fall with the same velocity. The leaden ball will be propelled with a rapidity much greater than that which affects the feather. But in this case the operation of gravitation is modified by the resistance of the air, which is much greater upon the feather than upon the leaden ball. That two such bodies would descend with the same velocity if relieved from the interference of the air, may be shown by the experiment which is familiarly known as that of *the guinea and feather*.

236. *Guinea and feather experiment.* — Let a glass tube, A B (*fig.* 35.), of wide bore, as, for example, three or four inches, and of five or six feet in length, be closed at one end, B, and supplied with an air-tight cap and stop-cock at the other end, A. The cap being unscrewed, let small pieces of metal, cork, paper, and feathers, be put into it, the cap screwed on, and the stop-cock, closed. Let the tube be rapidly inverted, so as to let the objects included fall from end to end of the tube. It will be found that the heavier objects, such as the metal, will fall with greater, and the lighter with less speed, as might be expected. But that this difference of velocity in falling is due, not to any difference in the operation of gravity, but to the resistance of the air, is proved in the following manner. Let the stop-cock be screwed upon the plate of an air-pump, the cock being open, and let the tube be exhausted. Let the cock then be closed, and unscrewed from the plate. On rapidly inverting the tube, it will then be found that the feathers will be precipitated from end to end as rapidly as the metal, and

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that, in short, all the objects will fall together with a common velocity.

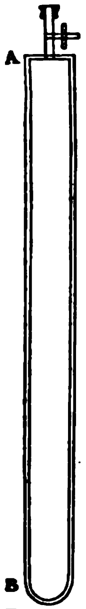

Fig. 35.

237. *Weight of bodies proportional to their quantities of matter.*—Since the attraction of the earth acts equally on all the component parts of bodies, and since the aggregate forces produced by such attraction constitute what is called the *weight* of the body, it is clear that the weights of bodies must be in the exact proportion of the number of particles composing them, or of their quantity of matter.

Hence, in the common affairs of commerce, the quantities of bodies are estimated by their weights.

It will appear, hereafter, that the weight of a body, or the force with which it is attracted to the surface, is slightly different in different places upon the earth; but this is a point which need not be insisted on at present.

At the same place the weights are invariably and exactly proportional to the quantities of matter composing the bodies. If one body have double or triple the weight of another, it will have double or triple the quantity of matter in the other.

238. *Motion of a falling body accelerated.*—It is not enough for the purposes of science to know merely the direction of the motion which gravity impresses upon bodies; we require to know whether the motion be one having a uniform velocity; or, if not, in what manner does the velocity of the falling body vary?

If a man leap from a chair or table, he will strike the ground without injury. If the same man leap from a house-top, he will probably be destroyed by the fall. These, and innumerable similar effects, indicate that the force with which a body strikes the ground is augmented with the height from which it falls. Now, as this force depends on the velocity of the body at the moment it touches the ground, it follows that the velocity of the fall is augmented with the height.

In short, when a body is disengaged and allowed to descend in obedience to gravity, its velocity is gradually accelerated as it descends. Meteoric stones which descend from the upper regions of the atmosphere, strike the earth with such force that they are often known to penetrate in it a considerable depth.

239. *Force of fall not proportional to height.*—It might be naturally enough conjectured, that the force with which a body strikes the earth is proportional to the height from which it has fallen; and such an illustration has been accordingly used by orators in speaking of the severity of censure proceeding from high quar

ters; but, like many other ornaments of eloquence drawn from physical science, this is erroneous.

The force of the fall is not, as we shall now show, proportional to the height from which the body has descended. A body falling from a double height does not strike the ground with a double force.

240. *Analysis of the motion of a falling body.*—When a body, such, for example, as a leaden ball, is disengaged at any height, and delivered to the action of gravitation, the effect of this force upon it is to impart to it a certain velocity.

Now it is evident that the quantity of velocity which the attraction of the earth gives to the ball in one second of time must be equal to the force which it would give to it in another second of time.

Let us suppose, for example, that a moveable stage s (*fig.* 36.) is attached to a wall or pillar, and is so adjusted that the ball disengaged at B shall arrive upon the stage s precisely at the termination of one second. The body will then strike the stage with a certain force.

Let another stage s' be placed at the same distance from s as s is from B. If the ball, having been brought to rest by the stage s, is again disengaged, it will strike the stage s' at the end of another second, and with the same force; and if the stage s'' be fixed at an equal distance below s', the ball, having been brought to rest at s', and then disengaged, will strike the stage s'' at the end of the third second, and with equal force.

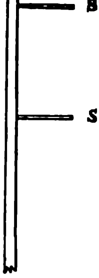

Fig. 36

In this case we have supposed that while the ball descends, the velocity it has acquired at the end of each successive second is destroyed by the resistance of the stages s, s', and s'', &c. But suppose that on arriving at s, at the end of the first second, the body was not deprived of the velocity it had acquired, but allowed to retain it in its descent, the retention of this velocity would not in the slightest degree prevent the action of gravity in imparting to it an equal quantity of velocity in the second second; therefore at the end of the second second the body would have the velocity with which it struck the stage s, *in addition to* the velocity which it had acquired at the end of the second second. In the second second, therefore, the body would descend through a much greater space than s s', and at its termination would have a velocity double that which it had at the end of the first second. In like manner, if the velocity acquired in the second second were not destroyed by the stage s', the body would at the end of the third second possess this velocity, in

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addition to the velocity which would be imparted to it by the action of gravity in the third second.

In fine, it follows that the action of gravitation imparts to a descending body a certain velocity in every successive second of time during its action; and consequently, the velocity which a falling body has at the end of ten or twenty seconds, is exactly ten or twenty times the velocity it had at the end of one second.

241. *Velocity acquired by a falling body augments with the time of the fall.*—This principle, in virtue of which the velocity imparted by gravity to falling bodies accumulates in them, is expressed as follows:—

The velocity acquired by a body in descending by the force of gravity, increases in proportion to the time of the fall.

242. *Uniformly accelerated motion and uniformly accelerating force.*—A motion, the velocity of which is thus augmented in proportion to the time counted from its commencement, is called *uniformly accelerated motion*, and the force which produces such a motion is called *uniformly accelerating force.*

Gravity, therefore, acting on bodies near the surface of the earth, is a uniformly accelerating force.

Since a body in falling moves with a velocity gradually and uniformly accelerated, its average or mean velocity will be that which it had precisely at the middle point of the interval which elapses during its fall. Thus, if a body fall during ten seconds, the average speed will be that which it had at the end of the fifth second. This is evident, inasmuch as, the speed imparted in each successive second being the same, the average of all the speeds at the end of each number of seconds, counted from the commencement, will necessarily be that which it had at the middle point of the time.

It follows from this also, that the final speed acquired by a body at the end of any time will be double the average speed counted from the commencement of its fall. This is evident, since, the velocity being proportioned to the time, the final speed is necessarily double that which is acquired in half the time, which is, as has been just shown, the average speed.

243. *A body falling during any time acquires a speed which would in the same time carry it over twice the space through which it has fallen.*—It follows from this also, that if a body were to move with its final velocity continued uniformly, it would, in a time equal to that of the fall, move over a space equal to double that through which it had fallen; for the final speed being double the average speed, the space described with the former will be double the space described with the latter in the same time.

To obtain a more exact estimate of the manner in which the descent of a heavy body is accelerated, it will be useful to investigate the spaces through which a body moves in its descent during every successive second of time.

244. *Analysis of the heights fallen through in successive seconds.*—Let us express by H the height through which a body falls from a state of rest in one second. At the end of such second, the body has acquired a velocity, in virtue of which it would, in another second, without the further action of gravity, move through a space 2 H; but during the next second gravity would cause the body to descend through another space equal to H, supposing it to move from a state of rest. Therefore during the next second the body is moved through a space equal to three times H; that is to say, twice H in virtue of the velocity acquired at the end of the first second, and a space H in virtue of the action of gravity upon it during the next second.

Let us now consider the motion of the body during the third second. At the end of the second second, the body having fallen through a height expressed by 4 H, has acquired a velocity in virtue of which, without any further action of gravity, it would move through a space equal to 8 times H in two seconds, and 4 times H in one second; but in addition to this gravity also, in the third second, would move it through a space H; and from these two effects combined, the body in the third second would descend through a space expressed by 5 H. But we have seen that in the first two seconds it has fallen through a space expressed by 4 H, and therefore at the end of the third second it will have fallen through a height from the state of rest expressed by 9 H.

Pursuing its course further, we find that it begins its motion during the fourth second with a velocity such as would make it, in three seconds, without the further aid of gravity, move through a space equal to double that which it had fallen through from a state of rest, that is to say, 18 H; consequently, with this velocity, it would move in the fourth second through a space equal to 6 H; but, in addition to this, the action of gravity carries it in the fourth second through the space H, and by these combined effects it must move in this second through a space equal to 7 H. In the same manner, it may be shown that the space through which it moves in the fifth second is 9 H, while the space through which it moves in the first five seconds is 25 H; and the space through which it moves in the sixth second is 11 H, while the space through which it descends from a state of rest in the first six seconds is 36 H; and so on.

245. *Tabular analysis of the motion of a falling body.*—In the following table is expressed, in the first column, the number of seconds, or other equivalent intervals of time, counted from the commencement of the fall. In the second column is exhibited the space through which the falling body moves in each successive interval, the unit being understood to express the space through which a body falls *in the first* second of time. In the third column is expressed the

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velocity which the body has acquired at the end of each interval, counted from the commencement of the fall, and expressed by the space which, if such velocity continued uniformly, the body would describe in one second. In the fourth column is expressed the total heights from which the body falls from a state of rest to the end of the time expressed in the first column.

TABULAR ANALYSIS OF THE MOTION OF A FALLING BODY.

Number of Seconds in the Fall, counted from a State of Rest.	Spaces fallen through in each successive Second.	Velocities acquired at the End of Number of Seconds expressed in First Column.	Total Height fallen through from Rest in the Number of Seconds expressed in First Column.
1	1	2	1
2	3	4	4
3	5	6	9
4	7	8	16
5	9	10	25
6	11	12	36
7	13	14	49
8	15	16	64
9	17	18	81
10	19	20	100

Although all the circumstances attending the descent of bodies falling freely are included with arithmetical precision in the above table, we may nevertheless render it more easy to obtain a clear conception of these important physical phenomena by the annexed diagram (*fig.* 37.), in which the divided scale represents the vertical line along which the body is supposed to fall, O being the point from which it commences its descent. The points which it successively passes at the termination of 1, 2, 3, 4, 5, 6, and 7 seconds respectively are marked I, II, III, IV, V, VI, VII. The figures of the scale indicate the total heights through which the body has fallen at the end of each successive second, the unit being the height through which the body falls in the first second. The spaces included between brackets on the right of the diagram are those through which the body falls in each successive second. It will then be apparent, first, that the body is accelerated in its motion, inasmuch as the spaces through which it falls in each successive second are evidently increasing; secondly, that the space through which it falls in any number of seconds is expressed by the square of this number, the unit being the space fallen through in the first second; thirdly, that the spaces fallen through in each successive second are expressed by the odd numbers with reference to *the same unit.*

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A direct experimental verification of the results exhibited in the preceding table and diagram, would be attended with several practical difficulties. The heights through which a body falls by gravity, acting freely in several seconds, are considerable, and a great velocity is soon acquired. The resistance of the air disturbs the result, and some difficulty would be found in observing, with sufficient precision, the points at which the falling body would be found at each successive second of time.

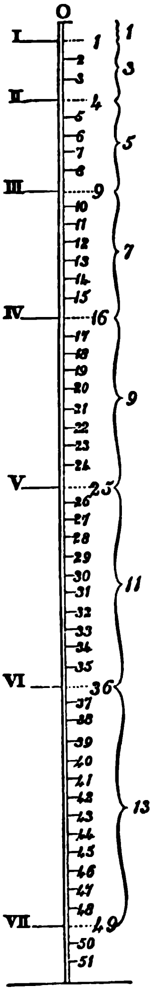

Fig. 37.

246. ***Attwood's machine for illustrating experimentally the phenomena of falling bodies.*** — This and other practical difficulties have, however, been surmounted by a beautiful and useful experimental apparatus, called from its inventor "Attwood's machine." By this apparatus, the intensity of the force of gravity can be diminished in any desired proportion without divesting it of any of its characters of a uniformly accelerating force. Thus we can make the falling body descend at so moderate a rate, that the effect of the atmospheric resistance becomes imperceptible, and the height, and all the circumstances attending the fall, can be observed with the greatest precision.

This contrivance consists of two equal cylindrical weights, W, W′ (*fig.* 38.), connected by a fine silken thread, which passes in a groove over a nicely-constructed wheel R, turning on a horizontal axis, so as to be subject to an imperceptible friction. This wheel, and the stand which supports it, are placed upon a bracket A B, attached to a wall, or supported on a pillar, at a convenient height. Adjacent to the thread supporting one of the weights W, there is a divided vertical scale, by which the circumstances attending the descent of the weight can be noticed and measured. When the weight W is brought to the highest point of the scale, the weight W′ will be near the ground; but the weight of the thread is so insignificant, that though unequal portions of it hang on each side of the wheel R, the difference of their weights produces no perceptible effect, and, accordingly, the two equal weights W W′ rest in equilibrium.

Now, if a small additional weight *w* be placed upon the top of the cylindrical weight W, it will cause the weight W to descend, and the weight W′ to rise; and this descent will have all the characters of a *uniformly accelerated* motion, for the force of gravity impresses on the

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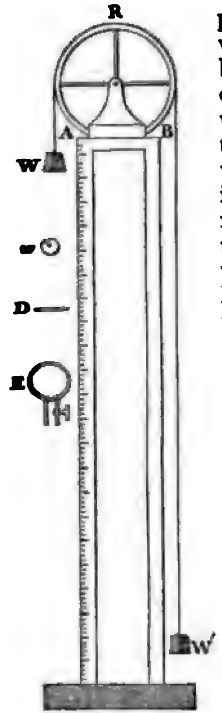

Fig. 38.

preponderating weight *w* the same moving force which it would impress upon it if it were free; but this moving force is, by the very condition of the apparatus, shared by *w* and the equal weights W and W', so that instead of imparting to *w* the velocity which such weight would have were it free, the velocity of the augmented moving mass, consisting of *w*, W, and W', will be diminished in precisely the same proportion as the mass moved is increased; therefore, the weight W bearing upon it *w*, will fall with a velocity so much less than that with which *w* would fall, were it free, as the combined weights W', W, and *w* are greater than *w* alone. But the other circumstances attending the descending motion will be precisely similar to those which attend the descending motion of any falling body. The machine will in fact, present a miniature representation of the phenomena of falling bodies; the effects will be the same as though the attraction of the earth upon a falling body were diminished to such an extent, that the velocity of the descent would be reduced to that with which the weight *w* falls.

Now, as we can adopt a preponderating weight *w* as small as may be desired, it is clear that we may reduce the velocity of the fall in so great a degree, that all the circumstances attending the motion during the descent can be deliberately and accurately observed.

Let us suppose, for example, that the weights W and W' are each twenty-four ounces, and that the preponderating weight *w*, placed upon W to produce its descent, is a quarter of an ounce. The total mass moved, therefore, by the action of gravity impressed upon the weight *w*, will be 193 times the weight *w*, for the weights W and W', taken together, are forty-eight ounces, that is to say, 192 quarters of an ounce; and the weight *w*, which is one quarter of an ounce, being added to this, will make a total of 193 quarters of an ounce.

The attraction of gravity, therefore, instead of imparting velocity to one quarter of an ounce, has to move 193 quarters of an ounce, and, consequently, the velocity it imparts per second will be 193 times less.

We have here, with a view to simplify the explanation, avoided all reference to the motion imparted to the wheel over which the strings pass; but it will be evident that the force impressed by gravity on the preponderating weight *w*, must be shared with the mat-

[redacted]

ter of the wheel, as well as with the weights W and W'. If the matter of the wheel were all collected at its edge, it would then be moved with the same velocity as the weights W and W' and in this case it would be only necessary to consider the weight of the wheel as forming part of the masses W and W', and therefore to diminish the latter so that the total weight of W and W' and the wheel should make up forty-eight ounces.

But as the mass of the wheel is not all collected at its edge, it does not all receive the same velocity, but, on the contrary, its different parts are moved with less velocities the nearer they are to its centre. This difference of moving force imparted to different parts of the wheel, requires to be allowed for, by calculating how much matter collected at the edge of the wheel would have an equal moving force. Such a calculation, though presenting no difficulty, and subject to no inaccuracy or doubt, would involve mathematical principles and operations which cannot be conveniently introduced here; and we may therefore assume, that the momentum imparted to the wheel is represented by an equivalent portion of the forty-eight ounces assigned to the weights W and W', and that, in fact, the real weights of these must be a little less than those assigned to them, the difference being represented by the effect of the wheel.

Being provided with a pendulum beating seconds in an audible manner, and taking the thread which sustains the weight W' between the fingers, let the weight W be elevated until its upper surface coincides with the zero of the scale. Listening attentively to the beats of the pendulum, let the thread be disengaged at the moment of any one beat.

It will be found that, at the moment of the next beat, the weight W will have fallen precisely *one* inch. During the second beat it will have fallen through precisely *three* inches more, during the third beat it will have fallen through *five* inches, during the fourth beat it will have fallen through *seven* inches, during the fifth beat it will have fallen through *nine* inches, and so on. Now, if these distances be compared with those given in the second column of the preceding table, they will be found to correspond; the spaces through which the weight W descends in successive seconds being, as shown in this table, expressed by the odd numbers.

In the same manner, it will appear that the height through which the weight W falls during the first second, being one inch, the height through which it falls during the first two seconds will be four inches, the height through which it falls during the first three seconds will be nine inches, the height through which it falls during the first four seconds will be sixteen inches, the height through which it falls during the first five seconds will be twenty-five inches, and so forth.

These numbers correspond with and verify those given in the *fourth column* of the preceding table.

[redacted]

7. *The heights from which a body falls are proportional to the res of the times of the fall.*—It is evident that these numbers, express the heights through which the bodies fall in any num- f seconds, counted from the commencement of the motion, are quares of the numbers of seconds; and hence we have the fol- g general principle:—

hen a body is moved by a uniformly accelerating force, such as ity, the spaces through which it moves, counted from the commencement of the motion, will be proportional to the squares of the , and the spaces through which it moves in equal intervals of will be as the odd numbers.

ese rules, which are of the highest importance, may be conve- tly reduced to arithmetical symbols. Let us express by *g* the through which a body, urged by a uniformly accelerating force a state of rest, would move in one second, a space which, in the of gravity, is 16ft. 1in., or 193 inches. Thus it is evident, what has been stated, that we shall find the space which the would move through in any given number of seconds, counted the commencement of its motion, by multiplying *g* by the re of this number of seconds.

48. *Formulæ expressing the heights, velocities, and times.*—ce, in general, if T express the number of seconds during which body has been moving from a state of rest, $T^2 \times g$ will express entire space through which the body has moved in the number econds expressed by T. If this space, then, be expressed by H, hall have

$$H = T^2 \times g.$$

n like manner, since it has been established that the velocity ch is gained in falling during one second, is such, that in each nd the body would with that velocity move through a space equal wice that through which it had fallen, it follows, that the velocity uired in one second is 2 *g*; in other words, it is such, that a body ing with that uniform velocity would move through a space ex- sed by 2 *g* in each second.

But it has also been shown, that the velocity augments in propor- to the time, and that the velocity in two, three, four, and five nds is two, three, four, and five times the velocity in one se- d. To find, therefore, the velocity acquired in any number seconds, we shall only have to multiply 2 *g* by that number econds. If, then, T express the number of seconds during which body has been falling, and V the velocity which it has gained in time T, we shall have

$$V = 2\,T \times g.$$

The two preceding formulæ include the whole theory of falling ies in *vacuo*. *From these may be deduced* the following formula, by which the velocity which is acquired in falling through any given height is known: —

$$v^2 = 4h \times g.$$

$$\text{or, } v = 2\sqrt{h \times g}.$$

This formula, expressed in ordinary language, is as follows:—

RULE TO CALCULATE THE VELOCITY ACQUIRED IN FALLING THROUGH ANY GIVEN HEIGHT. — *Multiply the height expressed in feet by* $16\frac{1}{12}$, *extract the square root of the product, and multiply the result by two.*

It remains now to show, that by Attwood's machine the numbers given in the third column of the preceding table may be verified; that is to say, to demonstrate, by direct experiment, that the velocity imparted to the body in its descent increases in the direct proportion of the time of the fall.

To accomplish this, the following arrangements are made. The preponderating weight used to produce the descent of W has the form of a long narrow bar D (*fig.* 38.), which is laid across the upper surface of the cylindrical weight W. A ring E, long enough to allow the weight W to pass through it, but not long enough to allow the bar resting on this weight to pass, is attached to the scale at the division marked 1. If the weight be now brought to such a position that its upper surface shall coincide with the zero of the scale, and if it be let fall at a moment corresponding with one beat of the pendulum, its upper surface will arrive at the ring E at the moment of the next beat, and the ring which allows the weight W to pass freely through will catch the bar, which will rest upon it. After the top of the weight, therefore, has passed the ring, the weight W being relieved from the bar, by whose preponderance its motion was accelerated, will continue to move downwards without further acceleration, with the velocity it had acquired at the end of the first second, such velocity being now continued uniform. If, then, the descent of this weight uniformly downwards be compared with the beats of the pendulum, it will be found to move uniformly at the rate of two inches per second.

Thus, we infer, first, that the velocity imparted at the end of the first second is such as to make the weight W move uniformly in one second double the space through which it has fallen, and that such velocity is at the rate of two inches per second.

Let the ring be now moved to the fourth division of the scale, and the bar being put upon the weight W, let the experiment be repeated.

It will be found that at the end of two seconds the bar will strike the ring and the weight will pass below it, moving with a uniform velocity; and by comparing its motion along the scale with the beats of the pendulum, it will be found that this velocity is at the rate of four inches per second

[redacted]

Again, let the position of the ring be fixed at the ninth division of the scale, and replacing the bar, let the experiment be once more repeated. It will be found that the bar will strike the ring at the end of the third second, and that the weight W, when disengaged from the bar, will continue to descend with the uniform velocity of six inches per second.

The same experiment may be repeated for as many seconds as the height of the scale may admit, and like results will be obtained.

We may thus obtain a complete verification of the numbers contained in the third column of the preceding table.

249. *Calculation of the height from which a body falls in one second.*—From these experiments we are enabled to calculate the height through which a body would fall in one second of time by the effect of the force of gravity, and independently of any influence from the resistance of the air.

It appears from what has been stated, that when the magnitude of the weights W′, W, and *w* was so adjusted that the height of the descent was 193 times less than the height with which *w* would fall freely, the height through which it fell was one inch. It consequently follows, that if W were submitted to the unimpeded action of gravity, it would fall through 193 inches, or 16 ft. 1 in., in the first second.

250. *Method of calculating all the circumstances of the descent of a falling body.*—The table at page 113, compared with this result, will show all the circumstances attending the descent of bodies falling freely; 16 ft. 1 in. being the unit of the table. Thus, if we desire to ascertain the height from which a body would fall in five seconds, we take the number in the fourth column of the table opposite 5 seconds, which is 25, and multiply it by 16 ft. 1 in., the product, which is 402 ft. 1 in., will be the height required.

In the same manner, if it be required to determine what space a falling body would descend through in the fifth second of its motion, we take, in the second column of the table, the number opposite 5 seconds, which is 9; we multiply 16 ft. 1 in. by this number, and find the product, which is 144 ft. 9 in., which is the space required.

In like manner, if it be required to determine with what velocity a body would strike the ground after falling during an interval of five seconds, we take the number in the third column of the table opposite 5 seconds, which we find to be 10, and we multiply 16 ft. 1 in. by this number. The product, which is 160 ft. 10 in., will be the velocity required; and we infer that the body thus falling would have, when it strikes the ground, a velocity of 160 ft. 10 in. per second.

It will be observed that the numbers in the first column of the table now referred to, and which express the time of the fall, are the square roots of the numbers in the fourth column, which express the height from which the body falls. We have therefore this general principle of uniformly accelerated motion.

When a body is moved by a uniformly accelerating force, the times required to move through any given space are proportional to the square roots of those spaces.

By the aid of this rule, and the results already obtained, we are enabled to ascertain the time which a body would take to fall from any given height. Thus, if a body be supposed to fall from a height of 10,000 feet: Find the number of times which 16 ft. 1 in. are contained in 10,000 feet, which is done by dividing 10,000 by $16\frac{1}{12}$. The quotient is 621·76.

This number is then the square of the number of seconds in the time of the fall. The square root of this obtained from a table of square roots being 24·9, we infer that the time a body would take to fall through the height of 10,000 feet is 24·9 seconds.

In the same manner it follows, that since the velocity acquired by a body in its fall is proportional to the time of the fall, and since the time of the fall itself is proportional to the square root of the height, the velocity acquired must also be proportional to the square root of the height.

If we would, therefore, determine the velocity or force with which a body falling from a given height would strike the ground, independently of the effect of the resistance of the air, we are enabled to do so by these principles.

Thus, let it be required to determine the force with which a body falling from the height of 10,000 feet would strike the ground. It has been just shown that the time of the fall would be 24·9 seconds, and it has been already demonstrated that the velocity acquired by the body would move it uniformly over a space equal to double the height through which it falls, and in the same time. Therefore, the velocity in this case would be such that in 24·9 the body would move through 20,000 feet; and consequently, by dividing 20,000 by 24·9, we shall obtain the velocity in feet per second, which appears, therefore, to be 803·2 feet per second.

251. *Force with which a body falls is as the square root of the height* — It appears, therefore, that the velocity or force with which a falling body strikes the ground increases in a much less proportion than the height from which it falls. If the height be augmented in a fourfold proportion, the force of the fall will only be augmented in a twofold proportion; and if the height be augmented in a ninefold proportion, the force of the fall will only be augmented in a threefold proportion; and so on.

252 *Why a fall from great heights is not so destructive as might be expected.* — This explains a fact of not unfrequent occurrence, and which sometimes produces surprise.

Persons sometimes fall or leap from such heights as would seem

[redacted]

to render their destruction inevitable, yet they are frequently found to escape without considerable injury. This is explained by the fact that the momentum, or shock produced by the fall, increases in a proportion so very much less than the height. A man can leap from a height of five feet with perfect impunity; if, however, he leap from a height of ten feet, the force with which he will strike the ground, instead of being doubled, will be increased in a proportion less than one half; and if he leap from a height of twenty feet, the force with which he strikes the ground will be only doubled.

253. *Case of a person falling down a coal-pit.* — A further mitigation of the shock produced by a fall arises from the resistance of the air, which further diminishes the velocity acquired. A case occurred some years ago, in which a boy, dressed in a smock frock, accidentally fell down the shaft of a coal-pit having a depth of nearly 100 feet. It was expected that he would have been found dead at the bottom. On searching, however, he was found there almost uninjured. It is probable, that in this case, the frock he wore afforded a resistance to the air, somewhat resembling a parachute, which, combined with the principle already explained, that the velocity augments in a very much less proportion than the height, explained his safety.

254. *Retarded motion of bodies projected upwards.* — All the circumstances attending the accelerated descent of falling bodies, which have been explained in the present chapter, are exhibited in a reversed order when a body is projected upwards. Gravity then acts as a uniformly retarding, instead of uniformly accelerating force, depriving the body so projected of equal quantities of velocity in equal times; and further, it is apparent that the velocities which the force of gravity thus destroys in a body projected upwards in any given time are exactly equal to those which it would impart to a body in the same time when falling freely.

Thus, if a body be projected vertically upwards with the velocity which it would acquire in falling freely during one second, the body so projected will rise exactly to the height from which it would have fallen in one second, and at any point of its ascent it will have the velocity which it would have at the same point if it had descended.

To determine, therefore, the height to which a body will rise projected upwards with a given velocity, it is only necessary to determine the height from which a body would fall to acquire the same velocity.

In like manner, to determine the time which a body would take to rise to a certain height when projected upwards, it is only necessary to determine the time which a body would take to fall freely from the same height.

255. *Motion down an inclined plane.* — A plane and hard surface, which is neither in the vertical nor horizontal position, is called

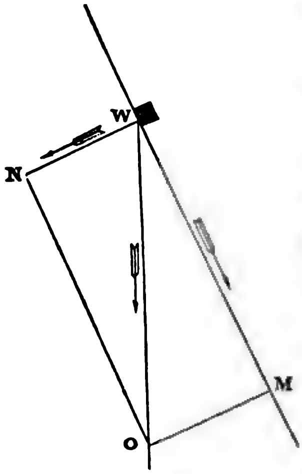

Fig. 39.

an inclined plane. In *fig.* 39., if the line W O be vertical, then W M will represent an inclined plane.

Bodies which descend upon inclined planes move with a uniformly accelerating force similar to that of gravity, omitting, as usual, the consideration of friction, and the resistance of the air.

Let W be a body placed upon the inclined plane. The force of gravity acts upon it in the vertical direction W O. Let this line W O, so representing the force of gravity, be considered as the diagonal of a parallelogram, of which W N and W M are sides; the side W N being perpendicular to W M. The entire force of gravity, therefore, represented by W O, and acting on the body W, will, by the principle of composition of forces, be equal to the two forces represented by the sides of the parallelogram W M and W N. But W N, being perpendicular to the plane, is counteracted by it, and exhibits itself merely in pressure upon it. The component W M, however, being in the direction of the plane and downwards, will cause the body to move down the plane.

The proportion of this accelerating force down the plane to that of gravity acting freely in the vertical direction, will, therefore, be that of the lines W M to W O.

If W O be the height through which a body would fall vertically in one second, then W M will be the distance through which the body would fall in the first second on the inclined plane.

It is evident, therefore, that by taking W O equal to 193 inches, the distance W M will be actually that down which the body W, independently of friction, &c., would fall in the first second.

If it be desired to ascertain the force with which the body W presses on the inclined plane, let W O be taken so as to consist of as many inches as there are pounds in the weight W.

Then W N will consist of as many inches as there are pounds in the pressure which W exerts on the plane.

256. *Motion on inclined plane uniformly accelerated.* — The motion down an inclined plane, therefore, being uniformly accelerated, like gravity, but only mitigated in its intensity in a certain ratio, depending on the inclination of the plane, all the circumstances which have been already explained in reference to the accelerated motion

[redacted]

of bodies falling freely, will be similarly exhibited in the motion down an inclined plane.

Let W M (*fig.* 40.) be an inclined plane, and W O the vertical line, and let us suppose two bodies dismissed at the same moment from W, one falling down the vertical line W O, and the other down the line W M. Let I, II, III, IV, V, be the points upon the vertical line W O, at which the body is found at the end of one, two, three, four, and five seconds.

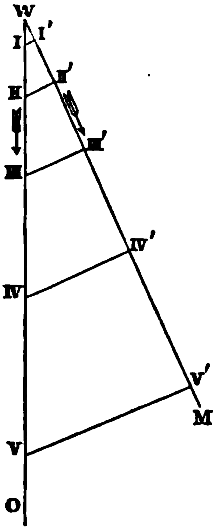

Fig. 40.

If from these points lines be drawn perpendicular to W M, the points I', II', III', IV', V', where these perpendiculars will meet the inclined plane, will be those at which the body falling down such inclined plane will be found at the same epochs; that is to say, at the end of the first second the one body will be found at I and the other at I', at the end of the second second the bodies will be found respectively at II and II', at the end of the third second at III and III', and so on.

The force down the inclined plane is just so much less in intensity than the force of gravity, as the spaces W I', W II', W III', &c. are respectively less than W I, W II, W III, &c. Consequently, it is evident that these spaces, being in the proportion of the forces, will be described in the same time, as, indeed, has been already proved.

In this manner, therefore, the circumstances of the motion down an inclined plane may always be determined with reference to the circumstances of the motion down a vertical line. If it be desired to ascertain the points at which a body falling down an inclined plane will acquire the same velocities which it acquired in one or more seconds in falling freely in the vertical direction, it is only necessary to consider that the more feeble force down the plane requires a proportionally greater space to produce a given velocity. If, then, W M and W O (*fig.* 41.) represent, as before, an inclined plane and a vertical line, and if, as before, I, II, III, IV, V represent the points at which the body, falling vertically, would be found at the end of one, two, three, four, and five seconds, then the points on the plane where the same velocity would be attained as the body had at the points, I, II, III, IV, and V, will be determined by drawing lines from the points, I, II, III, IV, and V respectively in the horizontal direction; because, by these means, the line W V on the plane will be just so much longer than the line W I as the force of gravity, acting freely, is more intense than the force down the in-

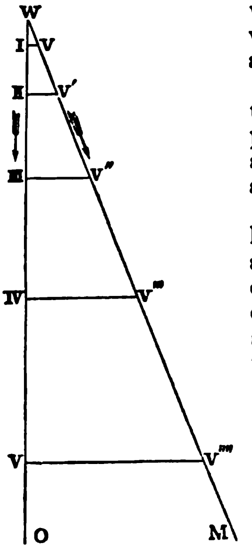

Fig. 41.

clined plane; consequently, the velocity which will be acquired at V on the plane, will be the same as the velocity acquired at I in falling freely.

In the same manner, it will appear that the velocities acquired on the plane at the points V', V'', V''', V'''', will be the same as the velocities acquired in falling freely at the points II, III, IV, and V.

257. *The motion of projectiles.*—We have considered the case where a body, acted on freely by the force of gravity, is either allowed to fall vertically downwards, or is projected vertically upwards. We shall now consider the other cases, in which a body is projected in any other direction, not vertical, and then abandoned to the action of gravity,—a problem which forms the foundation of the doctrine of projectiles.

The solution of this problem follows immediately from the principles which determine the motion of a body falling freely, as explained in the present chapter, and the composition of motion.

258. *Case of a projectile shot horizontally.*—Let us first take the case in which a body W (*fig.* 42.), as, for instance, a ball shot from a cannon, is projected in the horizontal direction W M.

If the force of gravity did not act on it, it would move forwards towards M with the velocity of projection continued uniform, and, in virtue of such motion, would pass over equal spaces in equal times. Thus if, by the velocity of projection, the body would move from W to I' in the first second, it would move from I' to II' in the next second, from II' to III' in the following second, and so on, these successive spaces being equal.

But if, on the other hand, the body, without being projected at all, were disengaged at W, and left to the action of gravity alone, it would, as has been already explained, descend vertically, and would be found at the points I, II, III, IV, V at the end of the successive seconds, the distance being, as already explained, represented by the numbers 1, 4, 9, &c.

Now, the body leaving W, being submitted to both these forces simultaneously, will, by the composition of motion, be found at the end of each successive second at the extremity of the diagonal of a parallelogram whose sides represent these motions. Thus, at the end of the first second, the body will be found at the point 1, being

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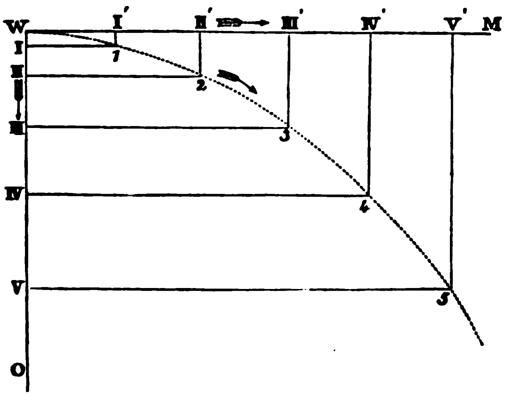

Fig. 42.

the extremity of the diagonal of a parallelogram whose sides are the space W I′, through which the body would move in virtue of the velocity of progression, and W I the space through which it would fall freely in the same time by gravity. If the force of gravity would have made it move over W I with a uniform motion, then the body, in moving from W, would follow exactly the diagonal of the parallelogram. But the force of gravity imparting to the body not a uniform, but an accelerated motion, first very slow and then more rapid, the body will pass from W to 1, not by a strict diagonal course, but by a curved line, as represented in the figure.

In the same manner, at the end of two seconds, the body will be found at 2. But it is actuated at the same time by two motions; first, the projectile motion, which, acting alone upon it, would carry it uniformly from W to II′; and, secondly, the force of gravity, which, acting alone upon it, would cause it to fall from W to II. At the end of two seconds it will therefore be found at the point 2, being the extremity of the diagonal.

But, as before, the motion from W to II not being uniform but accelerated, first slow but afterwards more rapid, the body will pass from W to 2, not along the diagonal, but over the curved line represented in the figure.

The same explanation will be applicable to its remaining course, and it will follow that the body will pursue the curved course from W to 5 in five seconds, in consequence of the combination of the projectile velocity imparted to it, and represented by W V′, combined with the descending motion imparted to it by gravity, and represented by W V.

259. *Case of oblique projection.*—In this case we have supposed,

for simplicity, the body to be projected in the horizontal direction; but the same principles will explain its motion, if projected in an oblique direction, such as W M (*fig.* 43.).

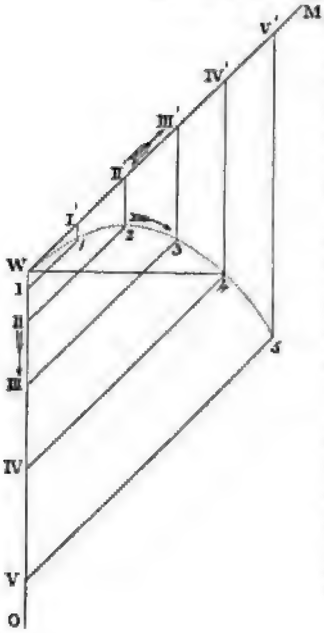

Fig. 43.

As before, let the space which the body would move over in one second, in virtue of the projectile force alone, gravity being supposed not to act upon it, be W I′. It would move over the equal spaces marked I′, II′, III′, IV′, V′, in the successive seconds.

On the other hand, suppose the body to be acted on by gravity alone, independently of the projectile force. It would then, as before, moving in the vertical line W O, be found at the end of the successive seconds at the points I, II, III, IV, V.

Now, by the principle of the composition of motion, the body will actually be found, in consequence of the simultaneous effects of the two motions imparted to it by gravity, and by the projectile force, at the end of the successive seconds, at the points 1, 2, 3, 4, 5, which are the extremities of the diagonals of parallelograms, whose sides are respectively the spaces which the body would describe in virtue of the projectile force, and of gravity acting separately. The course of the body will be the curved line represented in the figure, and not the straight diagonal, for the reasons already explained.

260. *Projectiles move in parabolic curves.*—The path which the projectile follows in this case is a curve, known in geometry as the *parabola*, the property of which is, that the sides of the parallelogram, whose diagonal determines its successive points, are related to each other as the successive whole numbers 1, 2, 3, 4, &c., and their squares 1, 4, 9, 16, &c.

261. *These conclusions modified by resistance of the air.*—It must, however, we repeat, be remembered, that these conclusions rest upon the supposition, that the body moves in a medium which offers no resistance to it, and which does not deprive it of any of the force imparted to it by projection or by gravity.

In the actual case, however, of all projectiles, the motion takes

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place through the atmosphere, which is a resisting medium, and, moreover, one of which the resistance varies, increasing in a certain high proportion with the velocity. The real path, therefore, of projectiles differs more or less from the parabola explained above. The deviation is not very considerable when the velocity of the moving body is not great; but when the projectile is driven with great velocity, as in the practice of gunnery, then the deviation from the parabolic path is so considerable, that the above theory becomes altogether inapplicable.

CHAP. VI.

CENTRE OF GRAVITY.

262. *Weight of a body is the aggregate of the weights of its molecules.*—If a body be prevented from moving in obedience to the force of gravity by a fixed axis passing through it, a fixed point from which it is suspended, or a surface placed beneath it, the effect of gravity upon it will be manifested by a pressure produced upon such axis, point of suspension, or surface.

This pressure is called the *weight* of the body.

As gravity acts separately upon all the component particles of a body, the weight of such body is composed of the aggregate of the weights of all its particles. This, which is manifest from what has been already explained, may be rendered still more clear, from considering that if a body be divided into parts, no matter how minute and numerous, each of these parts will have a certain weight, and the aggregate amount of their several weights will be exactly equal to the weight of the body of which they are the fragments.

Such a division may be carried to the most extreme practical limit of comminution by pounding, grinding, filing, and other processes known in the arts, and the weight will still be divided as the matter is divided; nor is it possible, even in imagination, to conceive any degree of comminution so great that the same principle will not prevail; and it may therefore be considered as established, that every individual atom which composes a body has weight, and that the weight of the mass is the sum of the weights of all its constituent atoms or molecules.

263. *Effect of cohesion on the gravity of the molecules.*—If the particles composing a body had no mutual coherence or other mechanical connection having a tendency to retain them in juxtaposition, each particle would obey the force of its gravity independently of the others, and they would fall asunder like a mass of sand. But if they be so connected by their mutual cohesion, as they are in fact in all

solid bodies, this cohesion will resist the tendency of their weights to separate them; they will maintain their juxtaposition, the body will retain its form, and the several forces with which gravity affects them will become compounded, so as to produce a single force or pressure, which is the resultant of all the separate forces impressed upon the particles.

264. ***Resultant of the gravitating forces of the molecules.*** — As this resultant enters as a condition into every mechanical question affecting bodies, it is of the greatest importance to investigate the conditions by which in every case its intensity and the line of its direction may be determined.

It has been already shown that the weights of all the particles composing a body act in directions parallel to a plumb-line, or perpendicular to a level surface. But it has been also demonstrated (158.) that when any number of forces act in the same direction in parallel lines, their resultant is a force acting in a line parallel to them, and in the same direction in this line, and that its intensity or quantity is equal to the sum of these forces.

The resultant, therefore, of the forces of gravity affecting all the particles of any mass of matter, is a single force acting vertically downwards, which is equal to the sum of all the forces affecting the particles severally, and therefore equal to the weight of the mass.

If, for example, A B, *fig.* 44., represent a mass of matter, and the small arrows pointing vertically downwards represent the direction of the gravitating forces of the particles composing such mass, then it follows, from what has been explained, that the resultant of all these forces, or a single force equal to them, will all have a direction parallel to them, such as D E, and will, in its intensity, be equal to their sum.

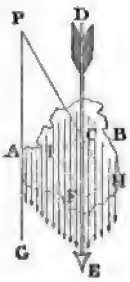

Fig. 44.

But this is not yet sufficient to indicate this resultant in a definite manner. We as yet only know that its direction is parallel to the common direction of the gravity of the particles; but innumerable lines may be imagined passing through the body vertically downwards, and the question still remains to be determined which of these lines is the direction of the resultant.

When the body in question has a determinate form and a uniform density, or even a density varying according to some known conditions, the principles of mathematical science supply methods by which the line of direction of the resultant may be determined; but we shall here adopt a more simple and generally intelligible method of explanation.

If we suppose the line represented by the great arrow D E (*fig.* 44.) to be that of the resultant, then it is evident that if any point such as C in that line be supported, the body will remain at rest, because

[redacted]

ınt of all the forces acting upon the body having the direc-
rill be expended in pressure on the fixed point c. The
refore, will be that the whole weight of the body will press
ıd the body will remain at rest.

ne would be true for any point whatever in the direction of
arrow. If, for example, D were a pin from which a thread
nded, and that this thread were attached to the body at any
the line D C, then the body would still remain at rest, the
.ght being expended in pressure upon the pin at D; for, as
e resultant of all the forces of gravity acting upon the com-
rticles of the body, would have the direction D E, and would
be supported by the fixed pin at D.

a point of support be selected which is not in the direction
sultant D E, such as P, and a string be carried from P to
of the body, such as c, then the body, although it will not
tted to descend vertically, in obedience to gravity, will not
ess remain at rest.

suppose the weight of the body to be expressed by the line
ıis line be taken as the the diagonal of a parallelogram whose
C H and C I, one in the direction of the cord, and the other
ıngles with it, — that portion of the weight which is repre-
y C H, and which is in the direction of the string, will act
fixed point P, and produce pressure upon it. The portion
ight which acts in the direction C I will move the body to-
e vertical line P G, which passes directly downwards from the
suspension. The body will therefore begin to move towards
ical line. If the body had been on the other side of the ver-
P G, it would still have moved towards it, and therefore in
on contrary to its present motion.

.ows, therefore, that if a body be supported by a fixed point,
t remain at rest unless the resultant D E of all the parallel
hich gravity impresses upon its particles pass through that

Experimental method of determining the resultant of the ting forces of the molecules. — We are thus supplied with a
l means of ascertaining the direction of the resultant of the
of all the component parts of a body with reference to any
oint taken upon it, as we have only to suspend the body by a
ttached to the given point, and allow it to settle itself at rest.
thus at rest, the resultant of the weights of all its particles
in the direction of the string by which it is suspended.

ıe same body be suspended by different points upon it, the
directions of the gravitating forces of its particles will differ
ence to the body, although they are the same in reference to
ction of the suspending string, being always parallel to it.

, for example, if an egg be suspended with its length vertical, the parallel forces which gravity impresses on its particles will be parallel to its length; but if it be suspended with its length horizontal, then the parallel directions of the gravity of its particles will be perpendicular to its length.

266. *There is a different resultant for every different point of suspension.* — Since in each case the resultant of these parallel forces will coincide with the direction of the string, it must in the one case pass through the egg in the direction of its length, and in the other case in a direction at right angles to its length.

In like manner, the body being supported by any point whatever taken upon it, the direction of the string will be different for each such point; and consequently, there will be an infinite variety of resultants of the gravitating forces of the particles of the body, according to the different points by which it may be suspended.

Now, a question arises, whether there is any relation between this infinite variety of resultants; for if such be not the case, the determination of the resultant of the gravitating forces of a body would be a problem which would present itself under an infinite diversity of forms and conditions for every individual body.

267. *All these resultants have a common point of intersection.* — This question may be solved by a very simple experiment, and its solution is attended with a remarkable and important result.

Take a solid body of any form, regular or irregular, and composed of a material which is easily perforated, without diminishing its mass, or considerably deranging its structure. Take, for example, a mass of putty of any form. Let this mass be suspended by a thread attached to a fixed point, which it may easily be, if previously surrounded by a thread forming a loop. When at rest, the resultant of the forces of gravity, acting upon all its particles, will be a vertical line penetrating its dimensions in the direction of the suspending thread. Take a needle, and pierce the putty in this direction. The hole which is thus made through it will represent the direction of the resultant of the gravitation of its particles.

Let the mass be now detached from the thread of suspension, and let it be again suspended, but in a different position, which may be easily accomplished by the loops of thread surrounding it.

The mass will again settle itself into a position of rest, and, as before the direction of the resultant of all its gravitating particles will be a vertical line in the exact direction of the suspending thread. Let the putty, as before, be thoroughly pierced in this direction with a needle.

Let the same experiment be repeated in three or four other different positions of the mass, so that we shall obtain several holes pierced through the body by the needle, representing the direction of the resultant of the gravitating forces, in the several positions in which the body was suspended.

[redacted]

Now a curious relation will be found to exist between the several directions in which the needle has pierced through the putty.

It will be found, in fact, that all these lines of perforation intersect, at a common point, within the dimensions of the body. This fact may be easily established.

Let a needle be inserted in any one of the perforations, and it will be found that another needle cannot pass through any of them, for its progress will be stopped by the needle already inserted. All the perforations, therefore, must intersect each other at a common point within the putty.

It appears from this experiment, that there is a certain point, within the dimensions of the body, through which the resultant of all the gravitating forces of the particles of the mass must pass, no matter in what position the body may be placed.

268. *Another experimental proof of this.*—This result, which is of high importance, may be further illustrated and verified in the following manner: —

Let a flat thin plate of metal, or a piece of card, of any form, however irregular, be pierced with small holes, at several points, so that it may be suspended upon a horizontal pin, the plate itself being vertical. When so suspended, it can only remain at rest, provided the resultant of the gravitating forces of its particles pass through the pin; for otherwise, as has been already explained, the body would move, in one direction or other, round the pin on which it is suspended.

If a plumb-line be suspended from this pin, it is evident that when the plate is at rest, the direction of the resultant of the gravitating forces must coincide with the direction of the plumb-line. Let a line then be traced upon the plate coinciding with the direction of the plumb-line.

Let the body be then detached from the pin, and let the pin be inserted in another hole. The body will now hang in another position, the resultant of the gravitating forces of its particles again coinciding with the plumb-line. Let the direction of the plumb-line be traced upon the plate as before. In fine, let this experiment be repeated, with all the holes pierced in the plate, and it will be found that the lines traced upon the plate, indicating the various directions of the resultant, of the gravitating forces of its particles, will intersect each other at one common point.

269. *This common point of intersection is called the centre of gravity.* — This common point, through which the resultants of the gravity of the particles of bodies pass, is called their *centre of gravity.*

A line drawn in the vertical direction, through the centre of gravity of a body, is called the *line of direction of the centre of gravity.*

270. *When the centre of gravity is supported, the body will remain at rest.* — If the centre of gravity of a body be supported on a

point, or axis, and the body is free to turn round such axis, the body will, in that case, remain at rest in any position in which it may be placed; for, according to what has been already stated, the resultant of the gravitating forces of all its particles must be in the direction of a vertical line passing through the centre of gravity, and the whole weight of the body may be considered as acting in that line. But, if the centre of gravity be suspended by a pivot, or an axis, then the whole weight of the body will press upon such pivot or axis, no matter what be the position in which the body is placed.

This may be easily verified by experiment.

Let the centre of gravity of any solid body be determined, by suspending it from different points, in the manner explained above, and let the body be placed upon a pivot or axis, passing through this point. It will be found to rest indifferently on such axis or pivot, in any position in which it may be placed.

This experiment may be easily performed with a piece of card or pasteboard. The centre of gravity being determined, let a pin be passed through it, and it will be found that the card will rest in any position upon the pin.

271. *When a body has a regular figure, its centre of magnitude is its centre of gravity.*—If a body, being of uniform density, have any regular figure, its centre of gravity will coincide with its centre of magnitude, for the matter composing the body will, in such case, be symmetrically arranged round that point; so that it is self-evident, that if this point be supported, the body will have no tendency to turn in any direction round it.

272. *Centre of gravity of a sphere.*—Thus, for example, it is evident, without experiment, that a ball or sphere of uniform density, such as a billiard-ball, has its centre of gravity at the centre of its magnitude. In like manner, a cube has its centre of gravity at the point where straight lines joining its opposite corners would intersect each other; that is to say, at its centre of magnitude.

273. *If a body have a symmetrical axis, the centre of gravity will be upon it.*—If the figure of a body be such, that the matte composing it is uniformly distributed round any line passing throug it, its centre of gravity must lie in that line, because, if it be suspended by a string in the direction of that line, it will remain a rest; since the gravity of its particles, acting equally on every sid of such line, will have no tendency to move it, it will equilibrate.

Thus, it is evident that the centre of gravity of a cone, being of uniform density, must be situate in its axis; that is, in a straight lin drawn from the point of the cone to the middle of its base.

In the same manner it may be shown, that the centre of gravit of solids of an oval figure will be in the axis of the oval; the cent *of gravity of* a cylinder will be at the middle point of its axis; th centre of gravity of a straight rod of uniform thickness will be at the middle point of its length, and at the centre of its thickness.

It will be easy, in this and all similar cases, to verify the conclusion, by suspending the body in the manner already described.

274. *Centre of gravity not always within the body.*—The centre of gravity of a body is not always placed within its dimensions. Thus, for example, the centre of gravity of a hoop is at its centre, an imaginary point, which does not constitute any part of the body in question.

In like manner, in all hollow bodies the centre of gravity is an imaginary point. Thus it is in the centre of a hollow sphere. The centre of gravity of an empty box or cask is within it, at an imaginary point.

If a piece of wire, which when straight has its centre of gravity at its middle point, be bent into a curved form, its centre of gravity will be an imaginary point within the concave part of the curve. In like manner, if the wire be bent into the form of a V, the centre of gravity will be an imaginary point within the angle of the V.

These conclusions may be verified, and the centre of gravity in all such cases found, by suspending the body in different positions in the manner already explained.

275. *Nevertheless it has the same properties.*—Although the centre of gravity in such cases be not a material point, and not included within the dimensions of the body, it nevertheless still possesses those properties which it would possess were it actually included within the mass of the body.

To verify this by experiment, let us suppose a bar of metal A B, *fig.* 45., bent into a curved form. Let its centre of gravity be determined by suspension. When supported by the point A, let A O be the direction of the plumb-line, and when supported by the point B, let B D be the direction of the plumb-line.

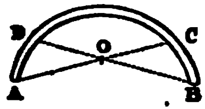

Fig. 45.

It follows, therefore, that the point O within the concavity of the circle where these two lines intersect, will be the centre of gravity. Let a light silk cord be attached to the points A and O, and stretched tight between them, and let another silk cord be stretched between the points B and D in the same manner. Now the point O, where these two cords cross each other, will be the centre of gravity.

Let a cord be tied to the junction of the strings at O, and let the upper extremity of this cord be attached to a fixed point, so that the wire may be thus suspended. It will be found that in this case, the hoop of wire will rest in equilibrium in any position in which it may be placed. In this case, the weight of the silk string, being insignificant in comparison with the weight of the wire, does not disturb the position of the centre of gravity, which still remains at O.

276. ***Centre of gravity takes the lowest position compatible with the conditions that affect the body.***—If a body, without being absolutely fixed in its position so as to be immoveable, be nevertheless partially restrained, so as to be capable of moving only under certain conditions, or within certain limits, then the centre of gravity will have always a tendency to move into the lowest position which the conditions under which the body is placed will admit of; and in all cases it can never remain at rest unless its line of direction, that is to say, a vertical line passing through it, should pass through a point of support.

277. ***Centre of gravity when at rest must be always above or below a point of support.***—It may therefore be assumed as a principle of the highest generality, that in all cases in which a body is at rest, a vertical line passing through its centre of gravity must also pass through a point of support. If the point of support, therefore, through which this line passes be placed above the centre of gravity, the body is said to be suspended; if it be placed below, it is said to be supported.

278. ***Centre of gravity when not supported oscillates.***—If a body be suspended from a fixed point by a string, it will remain at rest, as has been already explained, provided its centre of gravity be placed in a vertical line under the point of support. But if the body be drawn out of that position, so that the centre of gravity will be on either side of such vertical line, then the body when disengaged will fall from such position to the vertical line, and in consequence of its inertia will continue its motion beyond the vertical line until it comes to rest; it will then return to the vertical line, and thus oscillate from side to side.

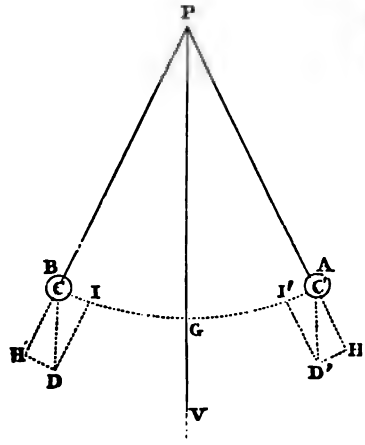

Fig. 46.

279. ***A pendulum.***—Such a body constitutes what is called the *pendulum*.

Let P, *fig.* 46., be the point of suspension. Let P B represent the string, and C the centre of gravity of the body. Let the weight of the body be represented by the vertical line C D. Let this be taken as the diagonal of a parallelogram, one of whose sides C H is in the direction of the string, and the other C I at right angles to it. The weight represented by the diagonal will thus, by the resolution of forces, be equal to two forces, one represented by C H and the other by C I. That which is represented by C H expends itself in pressure on the point of suspension; the other, represented by C I, will cause the body to move towards the vertical line P V, and in so moving the centre of gravity will describe the circular arc C G. When the centre of gravity arrives at G, it will be in the vertical line P V, passing through the point of suspension; and if the body were at rest it would remain there; but on arriving at G, the body has a certain velocity and moving force, which it will retain in virtue of its inertia, until deprived of it by some external agency. It will therefore continue to move to the right of G, and the centre of gravity will describe the circular arc G C′. In ascending this circular arc, the force of gravity has a tendency to destroy its velocity.

Let the weight of the body, as before, be represented by the vertical line C′ D′: it will be equivalent to the two forces represented by C′ H′ and C′ I′. The force C′ H′ is expended in pressure upon the point of suspension P; the other C′ I′ has a tendency to carry the centre of gravity C′ back to the point G, along the circular arc C′ G. This component of gravity, while the body moves from G to C′, gradually deprives it of its momentum, and if the momentum be entirely destroyed at the point C′, then this same component of gravity, C′ I′, will cause the body to return along the circular arc to the point G. In this manner the body would oscillate continually from side to side of the vertical line P V, the centre of gravity describing alternately equal arcs, G C and G C′. But the resistance of the air and other impediments have a tendency continually to diminish the length of the arcs, by which it departs from the vertical line, until at length the body loses its vibration and settles itself in such a position that the centre of gravity C will be quiescent in the vertical line P V.

280. *Conditions which determine the stability of a body.* — The stability of a body resting in any position is estimated by the magnitude of the force required to disturb and overturn it, and therefore will depend on the position of its centre of gravity with respect to the base.

If its position can be disturbed or deranged without raising its centre of gravity, then the slightest force will be sufficient to move it; but if its position cannot be changed without causing its centre of gravity to rise to a higher position, then a force will be necessary which would be sufficient to raise the entire weight of the body through the height to which its centre of gravity must be elevated; for, according to what has been already explained, the whole weight of the body may be considered concentrated at its centre of gravity.

281. *Stability of a pyramid.* — Let B A C, *fig.* 47., represent a pyramid, the centre of gravity of which is G. To turn this over the edge B, the centre of gravity must be carried over the arc G E, and

must therefore be raised through the height H E. If, however, the pyramid were taller relative to its base, as in *fig*. 48., the height H E, through which the centre of gravity would have to be elevated, would

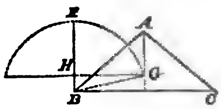

Fig. 47.

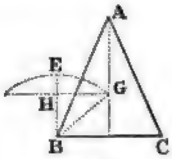

Fig. 48.

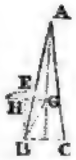

Fig. 49.

be proportionally less; and if the base were still smaller in reference to the height, as in *fig*. 49., the height H E would be still less, and so small, that a very slight force would throw the pyramid over the edge B. It is evident, from examining these diagrams, that the principle may be generalized, and that it may be stated that the stability of any body depends, other things being the same, upon the distance of the line of direction of its centre of gravity from the edges of its base. The nearer this direction is to one edge of the base, the more easily will the body be turned over this edge.

282. *Case in which the line of direction falls outside the base.*—If the line of direction of the centre of gravity fall outside the edge, as in *fig*. 50., then the weight of the body concentrated at G will be unsupported, and the body will fall over its edge.

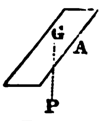

Fig. 50.

This will always take place, if the body be not attached to the ground at its base; but it happens, in some cases, that the body is so rooted to the ground at its base, that it will resist the tendency of its weight to make it fall, even though the line of direction of its centre of gravity should fall a little outside its base. Thus, we see trees not unfrequently leaning in such a position, that their centre of gravity obviously falls outside the limits of their trunk. Yet the trees nevertheless remain standing, the tenacity of the roots and their hold upon the soil being sufficient to resist the effect of their weight acting at the centre of gravity.

283. *Leaning towers of Pisa and Bologna.*—In the case of the celebrated leaning towers of Pisa and Bologna, although they are inclined considerably from the perpendicular, the lines of direction of their centres of gravity still fall within their bases.

The tower of Pisa is 315 feet high, and it is inclined so that if a plumb-line hang from the side towards which the inclination takes place, it will meet the ground at 12 ft. 4 in. from the base.

The tower of Bologna is 134 feet high, and a plumb-line similarly

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suspended would fall at 9 ft. 2 in. from the base. Nevertheless, these structures have stood, and will probably stand, as permanently as if they were erected in the true perpendicular, for the line of direction of their centre of gravity falls sufficiently within the base to render their overthrow impossible by any common force to which they will be exposed.

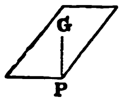

Fig. 51.

284. *Case in which the line of direction falls upon the edge of the base.* — If the line of direction of the centre of gravity, however, fall directly upon the edge, as in *fig.* 51., then the body will still stand, but it will be in a condition in which the slightest possible force will turn it over; as in this case it can be overturned without causing the centre of gravity to rise.

285. *Stability of a loaded vehicle.*—Hence appears the principle upon which the stability of loaded carriages or wagons depends. When the load is placed at a considerable elevation above the wheels, the centre of gravity is elevated, and the carriage becomes proportionally unstable. In coaches for the conveyance of passengers, the luggage is therefore very unsafely placed when collected on the roof, as is generally done. It would be more secure to pack the heavier luggage in the lower parts of the coach, placing light parcels on the top; for in such case the centre of gravity of the loaded vehicle would be in a lower position.

Drays for the conveyance of heavy loads are often constructed in such a manner, that the load would be placed below the axle of the wheels. If a wagon or cart, loaded in such a manner that its centre of gravity shall be in an elevated position, pass over an inclined road, so that the line of direction of the centre of gravity would fall outside the wheels, it would cause the vehicle to be overturned.

The same wagon will have a greater stability when loaded with a heavy substance which occupies a small space, such as metal, than when it carries the same weight of a lighter substance, such as hay, because the centre of gravity in the latter will be much more elevated.

286. *Stability of a table.*—If a large table be placed upon a single leg in its centre, it will be impracticable to make it stand firm; but if the pillar on which it rests terminate in a tripod, it will have the same stability as if it had three legs attached to the points directly over the places where the feet of the tripod rest.

287. *Stability of a body supported on several feet.* — When a solid body is supported by more points than one, it is not necessary for its stability that the line of direction should fall on one of these points. If there be only two points of support, the line of direction must fall between them. The body is in this case supported as effectually as if it rested on an edge coinciding with a straight line drawn from one point of support to the other. If there be three points of

support, which are not ranged in the same straight line, the body will be supported in the same manner as it would be by a base coinciding with the triangle formed by straight lines joining the three points of support. In the same manner, whatever be the number of points on which the body rests, its virtual base will be found by supposing straight lines drawn, joining the several points of support. When the line of direction falls within this base, the body will always stand firm; and otherwise not. The degree of stability is determined in the same manner as if the base were a continued surface.

288. *Gestures and motions of animals governed by the direction of the centre of gravity.*—All the attitudes, gestures, and movements of animals are governed with reference to the centre of gravity of their bodies. When a man stands, the line of direction of his weight must fall within the base formed by his feet. If A B, C D (*fig.* 52.) be the feet, this base is the space A B D C. It is evident that the more his toes are turned outwards, the more contracted the base will be in the direction E F, and the more liable he will be to fall backwards or forwards. Also, the closer his feet are together, the more contracted the base will be in the direction G H, and the more liable he will be to fall towards either side.

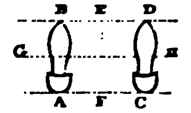

Fig. 52.

289. *Motion of the centre of gravity when a person walks.*—When a man walks, the legs are alternately lifted from the ground, and the centre of gravity is either unsupported, or thrown from the one side or the other. The body is also thrown a little forward, in order that the tendency of the centre of gravity to fall in the direction of the toes may assist the muscular action in propelling the body. This forward inclination of the body increases with the speed of the motion.

But for the flexibility of the knee-joints, the labor of walking would be much greater than it is, for the centre of gravity would be more elevated by each step. The line of motion of the centre of gravity in walking is represented by *fig.* 53., and deviates but little from a regular horizontal line, so that the elevation of the centre of gravity is subject to very slight variation.

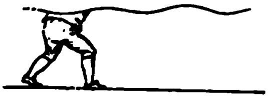
Fig. 53.

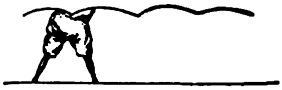
Fig. 54.

of knee-joint shown by the effect of wooden legs.—But knee-joint, as when a man has wooden legs, the

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centre of gravity would move as in *fig.* 54., so that at each step the weight of the body would be lifted through a more considerable height, and therefore the labour of walking would be much increased.

If a man stand on one leg, the line of direction of his weight must fall within the space on which his foot treads. The smallness of this space, compared with the height of the centre of gravity, accounts for the difficulty of this feat.

291. *Position of centre of gravity changes with every change of posture.*—The position of the centre of gravity of the body changes with the posture and position of the limbs. If the arm be extended from one side, the centre of gravity is brought nearer to that side than it was when the arm hung perpendicularly. When dancers, standing on one leg, extend the other at right angles to it, they must incline the body in the direction opposite to that in which the leg is extended, in order to bring the centre of gravity over the foot which supports them.

292. *Porter carrying a load.*—When a porter carries a load, his position must be regulated by the centre of gravity of his body and the load taken together. If he bore the load on his back, the line of direction would pass beyond his heels, and he would fall backwards. To bring the centre of gravity over his feet, he accordingly leans forward (*fig.* 55).

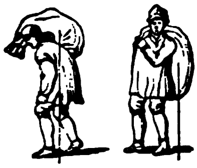

Fig. 55.

If a nurse carry a child in her arms, she leans back for a like reason.

When a load is carried on the head, the bearer stands upright, that the centre of gravity may be over his feet.

293. *Walking up or down a hill.*—In ascending a hill we appear to incline forward, and in descending to lean backward; but, in truth, we are standing upright with respect to a level plane. This is necessary, to keep the line of direction between the feet, as is evident from *fig.* 56.

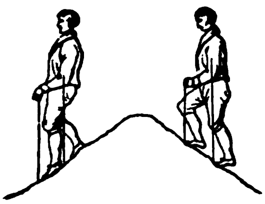

Fig. 56.

294. *Rising from a chair.*—A person sitting on a chair cannot rise from it without either stooping forward to bring the centre of gravity over the feet, or drawing back the feet to bring them under the centre of gravity.

If a person stand with his side close against a wall, his feet being close together, he will find it impracticable to raise the outside foot, for if he did, the line of direction of the centre of gravity of his body would fall outside the inner foot, and he would be unsupported.

295. *Case of quadrupeds.*—When a quadruped stands with his four feet on the ground, the centre of gravity of his body is over a point found by drawing the two diagonals of the quadrilateral formed by his feet; that is to say, if a line be drawn, joining his right fore foot with his left hind foot, and another joining his left fore foot with his right hind foot, then the centre of gravity of his body will be very nearly over the point where these lines cross each other. Strictly speaking, it will generally be a little nearer to his fore feet than this point. It will, however, still be very nearly on the centre of the quadrilateral base formed by his four feet; and, therefore, in a position to give complete stability to the animal.

When a quadruped walks, he raises his right fore and left hind foot (the former leaving the ground a little before the latter), the diagonal line joining his left fore foot and right hind foot supporting his weight. The centre of gravity of his body is a little in advance of this line; and his gravity, therefore, assists his forward motion. The left fore foot is raised a moment before the left hind foot is brought to the ground, and, in like manner, the right hind foot is raised immediately after the right fore foot comes to the ground. The effect of these motions is, that the weight of the animal is thrown alternately upon the two diagonal lines joining the right fore and left hind foot, and the left fore and right hind foot.

When a quadruped trots, he also raises his legs from the ground, alternately, by pairs, placed diagonally; but in this case the two feet leave the ground and return to it precisely together, and each pair springs from the ground a moment before the other pair returns to it, so that there are short intervals between the successive returns of the feet, by pairs, to the ground, during which the entire body is unsupported. The weight is, in these intervals, projected upwards by the spring of the legs, so that the centre of gravity of the body describes a succession of arcs concave towards the ground. It is this motion of the body which produces the action sustained by the rider of a horse in trotting.

When a quadruped gallops, he raises simultaneously his two fore legs, and by the muscular action of his hind legs he projects his weight forwards. During the spring the centre of gravity is unsupported, but is thrown forward, describing a circular arc, concave towards the ground.

When this arc has been completed, the fore legs reach the ground, and immediately afterwards the hind legs; and the centre of gravity is again momentarily supported, and the animal is in an attitude to repeat the same action.

296. *A cylinder rolled on a level plane.*—If a cylindrical body of uniform density be placed upon a horizontal plane, A B, *fig.* 57., its centre of gravity being its centre of magnitude C, the line of direction C P will necessarily meet the plane at the point where the

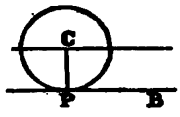

Fig. 57.

cylinder touches it, and the body will consequently remain at rest. If the cylinder be rolled upon the plane, the centre of gravity will be carried in a horizontal line, parallel to the plane represented by the dotted line in the figure. Since, therefore, in this motion the centre of gravity does not rise, any force applied to the body, however slight, will it to move, since no elevation of its weight is required. But, other hand, the body will have of itself no tendency to change sition, because the centre of gravity is not only supported, but se by no change of the body can it assume a lower position.

17. *An elliptic body on a level plane.* — If a board of uniform ity be cut into the form of an ellipse A B, *fig.* 58., and be placed upon a level surface D E, with the longer axis A B of the ellipse parallel to the surface D E, the centre of gravity C will then be vertically over the point P, at which the board touches the surface, and the body will be supported at rest. If the body be disturbed slightly from this position, the end A being depressed and the end B elevated, then the centre of gravity C will be elevated towards the point O; and if, on the other hand, the end B be depressed and the end A elevated, then the centre of gravity will be raised towards the point O′. In either case, this elevation of the centre of gravity will require the application of such a force as would be sufficient to raise the body through that height, whatever it be, through which the centre of gravity has been elevated; and if, after such elevation, the body be disengaged, and left to the free action of gravity, the centre of gravity will descend to the lowest possible position, that is to say, to the position represented in the figure, and will oscillate from side to side of this position until the vibrating motion be destroyed by the resistance of the air and by friction.

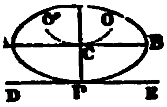

Fig. 58.

The centre of gravity will then rest in the position represented in the figure.

Let us now suppose that the same board is placed on the horizontal plane D E, with its longer axis vertical, as represented in *fig.* 59.

The line of direction C P of the centre of gravity will now pass through the point of support of the body, and consequently the board will be supported. But if in this case the body be slightly turned from its position to th right or to the left, the centre of gravity will descend t wards O or towards O′, and cannot resume the origin position at C until a force be applied to it which would sufficient to *raise the weight of the body* through the height to wh

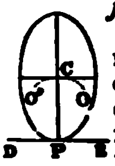

Fig. 59.

the centre of gravity has fallen. It is evident, therefore, that in this case, if the position of equilibrium C P be disturbed in the slightest degree by inclining the body a little either to the right or the left, the centre of gravity will move downwards, and the body will fall until it take the position represented in *fig.* 58., with its long axis horizontal.

298. ***Stable, unstable, and neutral equilibrium.*** — Now it will be observed, that in each of the three cases represented in *figs.* 57., 58., and 59., there is equilibrium, but this equilibrium is characterized in each case by particular conditions.

299. *Criterion of stable equilibrium.*—In the case represented in *fig.* 58., the equilibrium is called *stable*, because, if it be deranged either to the right or to the left, the body will of itself return to it, and settle definitively into it after some oscillation, the centre of gravity resuming its position over the point P. This position of stable equilibrium is determined by the condition, that no change of position can take place in the body without causing an elevation in the centre of gravity; or, what is the same, it is that position in which the centre of gravity is at the lowest point it is capable of assuming consistently with the conditions under which the body is placed.

The state of equilibrium represented in *fig.* 59. is called *unstable*, or tottering equilibrium. It is such a state of equilibrium, that if the slightest derangement takes place in the position of the centre of gravity, it will not return to the same point, but the body will assume another position, in which the centre of gravity will be in a state of stable equilibrium, as represented in *fig.* 58.

300. *Criterion of unstable equilibrium.* — Unstable equilibrium, then, is characterized by the quality that the centre of gravity is at the highest point which it can assume compatibly with the conditions in which the body is placed; and although it is vertically over the point of support, it is nevertheless in such a condition that the slightest derangement will cause it to descend, and the body to be overturned.

301. *Criterion of neutral equilibrium.* — The case represented in *fig.* 57. is an intermediate condition between these two extremes, and is called the state of neutral equilibrium. It is neither stable nor unstable. It is not stable, because the slightest force applied to the body will permanently change its position, it is not unstable, because the body will not be overturned by the action of its own weight, however its position may be changed.

302. EXAMPLE I. — *Children's toys.* — The effects of a variety of children's toys are explained by this principle. The centre of gravity is, by loading one extremity in a manner not perceptible to the eye, moved to a considerable distance from the centre of magnitude. The object, therefore, will only stand when the point which is the real centre of gravity is in the lowest position.

Thus, a figure made of some light substance, such as elder pith or cork, has a piece of lead attached to one of its extremities. If it be placed on the other extremity, the end being rounded, it will apparently, by a spontaneous movement, invert its position, and a sort of tumbler will be formed.

303. Example II. — *Feats of public exhibitors.* — Many of the feats exhibited by sleight-of-hand performers are explained by the principles of stable and unstable equilibrium. If any object, such as a sword, be supported on its point, it will be in unstable equilibrium so long as its centre of gravity is directly over its point; but as it cannot be maintained precisely so, the finger or other support of the point is moved slightly in one direction or another, so as to keep nearly under the centre of gravity, and to check the tendency of the sword to fall on the one side or on the other.

But these and similar feats are prodigiously facilitated if the object thus balanced is made to spin upon its point; for in that case the centre of gravity, though not vertically over the point of support, is continually revolving round a vertical line passing through the point of support, and the tendency which it has at one moment to make the body fall on one side is instantly checked by a contrary tendency when the revolving centre of gravity passes to the opposite side.

304. Example III. — *Spinning-top.* — It is in this manner that the common effect of a spinning-top is explained. It would be quite impracticable to make the top stand on its point if it did not revolve, or if it revolved very slowly; but if it have a very rapid motion of gyration, then it will stand steadily on its point.

It may be asked how the rapidity of the gyration affects the question. This is easily explained. If the centre of gravity revolve round the line so slowly that the time taken in half a revolution is so considerable as to allow it to fall to any considerable depth, then it cannot recover itself when it passes to the other side; but if the revolution be so rapid that half the time of one gyration is so small that the centre of gravity cannot fall through any sensible height, the top will maintain its position.

305. Example IV. — *Object spinning on point of a sword.* — Public exhibitors place a circular plate on the point of a sword, the point being placed as near the centre of the plate as possible. But, however near the centre it may be placed, it is not always possible to ensure its coincidence with the centre of gravity of the plate. If in this case the plate were at rest on the point of the sword, it would not be balanced, but would incline and fall on that side on which the centre of gravity would lie. The exhibitor, therefore, prevents this effect by giving to the plate a rapid motion on the point of the sword. The centre of gravity of the plate rapidly moves in a small circle round the point of support, and its tendency at one moment to fall down on one side, is checked the next moment by a contrary ten-

dency to fall down on the other side, and the plate accordingly spins rapidly on the point.

306. Example V. — *Cases in which centre of gravity seems to ascend.* — In some cases, the centre of gravity of a body apparently ascends; but this is always deceptive, and its real motion is invariably a descending one. Let a cylinder of wood, A B, *fig.* 60., be pierced by a hole O near its surface B, and let a cylinder of lead be inserted in this hole. The centre of gravity of the mass will then be, not at its centre of magnitude, but between that point and the centre of the cylinder of lead which fills the hole O, and will not be far removed from the centre of the lead, in consequence of the great comparative weight of that substance.

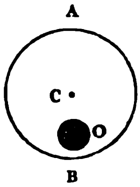

Fig. 60.

If such a cylinder as this be placed upon an inclined plane M N, *fig.* 61., in such a position that the line of direction O B of the centre of gravity shall fall above the point of contact P of the cylinder with the plane, the cylinder will roll up the plane, because its weight concentrated at the centre of gravity O, acting downwards in the line O B, will have a tendency to descend towards the plane, and will so descend, causing the body to roll towards N; and it will continue to descend until the cylinder take such a position that the line of direction of the centre of gravity O B shall pass through the point of contact of the cylinder with the plane.

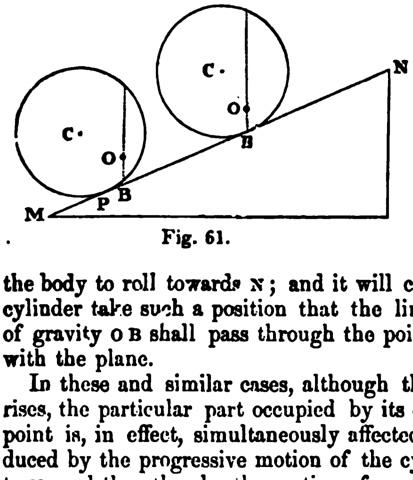

Fig. 61.

In these and similar cases, although the general mass of the body rises, the particular part occupied by its centre of gravity falls; that point is, in effect, simultaneously affected by two motions, one produced by the progressive motion of the cylinder up the plane from M to N, and the other by the motion of revolution of the cylinder round its centre C. In virtue of the former, the centre of gravity would rise; and in virtue of the latter, it would fall. The effect of the latter predominates until the centre of gravity comes into the vertical line, passing through the point of contact of the cylinder with the plane.

307. Example VI. — *Case of a globe rolled up an inclined plane by a person treading on it.* — A case of the ascent of the centre of gravity is sometimes produced by public exhibitors, the explanation of which may here be found instructive. The exhibitor

[redacted]

places a sphere of wood upon an inclined plane, and standing upon it, he places his feet on that side of the centre which is towards the elevation of the plane. Immediately the globe begins to roll up the plane, and the exhibitor, by moving his feet so as to keep them still near the highest point of the globe, but still on the side next the elevation of the plane, the globe continues to roll up the plane, the exhibitor dexterously maintaining his position as here described.

In this case there is a real ascent of the common centre of gravity of the globe and the body of the exhibitor. Now the question is, What force in this case produces this ascent? for it is evident that in the time during which it rolls to the top of the plane, the entire weight of the globe and the body of the exhibitor has been elevated through a perpendicular space equal to the height of the plane.

The force which accomplishes this is the muscular action of the feet of the exhibitor upon the surface of the globe. As the globe rolls up the plane, if the feet of the exhibitor pressed upon the same point of its surface, they would descend; and in that case the common centre of gravity of the globe and the body of the exhibitor, instead of ascending, would in fact descend, until the feet of the exhibitor would sink down to the surface of the plane; but this is prevented by the feet of the exhibitor continually stepping backwards upon the surface of the globe, so as to stand near the top; and thus, by moving his feet on the globe backwards continually towards the top of it, the exhibitor elevates the centre of gravity of his body, while the action of his feet upon the globe causing it to roll up the plane at the same time, raises the centre of gravity of the globe.

808. *Centre of gravity of fluids.* — In all that we have stated respecting the centre of gravity, we have supposed the body to be solid; but this quality also plays an important part in the phenomena of fluids. The centre of gravity of a fluid mass is determined by the same conditions as if it were solid. It is that point which would have the properties already defined, if the fluid mass were supposed to be congealed.

Thus the centre of gravity of the water forming a lake is that point which would have the properties already explained, if the water of the lake were converted into a mass of ice. It will appear hereafter, however, to possess, in reference to fluid bodies, many important characters.

809. *Centre of gravity of two separate bodies.* — The centre of gravity of two separate and independent bodies is that point between them which would possess the characters already defined, if the two bodies were united by a straight and inflexible rod which is itself devoid of weight.

This point may be determined by a very simple mathematical process. Let the centres of gravity of the two bodies in question be conceived to be connected by a straight line, and let a point be found upon this straight line which shall divide it into two parts, which shall be in the inverse proportion of the weights of the two bodies Then this point will be their common centre of gravity.

Thus let A and B, *fig.* 62., be the two bodies, and let *a*, *b* be their centres of gravity. Draw the line *a b*, and take upon it a point C, such that *b* C, shall bear to *a* C the same proportion as the weight of A bears to the weight of B. In this case, C will be the centre of gravity of the two bodies. Now if the line *a b* were a rigid rod devoid of gravity, the point C would have all the properties which have been already explained as belonging to the centre of gravity; thus, the bodies would balance themselves on C in any position.

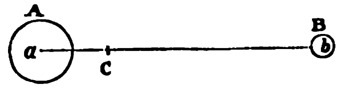

Fig. 62.

CHAP. VII.

CENTRIFUGAL FORCE.

810. *Force consequent on a rotatory motion.* — If a ball of metal or other heavy substance, placed upon a smooth and level surface, be attached to the extremity of a string, the other extremity of which is fastened to a fixed point upon the surface, and then whirled round in a circle, it is known, by universal and constant experience, that the string will be stretched with a certain force, which will be augmented as the velocity of the whirling motion is increased, or as the string is lengthened.

That such force is not produced by gravity is evident, inasmuch as the level surface upon which the body moves supports its weight.

811. *Centrifugal force.* — This force, which always attends matter that is moved round a centre, in what manner and under what form soever the motion be produced, is called *centrifugal force*, because it is manifested by a tendency of the matter which revolves to recede from the centre of revolution; this tendency in the case just mentioned being manifested by the force with which the string connecting the body with the fixed point is stretched. This tension resists the tendency of the ball to fly from the centre, and is therefore the measure of its centrifugal force.

812. *Centrifugal force a consequence of inertia.* — That centrifugal force is a mere effect of the inertia of matter, may be easily

It has been already explained, that in virtue of its inertia, if in motion, can only move uniformly in a straight line. If,

[redacted]

therefore, it be deflected from one straight line into another straight line, it must be by the action of some force impressed upon it at the moment of deflection; and if a body be continually deflected from a straight direction, which it must be if it move in a curve, then such body must be under the operation of a force continually acting upon it, producing such incessant change of direction.

Let P, *fig.* 63., be the fixed point to which the string is attached. Let A be the ball, and let A C F be the circle in which the ball is whirled round. Let A C be a small arc of this circle moved over in a given interval of time. Starting from A, the motion of the ball has the direction of the tangent A D to the circle, and it would move from A to D in the given interval of time, if it were not deflected from the rectilinear course; but it is deflected into the diagonal A C, and this diagonal, by the composition of forces, is equivalent to two forces represented by the sides A D, A B. But the motion A D is that which the body would have in virtue of its inertia; and therefore the force A B, directed towards the fixed point P, is that which is impressed upon it by the tension of the string, and which, combined with the motion A D, causes it to move in the diagonal A C.

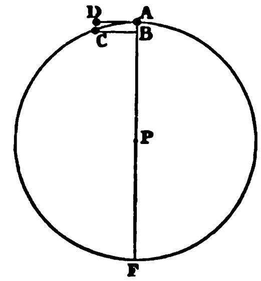

Fig. 63.

The tension of the string, therefore, is in fact a force directed to the centre P, which continually deflects the body from the tangent to the circle in which it has a constant tendency to move in virtue of its inertia.

313. *Method of calculating centrifugal force.* — It follows from the elementary principles of geometry, that the space A B, which is that which represents the force of the string upon the ball, or the centrifugal force, and which is in fact the space through which the body is moved in a given small interval of time by the tension of the string which measures the centrifugal force, is found by dividing the square of the number representing A C by the number representing the diameter A F of the circle, or twice the length of the string A P. But A C being the space described in a given time by the revolving body, is its velocity.

If we would then compare the centrifugal force of the body with its weight, we have only to compare the space which the body would be moved through by the centrifugal force acting alone upon it, with the space which gravity would move the same body through acting equally alone upon it.

Let us then express the physical quantities involved in this question as follows:

w = the weight of the revolving body.
c = its centrifugal force.
v = its velocity in feet per second.
R = length of string in feet.
g = 16$\frac{1}{12}$ feet, being the height through which w would fall freely in one second.

We shall have then the following proportions:—

$$w : c :: g : \frac{v^2}{2R};$$

and therefore we have

$$c = w \times \frac{v^2}{2R \times g}.$$

This formula, expressed in ordinary language, is as follows:

814. Rule to calculate centrifugal force, when the weight, velocity, and radius of rotation are given.—*The centrifugal force of a body revolving in a circle is found by multiplying its weight by the square of the number of feet which it moves through in a second, and dividing the product by the number of feet in the radius of the circle it describes, multiplied by 32⅙.*

But it is more convenient in practice to express the centrifugal force of a revolving body by reference to the number of revolutions it performs in a given time. Let us therefore express by N the number of revolutions, or fraction of a revolution, performed by the body in one second. The circumference of the circle which it describes, the length of the string being R, will be 6·283 R.

If, then, this be multiplied by N, we shall obtain the space through which the body moves in one second, or its velocity; and since the square of 6·283 is 39·476, we shall have

$$v^2 = 39{\cdot}476\, R^2 \times N^2;$$

and therefore we shall have the centrifugal force expressed by

$$c = 1{\cdot}227\, w \times R \times N^2.$$

In this formula it must be understood, however, that the length of the string must be expressed in feet or fractions of a foot, and that N must express the number of revolutions, or fraction of a revolution, made by the body in one second.

This formula, expressed in ordinary language, is as follows:

815. Rule to calculate centrifugal force, when weight, radius of rotation, and number of revolutions per second are given. — *To find the centrifugal force of a revolving body, multiply its weight by the number* 1·227. *Multiply this product by the number of feet in its distance from the centre round which it turns, and finally multiply this product by the square of the number of revolutions, or fraction of a revolution, which it makes round that centre in one second of time.*

[redacted]

Example. — Let it be required to find with what force a body weighing 2 lbs. would stretch a string 3 feet long, revolving four times per second. Multiply 2 lbs. by 1·227, and we obtain 2·454 lbs.; multiplying this by 3, the number of feet in the length of the string, we obtain 7.362. In fine, multiply this by 16, which is the square of 4, the number of revolutions per second, and we have 117·792. So that the centrifugal force with which the string is stretched would be $117\frac{4}{5}$ lbs. very nearly.

From the preceding conclusions it follows, that if two bodies of equal weights be whirled round their centres by strings or rods of the same length, their centrifugal forces will be in proportion to the squares of the number of revolutions which they perform in a given time. Thus, if one of them make three revolutions while the other makes two, the centrifugal force of the former will be to that of the latter as 9 to 4.

Again, if two bodies of equal weight are attached to centres by strings of different lengths, but perform the same number of revolutions in a given time, their centrifugal forces will be in proportion to the lengths of the strings. Thus, if one be attached by a string of two feet, and the other by a string of three feet, the centrifugal force of the former will be to the centrifugal force of the latter as 2 to 3.

In general, if two bodies of equal weight be at different distances from the centres round which they revolve, and also make a different number of revolutions in the same time, then their centrifugal forces will be as the products found by multiplying their distances from the centre by the squares of the number of revolutions which they make in the same time.

316. ***Application of whirling-table to illustrate experimentally these theorems.*** — These conclusions will be experimentally verified by an apparatus called a whirling-table, usually found in collections of philosophical apparatus.

A part of this instrument is represented in *fig.* 64, where C is a metallic bar, having two upright pieces *f′ f* at its ends, in which a polished metal rod is fixed parallel to C. On this rod a ball *g* slides, being perforated by a hole corresponding to the rod. At the centre C is a vertical frame-work, which contains a number of thin circular weights *h* placed one above the other, and supported on a sliding stage, which is capable of rising and falling. At the centre of the top of this stage, carrying the weights, is a hook, to which two strings are attached, which are carried over grooves in the pulley *k*, and then pass over corresponding grooves in the lower pulley *k′*, from which they are carried to the ball *g* to which they are attached

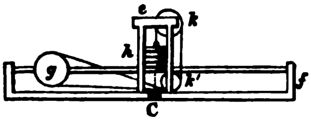

Fig. 64.

[illegible] with [illegible] the force necessary to [illegible] by varying the number of [illegible] the centre of its [illegible] a spindle, by which a [illegible] The apparatus is provided [illegible] like that represented [illegible] in connexion with any [illegible] which they make in a [illegible] By this apparatus, [illegible] can always be [illegible] centrifugal force is capable of [illegible] such that the centrifu- gal [illegible] the weights [illegible] but not to carry [illegible] When this takes place, the centrifu- gal [illegible]

[illegible] revolving bodies can be examined and [illegible]. By varying the distance of the weights [illegible] they revolve, the centrifugal forces [illegible] determined; and by varying [illegible] the effects of different angular motions, or of a different number of revolutions in a given time, can be ascertained. By this apparatus, therefore, the general principles which have been already established respecting centrifugal force can be verified.

[illegible]

1st. That when equal weights are at equal distances from the centre of revolution, their centrifugal forces will be proportional to the squares of the numbers of revolutions which they make in a given time.

2d. When they revolve in the same time, then the centrifugal forces will be in the direct ratio of their distances from the centre.

3d. When they are at different distances from the centre, and revolve in different times, then their centrifugal forces will be in the ratio of the products found by multiplying their distances from the centre by the squares of the numbers of revolutions which they make in a given time.

4th. But if the weights be unequal, then let the proportion of the centrifugal forces which they would have if they were equal be first und, and let the numbers expressing them be multiplied by the mbers expressing the weights; the product will then express the portion of the centrifugal forces.

These propositions involve the whole theory of centrifugal force.

81 [illegible] ntrifugal forces of bodies revolving in the same time [illegible] on centre of gravity are equal.*—If two bodies re- [illegible] mon centre in the same time, but at different dis- [illegible] ir centrifugal forces would be in the proportion of

[redacted]

e distances if they were equal, the body at the greater distance ng a greater proportional centrifugal force. But if the body at lesser distance be increased in weight, so as to exceed the other he same ratio as the distance of the other from the centre of rity is greater, then the centrifugal forces will be equal; for what centrifugal force of the lesser gains by distance, that of the ıter gains by weight. Thus, if one of the bodies weigh three ces, and the other five ounces, and if further the latter be at three ıes from the centre, the other being at five inches from it, they l have the same centrifugal force, provided they revolve in the ıe time. This, which is an important proposition, may be experintally proved by the whirling-table.

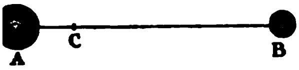

Fig. 65.-

Let A and B, *fig.* 65., be two bodies connected by a wire, and let a point C be taken upon this wire, in such a position that B C shall bear to A C the same proportion as the weight of A bears to the weight B; then let the wire be attached to the spindle of the whirlingəle at C, so that the balls shall be made to whirl round C. It will found that the wire will maintain its position, although free to ›ve in the direction of its own length, showing that the centrifugal ce exerted by the greater ball A upon the wire in the direction C A, equal to the centrifugal force exerted by the lesser ball B, upon the re in the direction C B. The lesser ball B, therefore, gains as much ntrifugal force by its greater radius B C, as the greater A gains by superior weight.

The point C, which divides the distance between the balls in the verse ratio of their weights, is, as has already been shown, their mmon centre of gravity; and it therefore follows, that if two bodies volve round their common centre of gravity in the same time, they ll exert equal centrifugal forces upon it.

318. *Examples of centrifugal force.*—Examples are presented of e effects of centrifugal force in almost all the motions which fall thin our daily observation.

319. Example I.—*Turning rapidly round a corner.*—A horsean, or a pedestrian passing round a corner, moves in a curve, and nsequently suffers a centrifugal force directed from the centre of e curve, which increases with his velocity, and which impresses on s body a force directed from the corner. He resists this force by clining his body towards the corner. An animal made to move in ring, as is customary in training horses, inclines his body towards e centre of the ring.

320. Example II.—*Horse moving round a circus.*—In all the uestrian feats exhibited in the circus, it will be observed, that not ly the horse, but *the rider, inclines his body* towards the centre,

and according as the speed of the horse round the ring is increased, this inclination becomes more considerable. When the horse walks slowly round a large ring, the inclination of his body is imperceptible; if he trot, there is a visible inclination inwards; and if he gallop, he inclines still more; and when urged to full speed, almost lies down upon his side, his feet acting against the partition which separates the circus from the adjacent parts of the theatre.

In all these cases, the facts are explained by considering that the centrifugal force and the weight of the horse are compounded together, and form a resultant which is directed upon the ground, and is represented by the pressure of the horse's foot.

321. *Method of calculating the inclination of the horse towards the centre.* — The actual amount of the centrifugal force, and the proportion which it bears to the weight of the animal, or other body which is moved in the circle, can be determined by the principles already explained, if the radius of the circle and the velocity of the body moving in it are known; and from these may be calculated the inclination which the body of the animal must assume in order to be whirled round the circle without falling outwards by the effect of the centrifugal force.

Let C (*fig.* 66.) be the centre of the ring round which the animal moves. Let C F be its radius, F being therefore the point at which the feet of the animal would act. Take the line F A, perpendicular to F C, and consisting of as many inches as there are pounds weight in the animal. Take A B parallel to F C, consisting of as many inches as there are pounds weight in the centrifugal force. Then F B will represent the inclination which the animal must assume in order to prevent it from falling either outwards or inwards. A less inclination than this would cause him to fall outward, and a greater inwards.

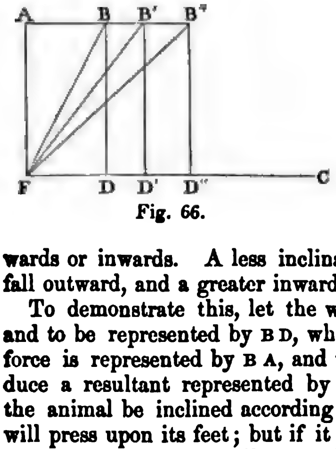

Fig. 66.

To demonstrate this, let the weight be conceived as acting at B, and to be represented by B D, which is equal to A F; the centrifugal force is represented by B A, and these two forces combined will produce a resultant represented by the diagonal B F. If the body of the animal be inclined according to this line B F, then the resultant will press upon its feet; but if it be inclined at a less angle, the resultant will cause it to fall outwards, and if at a greater angle it would cause it to fall inwards.

If the centrifugal force be increased, as will be the case if the animal moves with increased speed, then it would be represented by A B′, and the resultant of it, and of the weight, would be represented

[redacted]

y B′ F. Again, if the centrifugal force be further increased, and epresented by A B″, the weight being represented by B″ D″, the resultant of these will be represented by B″ F, which line must then be he inclination of the body of the animal. It is clear, then, that an ncrease of the centrifugal force, which arises from increased speed, ill cause the body of the animal to incline more and more towards he centre of the circle.

For example, if a horse move in a ring of 60 feet diameter, with speed of 15 feet per second, the ratio of the centrifugal force to his eight will be that of the square of 15, or 225, to 60 × 32¼, which s equal to 1930; the ratio, therefore, of the centrifugal force to the eight is 1 to 8½ very nearly. We shall therefore find the inclination corresponding to this, by taking A F (*fig.* 66.) equal to 8½ inches, nd A B equal to 1 inch: the line F B would represent the inclination f the horse.

322. Example III. — *Carriage turning a corner.* — A carriage ot having voluntary motion cannot make this compensation for the disturbing force which is called into existence by the gradual change of direction of the motion; consequently it will, under certain circumstances, be overturned, falling, of course outwards, or *from* the corner. If A B be the carriage, and C (*fig.* 67.) the place at which the weight is principally collected, this point C will be under the influence of two forces; the weight, which may be represented by the perpendicular C D; and the centrifugal force, which will be represented by a line C F, which shall have the same proportion to C D as the centrifugal force has to the eight. Now, the combined effect of these two forces will be the ame as the effect of a single force represented by C G. Thus, the pressure of the carriage on the road is brought nearer to the outer wheel B. If the centrifugal force bear the same proportion to the weight as C F (or D B), *fig.* 68., bears to C D, the whole pressure is thrown upon the wheel B.

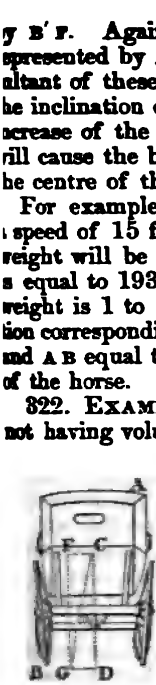

Fig. 67.

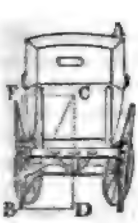

Fig. 68.

If the centrifugal force has to the weight a greater proportion than D B has to C D, then the line C F, which represents it (*fig.* 69.), will be greater than D B. The diagonal C G, which represents the combined effects of the weight and centrifugal force, will, in this case, pass outside the wheel B, and therefore this resultant will be unresisted. To perceive how far it will tend to overthrow the carriage, let the force C G be resolved into two; one in the direction of C B, and the other, C K, perpendicular to C B. The former, C B, will be resisted by the road, but the latter, C K, will tend to lift the carriage over *the external wheel. If the* velocity and the curvature of the course be continued for a sufficient time to enable this force C K to elevate the weights so that the line of direction shall fall on B, the carriage will be overthrown.

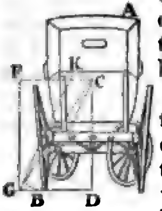

Fig. 69.

It is evident, from what has been now stated, that the chances of overthrow, under these circumstances, depend on the proportion of B D to C D, or, what is to the same purpose, of the distance between the wheels to the height of the principal seat of the load. It was shown in the last chapter, that there is a certain point, called the centre of gravity, at which the entire weight of the vehicle and its load may be conceived to be concentrated. This is the point which, in the present investigation, we have marked C. The security of the carriage, therefore, depends on the greatness of the distance between the wheels and the smallness of the elevation of the centre of gravity above the road; for either or both of these circumstances will increase the proportion of B D to C D.

323. Example IV. — *Stone in a sling.* — If a stone or other weight be placed in a sling which is whirled round by the hand in a direction perpendicular to the ground, the stone will not fall out of the sling, even when it is at the top of its circuit, and consequently has no support beneath it. The centrifugal force in this case acting from the hand, which is the centre of rotation, is greater than the weight of the body, and therefore prevents its fall.

324. Example V.—*Glass of water in a sling.*—In like manner, a glass of water may be whirled so rapidly, that even when the mouth of the glass is presented downwards, the water will still be retained in it by the centrifugal force.

325. Example VI.—*Bucket of water whirling.*—If a bucket of water be suspended by a number of threads, and these threads be twisted by turning round the bucket many times in the same direction, on allowing the cords to untwist the bucket will be whirled rapidly round, and the water will be observed to rise on its sides and sink at its centre, owing to the centrifugal force with which it is driven from the centre. This effect might be carried so far that all the water would flow over, and leave the bucket empty.

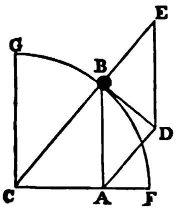

Fig. 70.

326. *Case of a body moving down a convex surface.* — If a body B (*fig.* 70.) move down a curved surface G F, whose centre is at C, it may acquire such a velocity that the centrifugal force will cause it to leave the surface, and to be projected forwards to the ground. The conditions under which this would take place are easily explained.

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ıe weight of the body be expressed by B A, and its centrifugal B E, and let the parallelogram be completed. Then the dia- D will be the resultant of these two forces, and will be the rhich the body has a tendency to move. So long as this dia- ırms an acute angle with B C, the body will remain on the ırface; but so soon as it forms a right angle with B C, then ecome a tangent to the surface and the body will fall off.

Centrifugal forces of a solid body revolving on a fixed axis. most important classes of problems in mechanical science in he principles determining the centrifugal force are practically , are those which relate to solid bodies revolving on an axis. lid body be pierced by a straight and round hole, in which a ıcal rod is inserted, and the body be made to turn rapidly this rod as an axis, each particle of matter composing the ill revolve in a circle round such axis; all these circles will :ribed in the same time, and consequently the centrifugal)f the particles exerted upon the axis will be in proportion to istances from the axis, such distances being the radii of the which they describe respectively round it.

the particles of the body which are at the same distance from is will therefore exert equal centrifugal forces upon it, and rhich are at greater distances will exert centrifugal forces pro- ıally greater than those at less distances. The particles dis- d round the axis will produce forces directed from the axis in ection of perpendiculars connecting them with the axes respec-

r it is evident that as many different forces will thus be exerted he axis as there are different particles of matter composing the f the revolving body.

r cases are presented which may arise in the combination of 'orces.

. *First case, in which the centrifugal forces are in equili-* —They may be in equilibrium; that is to say, the centrifugal exerted by all the particles composing the body on the axis ıeutralize each other. In this case the axis would suffer no in consequence of the centrifugal forces, and the body would)und it without producing any effect upon it.

ıuch a case, if the axis were withdrawn from the hole in which ıserted, the body would still continue to spin as before, because, the axis suffered no pressure or strain from the revolving mat- ı presence or absence can make no difference in the motion.

). *Second case, in which they have a single resultant.*—The 'ugal forces produced by the particles of the revolving mass e such as to be represented by a single force applied at some of the axis, and at right angles to it. In this case the axis uffer a *corresponding pressure or strain* at this point. If this point could be fixed, the remainder of the axis might in that case be withdrawn, because, since no other point of it suffers any strain from the effect of the motion, its presence or absence can produce no difference in the motion.

330. *Third case, in which they are equivalent to a couple.*—The centrifugal forces may be such that their combined effect cannot be represented by a single force, but may be represented by two equal or parallel forces acting on two different points of the axis, and in contrary directions. This combination is what has been already called a *couple*, and its effect is to twist the axis round some point intermediate between the two contrary forces. In this case the axis could not be withdrawn unless two fixed points were provided, representing the points to which the opposite forces of the couple are applied.

331. *Fourth case, in which they have the effect of a single force and a couple combined.*—The combination of forces produced by the revolving matter may be such as to be incapable of being represented either by a single force or by a couple. In this case it can be proved that their combined effects will be represented by a single force and a couple taken together. The effect, in such a case, is a pressure upon the axis at right angles to its length at the point where the single force is placed, and a tension or twist produced at the same time by the couple.

These are all the possible cases which can be presented by a solid body revolving on a fixed axis.

332. *Examples of the application of these principles.*—Their complete analysis and demonstration would require the use of the principles and formulæ of the higher parts of mathematical science, the introduction of which would not be suitable to the purpose of the present treatise. We shall therefore limit ourselves to some examples which will convey a sufficiently clear notion of the general effects produced by the rotation of solid bodies on fixed axes.

333. Example I.—*A ring revolving round its centre in its own plane.*—If a series of particles of matter placed in the circumference of a circle are made to revolve by a common motion round an axis, passing through such circle and perpendicular to its plane, their centrifugal forces will be evidently in equilibrium, and no pressure on the axis will be produced. A circular series of such particles is represented in *fig.* 71.: the radii represent the direction of the centrifugal forces, which are all equal, because the particles are equal and the distances from the centre are equal.

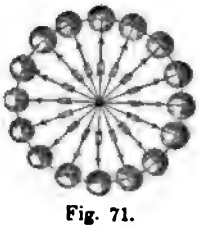

Fig. 71.

It is evident on inspection that these forces equilibrate round the centre, and that the central point, therefore, would suffer no pressure in one direction rather than in another.

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334. Example II.—*A flat circular plate.*—A flat circular plate of uniform thickness and density may be considered as consisting of a series of concentrical rings of such particles. If such a plate be revolved round an axis passing through its centre, and perpendicular to its plane, the centrifugal forces of the particles will be in equilibrium, and no pressure will be produced on the axis.

335. Example III.—*A cylinder revolving round its geometrical axis.*—A cylinder may be considered as composed of a number of such circular plates placed one upon the other, and the axis of the cylinder will be the line formed by the centres of these plates. If such a cylinder, therefore, revolve round its axis, the centrifugal force of its mass must be in equilibrium, because each separate plate being in equilibrium, the entire pile would necessarily also be in equilibrium.

336. Example IV. — *A solid of revolution revolving round its geometrical axis.*—There is an extensive and important class of solid bodies having a geometrical form, which gives to their axes this property. They are called *solids of revolution.*

Let A B D, *fig.* 72., be a triangle with a right angle at A. If this be supposed to revolve round the side A B as an axis, it will by its revolution generate a solid called a cone; that is to say, the space through which it would pass as it revolves, and which it would include within its sides in revolving if filled by any solid matter, would form a cone.

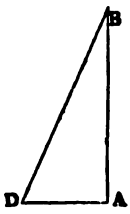

Fig. 72.

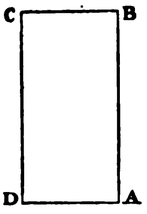

Fig. 73.

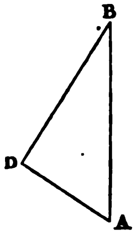

Fig. 74.

If a right-angled parallelogram, *fig.* 73., revolve round its side A B, it would generate in the same way a cylinder.

If a triangle A B D, *fig.* 74., revolve round a side A B, not adjacent to a right angle, it would generate the double cone.

If a semicircle (*fig.* 75.) revolve round its diameter A B, it would generate a sphere or globe.

If a semi-ellipse (*fig.* 76.) revolve round its shorter axis A B, it would generate an oblate spheroid, being a figure resembling an orange or a turnip.

If a semi-ellipse (*fig.* 77.) revolve round its longer axis, it would generate a prolate spheroid, being a figure resembling an egg, only that its ends are similar.

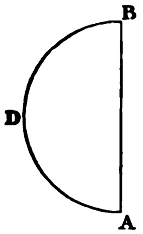

Fig. 75.

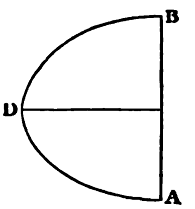

Fig. 76.

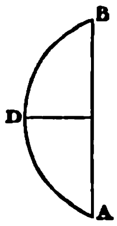

Fig. 77.

Now it is evident, that in these and all solid bodies whose figures are determined by the same principle, all sections made by planes perpendicular to their axes of revolution are circles through the centre of which such axes pass. Since the centrifugal forces of the particles of matter composing each of such sections are in equilibrium, the centrifugal force of the entire mass of the body is necessarily also in equilibrium, such mass being composed of these several sections. The body, in short, may be considered to be made up of a number of circular plates laid one upon another, varying in their diameter according to the form of the body.

If a solid of revolution, therefore, be made to revolve upon its geometrical axis, the centrifugal forces of its mass will be in equilibrium, and no pressure or strain whatever will take place upon its axis.

337. *Case of solids which have a symmetrical axis.* — The same reasoning will be applicable to all solids which have an axis round which the particles of such section made by a perpendicular plane are so arranged, that every particle of matter at one side has a corresponding particle at an equal distance at the other side; for in this case, every pair of such equidistant particles will exert equal and opposite centrifugal forces on the axis.

A great number of solid bodies, including the class of solids of revolution described above, fulfil this condition.

If the sections of a solid be all equal, and have any regular geometrical figure having a point within it forming its geometrical centre, and through which all lines drawn are bisected, then such a solid participates in the above property, and the centrifugal forces of its mass when revolving round such an axis will neutralize each other.

338. *Case of a rectangular prism.*—A column formed by laying one upon another equal flat plates of the same figure has this property. Such a column is called, in geometry, a rectangular prism.

339. *Case of a pyramid.* — A pyramid formed by laying upon each other similar plates not equal, but diminishing gradually in magnitude will have the same property.

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llows, therefore, that the axes of the rectangular prisms and ls whose bases have a centre of magnitude, enjoy the above y.

Case of axis of revolution passing through the centre of .—It will be observed, that the axis round which the centri-rce equilibrates in the examples given above, will pass through re of gravity of the bodies in question. It may therefore be whether such property belongs to all lines whatever passing the centre of gravity of a body; that is to say, whether, if is made to revolve upon an axis passing through its centre of the centrifugal force will be in equilibrium, and whether is will be free from strain or pressure.

easy to show that this will not be the case in general, and is only certain lines passing through the centre of gravity ave the property above mentioned.

rectangular plate having unequal sides, A B and B C, *fig.* 78., e to revolve round a line M N, passing through the middle points of the opposite sides A B and D C, the centrifugal forces will be in equilibrium, because every point on the one side of M N has a corresponding point at an equal distance on the other side.

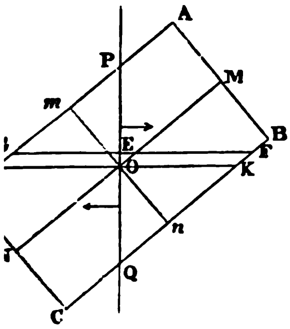

Fig. 78.

In like manner, the line *m n* passing through the middle points of the sides A D and B C, would enjoy a like property.

But if any other line, such as P Q, be drawn through the centre of gravity O, and the plate be made to revolve round such en it will be evident that the centrifugal force will not be in rium. For through a point such as E, let a line F G be drawn licular to the axis P Q. This line F G will be divided into un-arts at E, and the centrifugal force produced by the particles n E and F will be greater than the centrifugal forces produced particles between E and G. The same will be true of all rawn perpendicular to O P; and the combined effect of all the ugal forces acting in that part of the axis between O and P, to produce a greater strain on the side of the angle A than on e of the angle D, and the centrifugal force will have a resultant l towards the side of the angle A. By the same reasoning it may wn that the centrifugal forces of that part of the plate which is K L, will have a resultant directed to the side of the angle C, and this resultant will be equal to the resultant of the centrifugal forces above K L, directed to the side of the angle A.

It follows, therefore, that the axis P Q round which the plate revolves will be affected by forces having two resultants parallel to each other and in opposite directions, one perpendicular to O P, and directed towards the side A, and the other perpendicular to O Q, and directed towards the side C.

These two forces form a couple, and have a tendency to turn the axis of revolution towards the position *m n*, as represented by the arrows in the diagram.

341. ***Centrifugal forces round such an axis are either in equilibrium or are equivalent to a couple.***—This property is general, although it cannot be demonstrated in all its universality without the aid of the language and principles of the higher analysis.

It may be stated thus:—

In all bodies whatever, there are three lines passing through the centre of gravity which are at right angles to each other, each of which is so placed, in reference to the mass of the body, that the centrifugal forces produced by the revolution of the body round them respectively will be in equilibrium; and such lines, when the body revolves round them, will suffer no strain or pressure.

But if the body be made to revolve round any other line passing through the centre of gravity, the centrifugal force will produce a strain, which will be represented by two equal opposite and parallel forces acting upon the axis at opposite sides of the centre of gravity, and having a tendency to turn the axis in the position of one or other of the three axes of equilibrium here mentioned.

342. ***Axes of centrifugal equilibrium called principal axes.***—An axis round which the centrifugal force equilibrates is called a ***principal axis;*** and from what has been explained, it appears that there are three principal axes through the centre of gravity at right angles to each other.

343. ***Case in which all lines through centre of gravity are principal axes.***—There are some particular cases in which every line passing through the centre of gravity is a principal axis; such, for example, is the case with a sphere or globe of uniform density. Such a solid, whatever diameter it may revolve round, will be a solid of revolution, and the sections perpendicular to such diameter will be circles.

The same principle is true of all the regular solids. All lines, for example, passing through the centre of a cube are principal axes.

344. ***Case of an axis of revolution parallel to a principal axis through the centre of gravity.***—If a solid be made to revolve round an axis which does not pass through the centre of gravity, but which is parallel to one or other of the principal axes passing through that point, the centrifugal forces will not equilibrate round such axis, but

they will be represented by a single force perpendicular to it, and passing through the centre of gravity.

345. *Such axis called also a principal axis.* — Such an axis is called a principal axis, and in general there are three such axes corresponding with such points taken in the body, which are parallel respectively to the principal axis passing through the centre of gravity.

346. *Three principal axes at right angles to each other pass through each point.* — It may therefore be stated, in general, that if any point in a body be taken different from the centre of gravity, there are three lines passing through it at right angles to each other, round each of which, if the body is made to revolve, an effect will be produced by the centrifugal forces which can be represented by a single force, perpendicular to the circumference, and passing through the centre of gravity.

347. *Effect of revolution round a line which is not a principal axis.* — If the body be made to revolve round any line passing through a given point in it which is not a principal axis in the sense just referred to, then the centrifugal forces produced by such revolution cannot be represented either by a single force or by a pair of equal and opposite parallel forces, but will be represented by both of these together. This, therefore, is the character of all axes, round which a body would move, which do not pass through the centre of gravity, and are not parallel to either of the principal axes passing through that point.

348. *Experimental illustrations of these properties.* — Many of these important properties admit of being experimentally illustrated in a very striking manner by a simple apparatus.

Let a body be suspended by a thread, or, better still, by a skein composed of several threads, from a fixed point. Let this fixed point be so arranged, that a rapid rotatory motion may be given to it. This rotatory motion will soon be imparted to the body suspended to the threads by the twisting of the skein, and, after a time, the rotation of the body will become extremely rapid. It will first take place round the line, passing vertically through it in the direction of the skein, when it hangs quiescent. If this line happen to be one of the principal axes passing through its centre of gravity, it will continue to revolve round it; but if it be not one of these principal axes, the centrifugal force, as has been already explained, will produce an effect upon it, represented by two equal, opposite, and parallel forces tending to twist it. This effect will be soon rendered manifest in a remarkable manner: the body, when it revolves rapidly, will not continue in the same position which it had when it was quiescent. The line which was in the direction of the string will begin to take another direction, the point where the string is attached to it will not remain in its first position, and the body will throw itself into a position more or less at right angles to the string, and, in a word, will at length, after the revolving motion has become sufficiently rapid and continued, assume such a position, that a principal axis, passing through its centre of gravity, will become vertical, and the body will spin round it.

It is evident, therefore, that this effect takes place in spite of the opposing influence of the gravity of the body; for in the new position which the body assumes, its centre of gravity ceases to be represented by the skein. This experiment may be varied in a great variety of ways, exhibiting most instructive and amusing effects.

If a metallic ring be suspended by the skein by being attached to a point in its side, the ring when quiescent will hang with its plane vertical; but when the rotation becomes rapid, the ring will throw itself into a horizontal position, and will spin round a vertical axis through its centre, and perpendicular to its plane.

If the experiment be made with an oblate spheroid suspended by a point P (*fig.* 79.) in its equator P Q, its shorter axis A B being horizontal, it will, when it acquires a rapid motion, take the position represented in *fig.* 80., in which the shorter axis A B is vertical, and the equator P Q horizontal, and it will spin in this position round the principal axis A B, although the centre of gravity C is unsupported by the string.

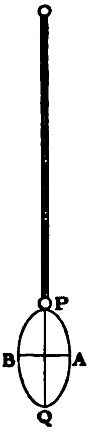

Fig. 79.

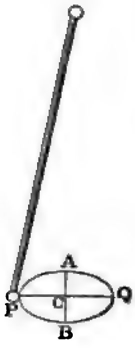

Fig. 80.

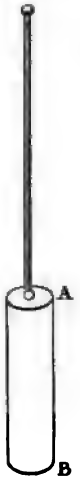

Fig. 81.

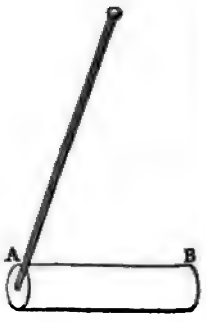

Fig. 82.

If a cylindrical rod A B (*fig.* 81.) be suspended by a point at the centre of one of its ends, and a rapid revolution be imparted to it, it will not continue in this position, but will assume the position represented in *fig.* 82., in which its length will be horizontal, and it will

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revolve round an axis passing through its centre, and at right angles to the length.

If a metallic chain, the ends of which are united, as in a necklace or bracelet, be suspended from any point, and put in rapid revolution, the chain will at first make some irregular gyrations, but after a time it will gradually open, and settle itself into the form of a precise circle, the plane of which will be horizontal, and the string will then be attached to a point in this circle. In short, the chain will assume the form of a solid ring in a horizontal plane.

CHAP. VIII.

MOLECULAR FORCES.

849. *The pores of bodies the region of molecular forces.*—It has been demonstrated in a former chapter, that the space included within the external surface of a body is not all occupied by the matter composing that body. We have seen that bodies, however dense or ponderous, are capable of being diminished in their volume by compression, or by diminution of temperature. It follows, therefore, that the component particles forming the mass of a body of a uniform density are uniformly distributed throughout its volume, each particle being separated from those around it by a space of greater or less extent unoccupied by matter.

These interstitial spaces are the regions which form the theatre of action of those important physical agents called molecular forces.

A multitude of phenomena, familiar to all observers, show that between the particles which compose the mass of a body, there exist attractive or repulsive forces, the sphere of whose action is in general limited to distances imperceptible to the senses, and which only admit of being proved by indirect means.

850. *Attraction of cohesion.* — The qualities of solidity and hardness, and, in general, those properties by which a body resists fracture or flexure, or any other derangement of its form, arise from the energy with which its component particles attract each other, and resist any force which tends to separate them.

Molecular attraction manifested in this manner is called the attraction of cohesion.

351. *Attraction of adhesion.* — If the surfaces of two bodies be brought into very close contact, it is found that they cannot be separated without the exertion of some force of greater or less intensity, according to the circumstances of the contact.

Molecular force manifested in this manner is called the attraction of adhesion.

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352. *Capillary attraction.* — Certain bodies being placed in contact with a fluid, the fluid will enter their dimensions and occupy their pores; as, for example, when a sponge or a lump of sugar is brought in contact with water.

The fluid in these cases rises in opposition to its gravity, and fills all the interstices of the sponge or the sugar.

Molecular force manifested in this manner is called capillary attraction.*

353. *Chemical affinity.* — When two bodies of different kinds are mixed together, their constituent particles will in certain cases unite, and form by their combination the constituent particles of a compound, differing in its sensible qualities from either of the components.

For example, if two gases called oxygen and hydrogen be mixed together in a certain proportion, and a light be applied to the mixture, an explosion will take place; the atoms of the two gases will unite one with another, and the entire mass will be converted into water, the weight of the water being exactly equal to the sum of the weights of the two gases.

In this case, each atom of the oxygen is attracted by an atom of hydrogen, and their combination forms an atom of water.

Molecular attraction manifested in this manner is called chemical attraction, or chemical affinity.

354. *The atoms of bodies manifest both attraction and repulsion.* — It has been shown that all bodies submitted to the action of mechanical forces of sufficient energy, are capable of being compressed and diminished in their volume. By such means, therefore, their component particles are forced into closer proximity.

But all of these resist such compression with a certain force, and most of them have a tendency to recover the volume which they had before compression.

This general fact indicates the existence of another force, contrary in its direction to the attraction of cohesion, the sphere of whose action is within that of the latter attraction. To explain this phenomenon, we are compelled to suppose that each atom composing a body is surrounded with a sphere of repulsion within which adjacent atoms cannot enter unless urged by a certain force.

But outside this sphere of repulsion there exists the sphere of attraction, by which such atoms attract all the surrounding atoms, which gives the character of solidity and hardness to the mass.

355. *Cohesion manifested in solids and liquids.* — The attraction of cohesion is manifested in solid bodies by the force which is neces-

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* So called from *capillus*, a hair; the magnitude of the pores being in this case estimated at about the thickness of a hair.

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to derange their form by fracture or flexure, or by any other ge of figure.

ıe same force is manifested in liquids in their tendency to form spherical drops, a globe being the greatest volume which can be ined within a given surface.

6. *Example of cohesion.*—Thus particles of water falling in the sphere attract each other, and collect in spherules, forming rain. ıch spherules after their formation be exposed to cold, they n and form hailstones.

a little mercury be let fall on a sheet of paper, it will collect in silvery globules, notwithstanding the tendency of the gravity particles to make it spread over the paper in fine dust.

numerable examples present themselves of this class of pheno- . The tear as it falls from the eye collects in a spherule upon heek; the dew forms a translucent globule on the leaves of a.

7. *Shot manufacture.* — The manufacture of shot presents one e most striking examples of this phenomenon in the arts. The in a state of fusion is poured into a sieve, the meshes of which mine the magnitude of the shot, at the height of about two red feet from the ground. The shower of liquid metal, after ng through the sieve, forms, like rain in the atmosphere, into rules, which before they reach the ground are cooled and ified.

ıese spherules form the common shot used in sporting, and the sion of their spherical form shows how regularly the liquid obeys eometrical law, that a sphere contains the greatest volume within en surface.

8. *Why liquids form spherical drops.* — This disposition of to affect the spherical form may be further elucidated in con- ing that any other figure which a body could take would neces- r place different parts of its surface at different distances from its al point, a circumstance which would be incompatible with the ined qualities of attraction of cohesion and fluidity.

y fluidity, all the particles forming the mass are free to move gst each other, and by the attraction of cohesion they are drawn d their common centre with the same force. To suppose that could rest at different distances from their common centre, would sarily involve the supposition either that the attraction by which are affected was unequal, or that the mass had not perfect ity.

ıis principle, which is so evident, may be inverted, and we may ne that in all cases where natural bodies are found in the spheri- orm, even though they be solid, they must have been at the h of their formation in a fluid state.

9. *The earth and planets were once fluid.* — Hence it is inferred

that the earth and the other bodies of the solar system were once fluid, and that our globe existed formerly in the universe in a liquid state.

360. *Mutual repulsion of the atoms of gas.* — In certain classes of bodies the sphere of repulsion which surrounds their particles is so extensive that the effect of cohesion cannot be directly manifested. This is the case with air and all bodies in the gaseous form.

If a quantity of air be included in a cylinder under a piston, and the piston be drawn upwards so as to increase the volume of the cylinder in a doubled proportion, the air will not remain occupying its former volume, as a liquid would do, but it will expand and fill the double volume under the piston. This effect is produced by the expansive force prevailing among the particles of the air. When the piston is raised, a vacuum would be left between the surface of the air and that of the piston. The repulsive force prevailing among the particles of the air being unresisted, takes effect, and the particles separate, the air expanding into a double volume. In this case it must be concluded, that the vacant spaces between the particles of air are twice as great as they were before the piston was raised. If the piston be again raised to double its present height, the same effect will take place. The air will again expand in virtue of the repulsive force prevailing among its particles, and the interstitial spaces separating the particles will be proportionably augmented.

There is no known limit to this expansive quality, and it consequently follows that the region through which the repulsive forces of gases act has a corresponding extent.

361. *Gases may be reduced to the liquid, and even to the solid state.* — By the combination of mechanical pressure and cold, several of the gases have been reduced to the liquid state, and analogy justifies the conclusion, that all gaseous bodies are capable of this change. In the liquid state, the attraction of cohesion is rendered manifest, as has been already shown. But we have a still further evidence of the attraction of cohesion amongst the particles of gases, inasmuch as some of them have been reduced to the solid state; and by analogy we may conclude that all are capable of this change. It has been already shown that the solid state is only a consequence of the attraction of cohesion.

362. *The existence of the attraction of cohesion between their atoms inferred.* — These and other phenomena lead to the conclusion, that in this case of the gaseous bodies, there is beyond the sphere of cohesion a sphere of repulsion. When the particles, either by the application of cold or compression, or both of these agencies, are brought into such close contact as to be within the sphere of cohesion, then they become a liquid or solid, as the case may be.

363. *Mutual repulsion ascribed to the influence of heat.* — The mutual repulsion found to prevail among the constituent particles of

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bodies is by some attributed to the agency of heat, and it is certain that the energy of this repulsion is increased or diminished, according as heat is imparted to or subtracted from bodies. In a solid body, such as a mass of gold, in its ordinary state, the attraction of cohesion between its particles greatly predominates over the influence of the repulsion already mentioned; but if heat be applied to this mass, the energy of the repulsion is gradually increased, until at length it becomes so nearly equal to that of cohesion, that the gravity of the particles overcomes that part of the cohesion not balanced by the repulsion, and the constituent parts of the mass no longer holding together in the solid form, the metal is converted into a liquid. If heat be still applied to this liquid, the temperature will rise and the liquid will expand; but after a certain quantity of heat has been imparted to it, the repulsive force between the particles themselves is so great, that, in spite of their gravity and of the attraction of cohesion, they separate and disperse into a vapor which possesses the qualities of gas, being capable of expanding without limit.

364. *The same body may exist in the solid, liquid, or gaseous state.*—Thus it appears that the same body may exist in the solid, liquid, and gaseous forms, according to the conditions under which it is placed in reference to heat.

365. *Adhesion of solids.*—If the surfaces of two pieces of metal, being rendered perfectly smooth, are brought into close contact by a strong pressure, they will adhere together with considerable force. That this adhesion is not due to atmospheric pressure can be demonstrated by showing that the adhesion will continue in a vacuum.

366. *Examples of this adhesion.*—In this case the superficial molecules of the two bodies are brought into contact so close as to be within the sphere of each other's attraction.

Innumerable examples of the adhesion of solid bodies are familiar to daily experience. We may write with chalk, or with a pencil, or charcoal on a wall or on a ceiling, although the effect of gravity would be to cause the particles abraded from the chalk, the lead, or the charcoal to fall from the wall or the ceiling.

Dust floating in the air sticks to the wall or ceiling, in spite of the tendency of its gravity to fall from them.

The force of adhesion of solid surfaces one to another may be ascertained by placing the adhering surfaces in a horizontal position, the lower one being attached to a fixed point, and the upper one connected with the arm of a balance. The weight necessary to separate them is the measure of the adhesion. If we desire to ascertain the amount of adhesion per square inch of surface, it is only necessary to divide such weight by the magnitude of the adhering surface expressed in square inches.

367. *Adhesion of wheels of locomotive to the rails.*—It is on the adhesion between *metallic surfaces* when pressed strongly together

that the efficacy of a locomotive engine depends. The driving-wheels press with a great weight upon the rails, and are made to revolve round their own centres by the force of the engine. If there were no adhesion, or even insufficient adhesion between the tire of the wheel and the rail on which it is pressed, the wheel would turn without advancing; and this actually does happen in cases where the rails are greasy, and very frequently when they are covered with a hoar frost, the contact being then interrupted, and the matter between the wheel and the rail not offering the necessary adhesion.

368. *Effect of lubricants.*—On the other hand, when the force applied to break the adhesion is directed perpendicularly to the adhering surfaces, a fluid or unctuous matter smeared upon the surface often increases the adhesive force.

Thus two metallic plates will adhere with greater force together if they are smeared with oil than when they are clean. This may partly arise, however, from the fact that the film of oil which covers them excludes air more effectually than could be accomplished in the case of surfaces so considerable by mere pressure.

369. *The bite in metal working explained.*—The effect known amongst workers in metal as the *bite* is the adhesion of two metallic surfaces brought into extremely close contact. It may be doubted whether this adhesion would not be diminished if some fluid were introduced between the surfaces.

370. *Effect of glues, solders, and like adherents.*—The adhesion of the surface of solids may be rendered more intense than even the cohesion of the particles of the solids themselves by interposing between them some substance in a liquefied form, which hardens by cold, and which when hard has a strength equal to or greater than that of the solids which it unites.

Glues, cements, and solders supply remarkable examples of this. Two pieces of wood glued together will break anywhere rather than at their joint. The processes of gilding and plating also supply examples of the adhesion of metals to each other.

371. *Silvering mirrors.*—The process of silvering mirrors is an example of the adhesion of metal to glass; and that of mortar in building is an example of the adhesion of earthy matters to each other.

Two pieces of caoutchouc, if pressed together upon freshly cut surfaces, will be found to unite as completely as if they composed one independent piece.

372. *Adhesion and repulsion between solids and liquids.*—Numerous examples indicate the force of adhesion between solids and liquids. If we plunge the hand in a basin of water, when we withdraw it the hand will be wet, a quantity of the water adhering to it in spite of the tendency of its gravity to drop from the skin.

Adhesion, however, does not invariably exist between solids and

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liquids. If we dip the hand in a basin of quicksilver, the liquid will not adhere, and the hand will not be wetted.

If a body covered with a coating of grease or other unctuous substance be immersed in water, it will not be wet on being withdrawn. The same is true of the plumage of water-fowl, to which water does not adhere.

If water be spilled on glass or paper, it spreads over it, and adheres to it; and if the glass or paper be held in an upright position, its surface continues to be covered with a stratum of water, manifesting the force of adhesion between these bodies; but if quicksilver be poured on glass or paper, it rolls freely about in spherical globules, and if the glass or paper be held in an upright position, it falls off, and no pàrt adheres.

If a rod of gold, silver, tin, or lead, be plunged in quicksilver, the liquid will adhere to the surface of the rod; but if a rod of iron or platinum be immersed in the quicksilver, it will not adhere to the metal. It appears, therefore, that the superficial particles of the former metals attract the particles of the quicksilver, but those of the latter do not.

If the water be poured into a gauze strainer, it will pass freely through; but if quicksilver be poured into the same strainer, it will be retained as effectually as though the strainer were a solid body.

If, however, the bottom of the strainer thus containing the quicksilver be brought into contact with water, the quicksilver will immediately pass through.

If appears from this, that the particles of the gauze, when dry, repel the quicksilver, and that the particles of the latter have a greater force of mutual cohesion than is equal to the gravity of particles having the magnitude of the meshes of the gauze, but that water has not the same repulsion for the quicksilver as the gauze; and accordingly, when the gauze becomes saturated with water, it no longer retains the liquid metal.

373. *The rope pump.* — The instrument called the cord or rope pump depends on the attraction of adhesion which takes place between the threads of the cord and water. Several endless cords placed close beside each other are extended between two pulleys, one at the top and the other at the bottom of the machine, so that when one of the pulleys is made to revolve, these cords turn, rising at one side and falling at the other. The lower part is submerged in the well from which the water is to be raised, so that a portion of the cords is below the surface of the water. The upper pulley being then made to revolve, the cords at one side descend into the water, and at the other side rise out of it. These are placed so close together, that the water adhering to two adjacent cords, fills the space between them; and thus a sheet of water is raised from the lower to the higher pulley by the ascending cords, and is discharged into a reservoir above

374. *Glass attracts water but repels mercury.* — If water be poured into a glass vessel, its surface will be slightly concave near the edges; the liquid being attracted by the sides, will rise at these points above its general level.

But if mercury be poured into the same vessel, the surface of the edges will be slightly convex, because a repulsion exists between the vessel and the mercury, or at least an absence of sufficient attraction prevails to counteract the mutual cohesion of the particles of mercury; the latter, therefore, have a tendency to cohere, and the parts of the fluid next the glass are accordingly depressed below the general level.

The same effect will take place with water, if the inner surface of the glass be smeared with any unctuous substance which repels water.

If water be poured from a glass vessel with perpendicular sides, and without any bend at its edge, it will, instead of falling straight to the ground, trickle down the outer surface of the glass, thus showing the adhesion of the water with the glass; but if mercury be poured from the same vessel, it will not trickle down, but will fall directly to the ground.

Nevertheless, a very feeble degree of adhesion prevails between surfaces and liquids which generally repel each other, as is indicated by the fact that small particles of water will adhere to oiled surfaces, and small particles of mercury to glass or paper, although generally the liquid is repelled.

375. *Properties of capillary tubes.* — A glass tube having an extremely small bore is called a capillary tube, by which it is implied that the magnitude of the bore does not exceed the diameter of a hair. The term, however, is applied to tubes having any bores not too large to produce the effects which we are now about to explain.

If such a tube be plunged in water, the water will enter the bore, and fill the tube to the level of the external water. If the tube be drawn from the water and held in a vertical position, a part of the water which filled its bore will gradually descend to its lower extremity, and form a large drop. When the magnitude of this drop becomes so great that its weight overcomes the cohesion of its particles, it will fall off, and another drop will be formed; but finally a column of water, of a certain height, will be sustained in the tube, and the drop at the lower end will not be detached.

The cohesion of the particles of the water forming the drop being greater than the weight of the drop, the drop will not fall off; but this does not explain the fact, that a column of water of a certain height is supported in the tube, notwithstanding the tendency of its gravity to make it descend.

376. *Capillary attraction in such tubes.* — This phenomenon is due to the attraction exerted by the surface of the glass on the inside of the tube upon the particles of water. Even though the drop were

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removed, this column would still be sustained, and no new drop would collect at the lower end.

It is found that the smaller the bore of the tube the greater will be the height of the column thus supported.

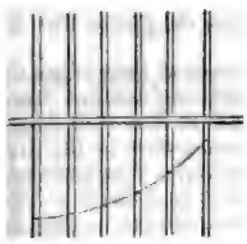

Fig. 83.

Let a series of capillary tubes gradually diminishing in their bores be inserted in holes parallel to, and at short distances from each other, in a small wooden rod, as represented in *fig.* 83. If this be plunged in water and drawn from it, each tube will sustain a column of the liquid, and the heights of each will gradually diminish as the bores of the tubes increase, so that a line drawn passing through the summits of the columns of water supported would form a curve, as represented in the figure.

The fact that the height of the column supported in a narrow tube is greater than that which is supported in a wider tube, may be thus explained.

Let A B, *fig.* 84., represent the diameter of the tube, and let A C and B C represent the distance to which the attraction of the sides of the tube at A and B extend. A particle of water therefore at C, submitted to the attraction directed to A and B, would be acted on by a force expressed by the diagonal C D.

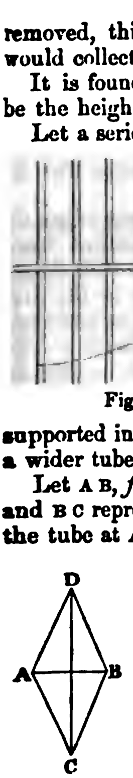

Fig. 84.

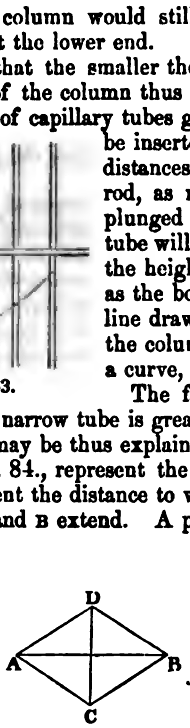

Fig. 85.

But if the diameter of the tube were greater, as represented at A B, *fig.* 85., the distance to which the attraction extends being still the same, the sides of the parallelogram A C and B C would be equal to A C and B C, *fig.* 84.; but the diagonal A B, *fig.* 85., being longer than A B, *fig.* 84., the diagonal C D, *fig.* 85., representing the attraction, would be less. Hence C D, *fig.* 84., representing the attraction in the narrow tube, will be greater than C D, *fig.* 85., representing the attraction in the wider tube.

In accordance with this conclusion, it is found practically that the heights to which columns of water will be sustained in different tubes are inversely as their diameters; that is to say, if the diameter of one tube be half that of another, the height to which the water rises in it will be double that of the other.

377. *Capillary attraction manifested between plane surfaces.*—Capillary attraction is likewise manifested between two plane surfaces brought close to each other. If two plates of glass, for example, be fixed parallel to each other without touching, but with a very small space between them, they will support a quantity of water between

and this resultant will be equal to the resultant of the centrifugal forces above K L, directed to the side of the angle A.

It follows, therefore, that the axis P Q round which the plate revolves will be affected by forces having two resultants parallel to each other and in opposite directions, one perpendicular to O P, and directed towards the side A, and the other perpendicular to O Q, and directed towards the side C.

These two forces form a couple, and have a tendency to turn the axis of revolution towards the position *m n*, as represented by the arrows in the diagram.

341. ***Centrifugal forces round such an axis are either in equilibrium or are equivalent to a couple.*** — This property is general, although it cannot be demonstrated in all its universality without the aid of the language and principles of the higher analysis.

It may be stated thus:—

In all bodies whatever, there are three lines passing through the centre of gravity which are at right angles to each other, each of which is so placed, in reference to the mass of the body, that the centrifugal forces produced by the revolution of the body round them respectively will be in equilibrium; and such lines, when the body revolves round them, will suffer no strain or pressure.

But if the body be made to revolve round any other line passing through the centre of gravity, the centrifugal force will produce a strain, which will be represented by two equal opposite and parallel forces acting upon the axis at opposite sides of the centre of gravity, and having a tendency to turn the axis in the position of one or other of the three axes of equilibrium here mentioned.

342. ***Axes of centrifugal equilibrium called principal axes.*** — An axis round which the centrifugal force equilibrates is called a *principal axis;* and from what has been explained, it appears that there are three principal axes through the centre of gravity at right angles to each other.

343. ***Case in which all lines through centre of gravity are principal axes.*** — There are some particular cases in which every line passing through the centre of gravity is a principal axis; such, for example, is the case with a sphere or globe of uniform density. Such a solid, whatever diameter it may revolve round, will be a solid of revolution, and the sections perpendicular to such diameter will be circles.

The same principle is true of all the regular solids. All lines, for example, passing through the centre of a cube are principal axes.

344. ***Case of an axis of revolution parallel to a principal axis through the centre of gravity.*** — If a solid be made to revolve round an axis which does not pass through the centre of gravity, but which is parallel to one or other of the principal axes passing through that point, the centrifugal forces will not equilibrate round such axis, but

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they will be represented by a single force perpendicular to it, and passing through the centre of gravity.

345. *Such axis called also a principal axis.* — Such an axis is called a principal axis, and in general there are three such axes corresponding with such points taken in the body, which are parallel respectively to the principal axis passing through the centre of gravity.

346. *Three principal axes at right angles to each other pass through each point.* — It may therefore be stated, in general, that if any point in a body be taken different from the centre of gravity, there are three lines passing through it at right angles to each other, round each of which, if the body is made to revolve, an effect will be produced by the centrifugal forces which can be represented by a single force, perpendicular to the circumference, and passing through the centre of gravity.

347. *Effect of revolution round a line which is not a principal axis.* — If the body be made to revolve round any line passing through a given point in it which is not a principal axis in the sense just referred to, then the centrifugal forces produced by such revolution cannot be represented either by a single force or by a pair of equal and opposite parallel forces, but will be represented by both of these together. This, therefore, is the character of all axes, round which a body would move, which do not pass through the centre of gravity, and are not parallel to either of the principal axes passing through that point.

348. *Experimental illustrations of these properties.* — Many of these important properties admit of being experimentally illustrated in a very striking manner by a simple apparatus.

Let a body be suspended by a thread, or, better still, by a skein composed of several threads, from a fixed point. Let this fixed point be so arranged, that a rapid rotatory motion may be given to it. This rotatory motion will soon be imparted to the body suspended to the threads by the twisting of the skein, and, after a time, the rotation of the body will become extremely rapid. It will first take place round the line, passing vertically through it in the direction of the skein, when it hangs quiescent. If this line happen to be one of the principal axes passing through its centre of gravity, it will continue to revolve round it; but if it be not one of these principal axes, the centrifugal force, as has been already explained, will produce an effect upon it, represented by two equal, opposite, and parallel forces tending to twist it. This effect will be soon rendered manifest in a remarkable manner: the body, when it revolves rapidly, will not continue in the same position which it had when it was quiescent. The line which was in the direction of the string will begin to take another direction, the point where the string is attached to it will not remain in its first position, and the body will throw itself into a position more attraction, both on the water and on the alcohol; but a stronger attraction on the former than on the latter. If, then, the chemical attraction of the water on the alcohol, and the alcohol on the water, be assumed to be equal, then the greater capillary attraction of the pores of the bladder for the water than for the alcohol, will explain the fact that more water passes into the vessel containing alcohol than alcohol into the vessel containing water.

In this case we have supposed the water to press downwards on the bladder within which the alcohol is contained. The same effect, however, would take place if the bladder were presented downwards, and the water pressed against it upwards. The same results would also ensue, if the vessel inclosed by the bladder contained water, and were immersed in a larger vessel containing alcohol.

387. *Motion of a glass bulb floating on water.* — If a hollow glass sphere, blown so thin as to be extremely light, be floated on the surface of water which half fills a glass vessel, it will, if placed near the side, move towards the side with an accelerated motion, and will soon touch the side and remain in contact with it. This movement is produced by the attraction of the water, which rises upon the side of the vessel, as already explained, upon the light glass bulb.

If the vessel be now filled without agitating the liquid in it, which may be done by introducing the water by means of a funnel, the lower orifice of which is at some depth below the surface, so as to fill it to the brim until the water becomes convex at the top, without however overflowing, the glass bulb will then move from the side of the vessel towards the centre; but the water being now convex instead of concave, that which touches the bulb on the inside rises higher upon it and attracts it.

Two small glass bulbs floating near each other will attract each other from the same cause; but if the glass bulb in either of these cases, and the sides of the vessel, be greased, so that the water shall not rise upon them, opposite effects take place.

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BOOK THE THIRD.

THEORY OF MACHINERY.

CHAPTER I.

GENERAL PRINCIPLES.

388. *What constitutes a machine.*—A machine is an instrument or apparatus by which a force applied at a certain point, and having a certain determinate intensity and direction, is made to exert a force at another point, more or less distant from the former, and generally different in intensity and direction.

Thus, for example, a horse moving on a horizontal road in a circle, is made to raise a weight vertically in the shaft of a mine, or water from the shaft of a well.

Men pulling at a rope in some direction more or less oblique, are enabled to raise a mass of heavy matter from the hold of a ship, and to transfer it to a distant wharf.

389. *The moving power.*—The force which is applied to and transmitted by a machine is technically called the power; the point at which it is applied is called the point of application; its direction is the line in which the force has a tendency to make the point of application move; and its intensity is usually expressed by a weight which, acting at the same point of application, would produce a like effect upon it.

The moving powers applied to and transmitted by machinery are infinitely various. In the capstan of a ship, the moving power is human force applied to it; in a common pump, the same moving force is used; a horse is the moving power applied to vehicles of transport on common roads, and a steam-engine on railways; the wind is the moving power applied to a sailing vessel and to a wind-mill; the momentum of water acting against the float-boards of a wheel, or its weight acting in the buckets, is the moving power of a water-wheel; the elastic force of steam acting on the piston in the cylinder, is the moving power of the steam-engine.

390. *The working point.*—That part of a machine which is im-

mediately applied to the resistance to be overcome, is called the *working point*.

391. *The weight.*—The resistance, whatever be its nature, to which the working point is applied, is technically called the *weight* or *load*.

In many cases, weight is the actual resistance which machines are applied to overcome; as, for example, in raising water from a well or from mines; also in raising ore. In some cases, the resistance to be overcome is friction, used for the purpose of fracturing and pulverising material substances. This is the case in flour-mills.

In some cases, the resistance to be overcome is the friction of surfaces and the resistance of the air. This is the case when carriages are moved on level roads or railways.

Whatever be the nature of the resistance, a weight which would produce an equivalent force acting against the moving power, may be assigned. Thus, for instance, if the traces of a carriage drawn by horses, or the chain connecting a locomotive with a railway train, be stretched by the resistance of the carriage or the train, a weight may be substituted, which, being suspended vertically from the traces or the chain, would produce the same tension. The resistance in such case is expressed by stating the amount of this weight.

392. *Various resistances and physical conditions omitted provisionally in the exposition of the theory of machinery.*—In the exposition of the theory of machinery, it is expedient to omit, in the first instance, the consideration of many circumstances, of which, however, a strict account must be subsequently taken, before any practically useful application of them can be made. A machine, such as we must for the present contemplate it, is a thing which can have no real or practical existence. Its various parts are considered to be free from friction. Thus the surfaces composing it, which move in contact with each other, are assumed to be infinitely smooth and polished; the solid parts are all considered to be absolutely rigid and inflexible.

The weight and inertia of the matter composing the machine itself are wholly neglected, and we reason upon it as if it were divested of these qualities. Cords, ropes, and chains are supposed to have neither stiffness, thickness, nor weight; they are regarded as mathematical lines, infinitely flexible and infinitely strong. The machine, when it moves, is assumed to encounter no resistance from the air, and to be in all respects circumstanced as if it were in a vacuum.

These suppositions being all false, it follows that none of the consequences immediately deduced from them can be true. Nevertheless, as it is the business of art to bring machines as near to this state of ideal perfection as possible, the conclusions which are thus obtained, though false in a strict sense, yet deviate from the truth in no considerable degree.

[redacted]

These conclusions may, in fact, be regarded as a first approximation to truth.

393. *Physical science consists of a series of approximations to truth.* — The various effects which have been previously neglected are afterwards taken into account.

The roughness of surfaces, the imperfect rigidity of the solid parts, the imperfect flexibility of cords and chains, the resistance of the air and other fluids, and the effects of the weight and inertia of the machine itself, are afterwards severally examined, their properties explained, and the manner in which they modify the transmission of the power to the weight developed. These modifications and corrections being applied to the conclusions obtained, a second approximation to the truth is made, but still only an approximation; for in investigating the laws which govern the several effects last mentioned, we are compelled to proceed upon a new group of false suppositions.

To determine the laws which regulate the friction of surfaces, it is necessary to assume that the surfaces are uniformly rough and subject to uniform pressure; that the solid parts which are imperfectly rigid, and the cords and chains which are imperfectly flexible, are constituted throughout their entire dimensions of a uniform material, so that the imperfections do not prevail more in one part than in another. Thus all irregularity is left out of account, and a general average of the effects taken. It is obvious, therefore, that, even in this second system of reasoning, we have still failed in obtaining a result exactly conformable to the real state of things. But it is equally obvious that we have obtained one much more conformable to that state than had been previously accomplished; and, in fine, it is found that the conclusions thus obtained are sufficiently near the truth for practical purposes.

394. *This gradual approximation to truth not peculiar to mechanical science.* — The imperfections in our process of investigation, manifested in this laborious system of successive approximation to the truth, is not peculiar to Natural Philosophy. It pervades all departments of natural science. In Astronomy, the motions of the celestial bodies, and their various changes and appearances as developed by theory, assisted by observation and experience, consist of a like series of approximations to the real motions and appearances which take place in nature.

It is the same in Art. The first labors of the artist produce from the rude block of marble a rough and rude resemblance of the human form. The next attempts remove the greater inequalities and protuberances, and reduce the form to a closer resemblance to the original. It is not, however, until after a long succession of operations, in which smaller and smaller portions of the stone are detached, that the last labor of the chisel *of the master* completes the resemblance.

395. *How a machine is provisionally regarded.* — We shall there-

fore for the present consider the machine, by which the effect of the power is transmitted to the working point, as divested of weight and inertia; we shall consider all the pivots, axles, and surfaces which move in contact absolutely devoid of friction; we shall consider all cords, ropes, and chains to be absolutely and perfectly flexible, and to be moved in contact with the grooves and wheels without friction; and, in fine, we shall consider the machine itself, as well as the agent exerting the power, and the matter composing the weight, to move without resistance from the air or any other fluid.

396. *Mechanical truths improperly invested with the appearance of paradox.* — The exposition of the effects of machinery is often invested with the appearance of paradox. Astonishment is excited at what seems incompatible with the results of common experience, rather than admiration of the genius and skill by which simple and obvious principles are so applied as to produce unexpected results.

Thus it is stated that, by means of a machine, a power of comparatively insignificant amount is capable of supporting or raising a vast weight; as, for example, it is affirmed that the fingers of an infant pulling a thread of fine silk, which a pound weight could snap asunder, are capable by this or that machine of supporting or raising several hundred weight.

Statements like these, if literally understood, are fallacious; if rightly explained, they involve nothing which is not consistent with our habitual experience.

397. *Effects of fixed points or props.* — In every machine there are some fixed points or props, and the arrangement of the parts is always such, that all that portion of the weight not directly acting against the power is distributed among these props.

If the weight, for example, amount to 20 cwt., it is possible so to arrange it, that any proportion of it, however great, may be thrown upon the fixed points or props of the machine: the remaining part only can properly be said to be supported by the power, and this part so supported can never be greater than the power. Considering the effect of a machine in this manner, it appears that the power supports just so much of the weight, and no more, as is equal to its own force, and that all the remaining part of the weight is sustained by the machine.

The force of this observation will become more and more apparent when the conditions are explained, under which a power and weight can maintain each other in equilibrium, through the intervention of a machine, whether simple or complex.

398. *An indefinitely small power raising an indefinitely great weight involves no paradox.* — But if the power, instead of merely supporting the weight at rest, be employed to raise this weight a given height, it may be asked how it can be explained that a power indefinitely small can lift a weight indefinitely great.

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The paradoxical character of this statement arises, as is the case erally in such propositions, from the omission of an important dition. It is quite true that a feeble power is capable of raising reat weight, but it is necessary to add that in doing this, the feeble rer must act through a space just so much greater than that ough which the weight is raised, as the weight itself is greater n the power. Thus, if a weight of 1000 lbs. be raised one foot a power whose force is only equal to 1 lb., then such power in sing the weight must move through 1000 feet.

Now when this condition is stated, the proposition is stripped alto-ther of its paradoxical character.

There is nothing at all astonishing in the fact, that one thousand cessive exertions of a force of one pound, each exertion being de through the space of one foot, should raise a 1000 lbs. weight rough the height of one foot. There is nothing more surprising in ch a fact, than if the 1000 lbs. weight being divided into 1000 ual parts were raised by a thousand successive efforts of the power thout the intervention of any machinery.

399. *Effects of a machine under different relations of the power d weight.* — It will be necessary to consider the effect produced by eans of a machine under three distinct relations between the power d weight, viz.,

I. When the power equilibrates with the weight.
II. When the power is greater than that which equilibrates with the weight.
III. When the power is less than that which equilibrates with the weight.

400. *When the power and weight are in equilibrium, rest is not cessarily implied.* — The power and weight are said to be in equilibrium when they are so related to each other that when placed at st they will remain so. It is a great, but very common error, to ppose that equilibrium, as applied to a machine, necessarily implies st or the absence of motion. It is easy to show that if the power d weight, being in equilibrium, are put in uniform motion, they ill continue that uniform motion exactly as a mass of matter would in virtue of its inertia, if moving independently of any machine; r if we were to suppose that such motion would cease either suddnly or gradually, we necessarily also suppose a definite force apied to the machine to stop its motion. Since the power and weight e in equilibrium, they cannot of themselves stop or retard the mon. It is true that the motion will in practical application be grally retarded, but that will be the effect of friction and atmospheric sistance, both of which are at present excluded from considerion.

We cannot, on the *other hand,* suppose the uniform motion im*rted to the power and weight* to be accelerated, without supposing the application of some adequate force to produce such acceleration; the power and weight being excluded by the very condition of the equilibrium.

Let us suppose the power and weight to be connected with the machine by cords, by which they are suspended from their respective points of application, both being, as usual, represented by equivalent weights. Now the cords by which they are suspended will be stretched with the same force, whether the power and weight be at rest, or in uniform rectilinear motion; and consequently the relation between them in both cases must be the same.

401. *Equilibrium infers either absolute rest or uniform motion.*—The repose or the uniform motion of the power and weight are therefore the tests of equilibrium. Without these equilibrium cannot subsist, and with either of them it must subsist.

If a machine acted on by a power and weight be at rest, or be in uniform motion, the power and weight must be in equilibrium; and if the power and weight be in equilibrium, they must be either at rest or in uniform motion.

The most common state of machines which are under the operation of equilibrating forces, is that of uniform motion, and not that of rest, as commonly stated. If a wind or water-mill be in regular operation, its driving-wheel moving with a uniform speed, then the power of the wind or water will be in equilibrium with the resistance, whatever that may be. If a steam-engine be in regular operation, its piston will move at a uniform rate, and the force of the steam upon it will be in equilibrium with the resistance which it is applied to overcome. If a locomotive engine draw a railway train at a uniform speed, then the power exerted by the engine will be in equilibrium with the resistance opposed by the train.

402. *When the power more than equilibrates, accelerated motion ensues.*—Let us now consider the case in which the power is greater than that which equilibrates with the weight or resistance.

In this case the motion imparted to the object moved will be accelerated; for so much of the power as would equilibrate with the weight or resistance would impart, as has been already shown, a uniform motion to the object moved. The surplus power above this amount, therefore, must be employed in accelerating the motion.

403. *All machines are in this state when started.*—For example, if a locomotive engine exert a greater power than is equivalent to the resistance opposed by the train which it moves, then such surplusage of power can only act upon the inertia of the train, and will impart to it an equivalent amount of moving force. So long as this surplus power, therefore, acts, the mass of the train will receive from it a corresponding augmentation of its momentum, and consequently will receive a proportionate increase of speed. If the resistance, *however, opposed* by the train to the moving power augments with

[redacted]

sed, then it may at length become equal to the amount of the
g power; and when it does, their equilibrium is established,
e train is moved by the power at a uniform speed.
e conditions are by no means imaginary. They are realized
y case in which a train is started from a state of rest, and in
when any machine whatever is first put in motion.
Analysis of the effect in this case.—The power in com-
its action must necessarily be greater than the resistance
by the load; for if it were not, it would only equilibrate
resistance, and no motion would ensue. The surplus power
ed by the momentum acquired by the moving mass; and as
ity augments, more and more momentum is imparted. The
will at length become uniform, either because the energy of
r will be diminished, so as to become equal to the resistance,
e the resistance will be augmented, so as to become equal to
r; or, in fine, as most generally happens in practice, both of
cts are combined, the resistance increasing and the power
ng. This is always the case, therefore, when a machine is
by a surplus power, and when, on the other hand, there is
an ordinary resistance on the side of the machinery and of
When first starting, the velocity being inconsiderable, the
of the air and other agencies depending upon speed is less.
elocity increases, these resistances augment. This augment-
resistance, however, as the speed increases, is generally
s than the diminution of the moving power; in short, a con-
surplus power is generally necessary, at starting, to impart
ad and to the moving parts of the machinery the necessary
m. But after this momentum has once been imparted, then
emains for the power but to balance the resistance of the
perly so called.
xcess above the equilibrating power and the accelerated mo-
reciprocal consequences. Such excess necessarily infers the
d motion of the load, and the accelerated motion of the load
such excess.
When the power is too small to equilibrate, the motion is re-
—Effects directly the reverse of these are developed when
r applied is inferior to that which would equilibrate with the

s suppose, in this case, the machine to have been in uniform
nd therefore the power and weight to have been in equili-
Let the power then be diminished by any amount, however
he moment this diminution takes place equilibrium is destroy-
ower becomes inferior to the resistance, and there is an action
ction contrary to that of the power, and therefore contrary to
the motion which the load had already acquired, equivalent
nt to the difference between the resistance and the power.

This force will act against the momentum of the load, and will continually diminish it, until, at length, it brings the load to rest. From the moment, therefore, that the power becomes less than the resistance, the motion of the load will be gradually retarded.

The inferiority of the power to the resistance, and a gradually retarded motion, are therefore reciprocal consequences of each other.

It may be useful to illustrate still further these effects, which are of considerable importance in practice.

Let us suppose the resistance which a machine is employed to overcome to be represented by the weight A, *fig*. 86.; and let the power which acts against such resistance through the intervention of the cord A B C be represented by the force of an animal H. When the animal is at rest before starting, the cord A B C H is stretched with a force exactly equal to that of the weight. When the power begins to move, a momentum is imparted to the weight through the intervention of the cord. The cord is therefore stretched with an additional force proportional to this momentum. The speed of the power gradually increases from the moment its motion commences until it attains that speed which is continued uniform. During this increase of the speed of the power H, a corresponding and continual increase of momentum is imparted to the weight A, and consequently, during this interval, the tension of the cord is constantly greater than the weight. When, however, the speed of the power H becomes uniform, then no further momentum will be imparted to the weight, and the force exerted by the power will diminish so as to become exactly equal to the weight. During this uniform motion the tension of the cord will be the same as it would be if the power and weight were at rest.

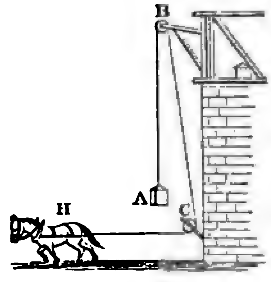

Fig. 86.

When the weight approaches its point of destination, and is about to be brought to rest, the power slackens its exertion, and, at the moment that it becomes less than the weight, a moving force takes effect equal in intensity to the difference between the power and weight directed from B to A. But against this there is the momentum of the weight directed from A to B in virtue of the uniform velocity with which it had been moving. The moving force, therefore, from B to A, represented by the difference between the power H and the weight A, will act against this momentum, and will gradually diminish it.

Although the upward motion of the weight, therefore, will continue after the diminution of the power, it will be gradually retarded, and after a certain interval will be altogether exhausted, and the weight will come to rest.

[redacted]

These effects take place in all machines whatever when they are started and stopped, and the circumstances and mechanical laws which govern them are precisely the same as in the illustration here given.

406. *The proper functions of a machine.* — The use of a machine is to adapt the power to the resistance. If the intensity, direction, and velocity of the power were identical with the intensity and direction of the resistance, and the velocity required to be imparted to it, then there would be no need of a machine; the power might be applied immediately to the resistance. But if a power of feeble intensity is required to act against a great resistance, then a machine must be interposed which will augment the intensity of the power. Or if a power moving in one direction be required to impart motion to a resistance in another direction, then a machine must be interposed which will transmit the effect of the power to a new direction. Or if a power having a certain velocity be required to impart a greater or less velocity to the resistance, then a machine must be interposed which will modify the velocity in the required proportion.

But even these, though the principal, are only a few of the infinite varieties of change and modification which machines are required to effect in the transmission of the power to the resistance. Independently of the directions, intensities, and velocities of the moving power and resistance, the character of the respective motions may differ in an infinite variety of ways: thus the moving power may be one which acts with a reciprocating motion between two points; as, for example, that of the piston of a steam-engine; and this moving power may be required to produce a continuous motion in a straight line, like the motion of a train along a railway.

The machine which connects such a power with such a weight must therefore be so constructed as to convert the reciprocating motion between two points into a continuous motion in a straight line.

The moving power may act in a straight line, while the resistance requires a circular motion.

Thus, the wind which acts upon the arms of a windmill is a continuous rectilinear force. The mill-stones to which that force is transmitted, revolve by a continual circular motion round vertical axes. The machinery of the windmill must therefore be adapted to convert the rectilinear force of the wind into the circular motion of the stones.

In every class of machines, and in every individual machine of each class, the relation between the velocities and directions of the power and weight, and the change produced on the character of the motion of the power when transmitted to the weight, depends solely on the structure of the machinery.

No variation in the magnitude of the power and weight can alter this relation. Thus the ratio of the velocities of the power and weight on a *lever* or an *inclined plane*, so long as their form and proportions remain the same, will be unaltered, and, whatever power or weight be applied to them, they will have this particular ratio.

407. *No machine can really add to the mechanical energy of the power.*—A machine being composed of inert matter cannot generate force, and consequently the working point cannot exert more force than is transmitted to it from the point of application of the power.

It will, in fact, exert less, because friction and other sources of resistance must intercept a portion of the action of the power in its transmission from its point of application to its working point; but as, for the present, the consideration of this species of resistance is neglected, and machines are considered as exempt from them, we shall assume that the influence of the power is transmitted undiminished to the working point. But it is important on the other hand to remember, that no *more* moving force can be so transmitted.

408. *Method of expressing the mechanical energy of the power and weight.*—Now the energy or momentum of the power is determined by multiplying the weight which is equivalent to it, by the space through which it is moved; and, on the other hand, the moving force imparted to the resistance is also estimated by multiplying the weight which is equivalent to this resistance by the space through which it is moved.

The moving force of the power is determined in the same manner as the moving force of a weight equivalent to it, and moving with the same velocity, would be determined. Thus, if we multiply the power, or its equivalent weight, by the space through which it moves in a given time; that is to say, by its velocity, we shall obtain a product which expresses its moving force or mechanical effect.

409. *Moments of power and weight.*—This product is called the *moment of the power.*

Thus if P express the power and p the space through which it moves in one second, then $P \times p$ will be its *moment.*

In like manner, the moving force imparted to the resistance at the working point will be expressed by multiplying the resistance, or the weight equivalent to it, by the space through which it is moved in a given time.

Thus if W be the weight, and w be the space through which it is moved in one second, then $W \times w$ will be the moving force of the weight, and this product is called the *moment of the weight.*

410. *The relation between these moments determines the state of the machine.*—The relation between the moments of the power and weight determines their mechanical state.

Three cases are here presented:

I. When the moment of the power is equal to the moment of the weight; that is, when

$$P \times p = W \times w.$$

[redacted]

II. When the moment of the power is greater than the moment of the weight; that is, when

$$P \times p \text{ is greater than } W \times w.$$

III. When the moment of the power is less than the moment of the weight; that is, when

$$P \times p \text{ is less than } W \times w.$$

In the first case, it is manifest that the power and weight will be in equilibrium, and that they will be either at rest or in uniform motion. For, since the moment of the power is the expression of its moving force, and since this moving force is transmitted without increase or diminution by the machinery to the weight, and since, by the supposition we have made, it is equal to the moving force of the weight, these two forces must balance each other, and therefore be in equilibrium.

411. *Equality of these moments determines equilibrium.* — The condition therefore of equilibrium is, that the moment of the power is equal to the moment of the weight, or

$$P \times p = W \times w.$$

412. *When the moment of power exceeds that of weight, the motion is accelerated in the direction of the power.* — If the moment of the power be greater than the moment of the weight, then the moving force of the power, exceeding that of the weight, and being transmitted to the working point undiminished, will prevail over it, and the power and weight must either have an accelerated motion in the direction of the power, or a retarded motion in the direction of the weight.

413. *When the moment of power is less than the moment of weight, the motion in direction of power is retarded.* — If the moment of the power be less than the moment of the weight, then the moving force of the power being transmitted to the weight and being less than the moving force of the latter, the latter will prevail, and therefore the power and weight must have either a retarded motion in the direction of the power, or an accelerated motion in the direction of the weight.

414. *In equilibrium, the velocity of the power is to that of the weight as the weight is to the power.* — If the moments of the power and weight be equal, we may infer that the power will bear to the weight the same ratio as the velocity of the weight bears to the velocity of the power, or

$$P : W :: w : p;$$

or, as it is sometimes expressed, the power and weight will be to each other inversely as their velocities.

This is another mode, then, of expressing the conditions under which the power and weight will be in equilibrium.

415. *Power always gained at the expense of time.*—It is this inverse proportion which is intended to be expressed, when it is said that power is never gained save at the expense of time; the meaning of which is, that if a small power work against a great resistance, the rate at which it moves the resistance will be just so much slower than that at which the power itself moves, as the resistance is greater than the power.

416. *All paradox thus removed from mechanical theorems.*—This condition of equilibrium, when rightly understood, removes all paradox from the statement of the effects of machinery. A small power working through a large space, raising a great weight through a small space, is merely an expedient by which a feeble power is enabled to accomplish its task, by a long succession of efforts, without dividing the weight. To raise the weight of a ton by a single effort one foot, would require a force equivalent to the weight of a ton. But if, by the intervention of a machine, a power is enabled to accomplish this object by 2240 distinct efforts, each effort working through one foot, then such power need not be more than one pound, or 2240 efforts made through the space of one foot, each effort exerting the force of one pound, will be mechanically equivalent to 2240 lbs., or one ton raised through one foot, and the effect produced will be the same as if the weight were actually divided into 2240 equal parts, and the power applied successively to raise each of these parts one foot.

417. *Utility of machinery not limited to make small powers overcome great resistance.*—A very inadequate estimate would, however, be formed of the objects and the utility of machinery, if we were to suppose them only directed to the particular class of problems which involve the motion of great weights or resistances by small powers. Cases innumerable occur, on the contrary, where small resistances are moved by great powers. For example, in a locomotive engine, while the piston in the cylinder moves once backwards and forwards, the train, which is the resistance overcome, is moved through a space equal to the circumference of the driving-wheel. Now, if we suppose the length of the cylinder, as frequently happens, to be one foot, and the circumference of the driving-wheel to be fifteen feet, then the velocity of the piston, or the power, will be to the velocity of the train, or the resistance, as 2 to 15; and consequently, the power which acts upon the piston must be greater than the resistance of the train, which is moved in the proportion of 15 to 2, omitting, as usual, the consideration of friction, &c. In like manner, in a watch or clock, the resistance of the object moved is merely that which is opposed to the motion of the hands on the dial-plate, while the moving power is the energy of the main-spring or of a descending weight. In both these cases, it is obvious that the power is vastly greater than the weight.

[redacted]

The object of machinery, therefore, may be stated generally as being the means by which the force and motion of the power are modified, so as to adapt them to the force or motion which is required to be imparted to the object moved.

418. *Case in which the point of application of the power and the working point do not move in the direction of the power and weight.* — In all that precedes, it has been assumed that the point of application of the power moves in the direction in which the power acts, and that the motion imparted to the working point is in a direction immediately opposed to the action of the weight or resistance. This, in fact, is what generally takes place in the practical construction and operation of machinery; for it is evident that if the point of application of the power were not free to move in the direction in which the power acts, a part of the power would necessarily be lost; and that if the working point did not move in a direction immediately opposed to the weight or resistance, a part of the force transmitted to the working point would be inefficient. But as, in certain cases, these conditions would not be fulfilled, it would be useful to state how the principles which have been established in the present chapter must be modified in such cases.

If by the construction of the machinery the point of application of the power moves in a line different from that in which the power acts, then the effective part of the power will be found by the parallelogram of forces.

Let A (*fig.* 87.) be the point of application of the power, and

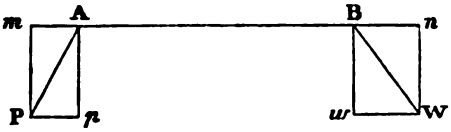

Fig. 87.

let B be the working point. Let A P represent the direction of the power, and B W the direction of the resistance or weight. Let A *p* be the direction in which the point of application is free to move, and let the working point B be free to move in the direction opposite to B *w*. Let right-angled parallelograms be formed, having for their diagonals A P and B W, representing the power and resistance, and having their sides in the direction A *p* and B *w*, in which the point of application and the working point are respectively free to move.

The power will then be equivalent to two forces represented by A *m* and A *p*; the latter, being in the direction in which alone the point of application can move, is alone effective; that part of the power represented by A *m* will necessarily be expended in pressure and strain upon the fixed points of the machine.

In like manner, the weight or resistance represented by B W is equivalent to two forces B *w* and B *n*; the force B *w* being in the direction against which alone the working point can act, is that portion of the weight or resistance which the working point will act against: the remainder of the weight will produce strain or pressure on the fixed point.

In the application of the principles which determine the relation of the power and weight in cases of equilibrium which have been established in the present chapter, the effective portion only of the power and weight will be to be taken into account. Thus the power is to be considered as represented by A *p*, and the weight by B *w*.

These principles will be rendered more clearly intelligible when they have been illustrated in their application to the simple machines.

CHAP. II.

CLASSIFICATION OF SIMPLE MACHINES.

419. *Machines simple and complex.* — Machines which are composed of two or more parts acting one upon another, are called complex machines.

Machines which consist only of one part are called simple machines.

The several parts composing a complex machine are themselves simple machines.

In a complex machine the effect of the power is transmitted successively through each of the parts composing it until it reaches the working point.

The effect of complex machines is determined by combining together the separate effects of the simple machines of which they are composed.

To estimate the effects of machinery, therefore, it will be necessary, in the first instance, to explain the principles of simple machinery.

420. *Classification of simple machines.* — Simple machines have been differently enumerated by different writers. If the object be to group in the smallest possible number of distinct classes those machines whose efficacy depends on the same principle, the simple machines may be comprised under the following three denominations: —

I. A solid body turning on an axis.

II. A flexible cord.

III. A hard and smooth inclined surface.

[redacted]

Notwithstanding the infinite variety of machinery, and of the parts composing it, it will be found that those parts may invariably be brought under one or other of the above classes.

421. *Condition of equilibrium of a machine having a fixed axis.* — In a machine composed of a solid body turning on an axis, all the parts are carried round such axis as a common centre, and describe circles round it in the same time. It is evident that the magnitude of these circles, and consequently the velocities of the different parts, will be proportional to their respective distances from the axis in which their common centre lies.

But since it has been already shown that when the power and weight are in equilibrium they must be inversely as the velocities of the points to which they are applied, it follows that any power and weight applied to such a machine will be in the inverse proportion of their distances from the axis when they are in equilibrium.

It must be understood, in the application of this important principle, that the power and weight are supposed to act in the direction of the motion of the parts to which they are respectively applied. If they do not act in this direction, then they must be resolved by the principle of the composition of force into two forces, one acting in the direction of the motion of the point of application, and the other in a direction being a point upon the axis.

This has been already explained (418.).

422. *Condition of equilibrium of a flexible cord.* — The second class of simple machines includes all those in which a force is transmitted by means of flexible threads, ropes, or chains. The principle by which the effects of these machines is estimated is, that the tension throughout the whole length of the same cord, provided it be flexible and free from the effects of friction, must be the same.

Thus, if a force acting at one end be balanced by a force acting at the other end, however the cord may be bent, or whatever course it may be compelled to take, by any cause which may affect it between its ends, these forces must be equal, provided the cord be free to move over any obstacles which may deflect it.

This class includes all the various forms of pulleys.

423. *Condition of equilibrium of a weight upon a hard inclined surface.* — The third class includes all those cases in which the weight or resistance is supported or moved upon a hard surface inclined to the direction in which the weight or resistance itself acts.

The effects of such machines may be estimated by the principles already explained.

The force of the weight or resistance being resolved into two other forces by the principle of the composition of force, one of these two forces will be perpendicular to the surface, and thus supported by its reaction; the other will be parallel to it, and will act against the power.

424. *Classification of the mechanic powers.*—The first class of simple machines above mentioned, consisting of a solid body revolving on an axis, is usually subdivided into two.

1st. The lever, which consists of a solid bar, straight or bent, resting upon a prop, pivot, or axis.

2d. A cylinder connected with a wheel of much greater diameter moving round a centre or axis. This combination is called the wheel and axle.

The second class includes the pulley.

The third class includes the simple machines, commonly known as the inclined plane, the wedge, and the screw; the last being, as will appear hereafter, nothing more than an inclined plane rolled round a cylinder.

The classes, therefore, of the simple machines, as they are generally received, and which are known as the mechanical powers, are the six following:—

I. The lever.
II. The wheel and axle.
III. The pulley.
IV. The inclined plane.
V. The wedge.
VI. The screw.

We shall accordingly explain these in the following chapters, and show the most important varieties and combinations of which they are susceptible.

CHAP. III.

THE LEVER.

425. *Levers: first, second, and third kinds.*—A straight and solid bar turning on an axis, is called a lever.

The arms of the lever are those parts of the bar extending on each side of the axis.

The axis is called the fulcrum or prop.

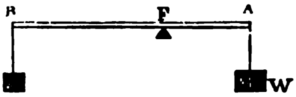

Fig. 88.

Levers are commonly divided into three kinds, according to the position which the fulcrum has in relation to the power and weight.

If the fulcrum be between the power and weight, as in *fig.* 88, the lever is of the first kind.

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If the weight be between the fulcrum and power, as in *fig.* 89., the lever is of the second kind.

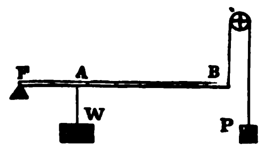

Fig. 89.

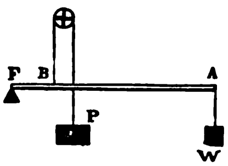

Fig. 90.

If the power be between the fulcrum and weight, as in *fig.* 90., the lever is of the third kind.

426. *Condition of equilibrium.*—Of whatever kind the lever may be, the conditions of equilibrium of the power and weight will be such that they are inversely as their distances from the fulcrum, this being the general condition of equilibrium for all machines which turn round a fixed axis (421.). It follows, therefore, that in *figs.* 88., 89., and 90., we shall have

$$\mathrm{P} : \mathrm{W} :: \mathrm{FA} : \mathrm{FB}$$

or, if p express the distance of the power from the fulcrum, and w the distance of the weight from the fulcrum, we shall have

$$\mathrm{P} : \mathrm{W} :: w : p;$$

or, what is the same,

$$\mathrm{P} \times p = \mathrm{W} \times w.$$

This statement, as will be perceived, is nothing more than a repetion of the general principle affecting machines which turn on an axis, in virtue of which forces upon them are in equilibrium when their moments round the axis are equal. The moment of the power is $\mathrm{P} \times p$, and the moment of the weight is $\mathrm{W} \times w$. The tendency of the power to turn the lever round its fulcrum in the direction of the power is expressed by the moment $\mathrm{P} \times p$, and the tendency of the weight to turn the lever in the contrary direction is expressed by $\mathrm{W} \times w$.

427. *Effect of power or weight varies as their leverage.*—It follows, therefore, that the tendency of the power to turn the lever would be augmented either by increasing the amount of the power P, or by increasing its distance p from the fulcrum. In either case the effect will be increased in a corresponding proportion. Thus, if we remove the power to double its distance from the fulcrum, we shall double its effect; and if we remove it to half its distance, we shall diminish its effect one half.

The distance of a force, whether power or weight, from the fulcrum, is called its leverage; and it is evident from what has been stated, that the effects of any force applied to a lever, will be proportional to its leverage.

428. *When power or weight is oblique to the lever.* — If the forces applied to a lever do not act perpendicular to it, their effect will be found by drawing from the fulcrum a perpendicular on their directions. This perpendicular will be their leverage. Thus in *fig.* 91, if the power act in the direction B P, draw F N perpendicular to the direction P B N; the power will have the same effect in turning the lever, as if it acted at N upon the lever N F. The moment of the power, therefore, in this case, will be found by multiplying it by F N, the perpendicular distance of its direction from the fulcrum.

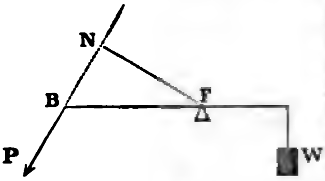

Fig. 91.

In general, therefore, the leverage of any force applied to such a machine is estimated by the perpendicular distance of the direction of such force from the fulcrum.

429. *Relation of power and weight in levers of first, second, and third kinds.* — In a lever of the first kind, the power and weight may be equal, and will be so when their leverages are equal.

The weight may be less than the power, and it will be so when it is at a greater distance from the fulcrum than the power.

The power may be less than the weight, and it will be so when it is at a greater distance from the fulcrum than the weight.

In a lever of the second kind, the weight, being between the fulcrum and the power, must be at a less distance from the fulcrum than the power, and must consequently be always greater than the power.

In a lever of the third kind, the power being between the fulcrum and the weight, will be at a less distance from the fulcrum than the weight, and consequently in this case the power must always be greater than the weight.

430. *Examples of levers of first kind.* — *Balance.* — Numerous examples of levers of the first kind may be given. A balance is a lever of this kind with equal arms, in which the power and weight are necessarily equal. The dishes are suspended by chains or cords from points precisely at equal distances from the fulcrum, and being themselves adjusted so as to have precisely equal weights, the balance will rest in equilibrium when the dishes are empty. To maintain this equilibrium, it is evident that equal weights must be put into the two dishes; the slightest inequality would give a preponderance to one or the other dish.

431. *Steelyard.* — A steelyard is a lever with unequal arms; the power, being represented by a sliding weight, is adjusted so that its

[redacted]

leverage may be changed at pleasure. These and similar instruments, are used for the purpose of weighing in commerce.

432. *Crowbar, poker, scissors, &c.* — A crowbar is a lever of the first kind. In this instrument, when used, for example, to raise a block of stone, the fulcrum, *fig.* 92., is another stone F placed near

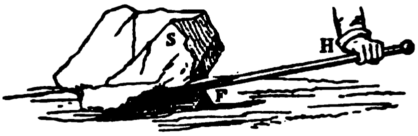

Fig. 92.

that which is to be raised, and the power of the hand H is placed at the other end of the bar.

A poker applied to raise fuel is a lever of the first kind, the fulcrum being the bar of the grate.

Scissors, shears, nippers, pincers, and other similar instruments are composed of two levers of the first kind, the fulcrum being the joint or pivot, and the weight the resistance of the substance to be cut or seized, the power being the fingers applied at the other end of the levers.

The brake of a pump is a lever of the first kind, the pump-rods and piston being the weight to be raised.

433. *Examples of levers of second kind. — Oar, rudder, chopping-knife, door, wheelbarrow, &c.* — Examples of levers of the second kind, though not so frequent, are not uncommon.

An oar is a lever of the second kind. The reaction of the water against the blade is the fulcrum. The boat is the weight, and the hand of the boatman the power.

The rudder of a ship or boat is an example of this kind of lever, and explained in a similar way.

The chopping-knife, *fig.* 93., is a lever of the second kind. The end F attached to the bench is the fulcrum, and the weight the resistance of the substance R to be cut.

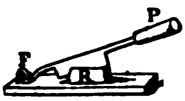

Fig. 93.

A door moved upon its hinges is another example.

Nutcrackers are two levers of the second kind, the hinge which unites them being the fulcrum, the resistance of the shell placed between them being the weight, and the hand applied to the extremity being the power.

A wheelbarrow is a lever of the second kind, the fulcrum being the point at which the wheel presses on the ground, and the weight being that of the barrow and its load collected at their centre of gravity.

The same observation may be applied to all two-wheeled carriag which are partly sustained by the animal which draws them.

434. *Examples of levers of third kind.* — ***Limbs of anima treadle of the lathe, tongs, &c.*** — Levers of the third kind, actii as has been explained, to mechanical disadvantage, the power bei always greater than the weight, are of less frequent use. They are ado ed only where rapidity and dispatch are required more than power.

The most striking examples of levers of the third kind are fou in the animal economy. The limbs of animals are generally lev of this description. The socket of the bone is the fulcrum, a stro muscle attached to the bone near the socket is the power, and t weight of the limb, together with whatever resistance is opposed its motion, is the weight. A slight contraction of the muscle in tl case gives a considerable motion to the limb: this effect is parti larly conspicuous in the motion of the arms and legs in the hum body; a very inconsiderable contraction of the muscles at the sho ders and hips gives the sweep to the limbs, from which the bo derives so much activity.

The treadle of the turning-lathe is a lever of the third kind. T hinge which attaches it to the floor is the fulcrum, the foot applied it near the hinge is the power, and the crank upon the axis of t fly-wheel, with which its extremity is connected, is the weight.

Tongs are levers of this kind, as also the shears used in sheari sheep. In these cases the power is the hand, placed immediat below the fulcrum or point where the two levers are connected.

435. *How to determine the pressure on the fulcrum of a lever.* The pressure on the fulcrum of a lever, when the power and wei are in equilibrium, is determined by the principle of the compositi of forces. In a lever of the first kind, the resultant of the pow and weight is a single force passing through the fulcrum, equal their sum; consequently, the pressure on such point will be equal the sum of the power and weight.

In a lever of the second and third kind, the power and weight a ing in contrary directions, will have a resultant equal to their diff ence passing through the fulcrum. This res tant will therefore express the pressure on t fulcrum.

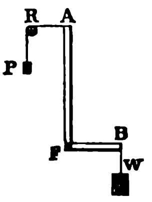

Fig. 94.

436. ***Rectangular lever.*** — In the rectangu lever, the arms are perpendicular to each oth and the fulcrum F, *fig.* 94., is at the right ang The moment of the power in this case is P m tiplied by A F, and that of the weight W mu plied by B F. When the instrument is in equi brium, these moments must be equal.

When the hammer is used for drawing a n it is a lever of this kind; the claw of the hammer is the shorter a

…ired to transmit an …ce, as for example, in …ressed upon the type by is driven upon it by a …vere force. The handle …r being pressed in the di… …he arrow, exerts a corres… …ssure on the point C, which …n the direction C D, perpen… … A B. This motion C D is …nto two by the parallelogram …, one in the direction C E, and …r in the direction C F; the …exerts pressure on the fixed …A, and the other acts upon the …R, by means of the joint G forc… …t downwards. As the joint C ad…nces, the angle A C G becomes more …d more obtuse, and the component …E of the force acting at B bears a …apidly increasing proportion to the force itself, so that when the levers A C and C G come nearly into a right line, the pressure exerted at B is augmented at G in an almost infinite proportion.

440. ***Beautiful example of complex leverage in the mechanism which connects the key and hammer in Erard's piano-forte.*** — In this instrument, the object is to convey from the point where the finger acts upon the key, to that at which the hammer acts upon the string, all the delicacy of action of the finger, so that the piano may participate, to a certain extent, in that sensibility of touch which is observable in the harp, and which is the consequence of the finger acting immediately on the string in that instrument, without the intervention of any other mechanism.

The combination of levers, by which the action of the finger is transmitted to the string, in Erard's pianoforte, is represented in *fig.* 99.

The key is represented at *b a c*, the centre or pivot on which it plays being *a*, and the ivory table upon which the finger acts being at *b*. The point to which its motion is communicated is at *c*.

This motion is transmitted by a double-jointed piece *d* to an intermediate lever *e f*, the pivot of which is at *e*. At the joint *f* is a rod *g* called the *sticker*, which carries up the hammer to the string. The hammer is supported by the head of the sticker *g*, and at the same time rests upon the oblique lever *i*, which latter is acted upon by the spring *h*.

When the key is pressed down by the finger at *b*, the piece *d* is raised, and by it the lever *f* and the sticker *g*. This lever raises the lever *i*, and acts upon the rod of the hammer at a point near the joint, making the hammer rise along the dotted curve so as to strike the string.

The proportions of this combination of levers are such, that, after the blow of the hammer on the string, the check *k* comes forward and receives the hammer in its fall, at about one third of its original distance from the string; so that while the finger continues to keep down the key, the hammer remains at a distance from the string, equal to one third of its distance when the key is not depressed.

In the meantime, the spring *h* has given way under the weight of the hammer, and under these circumstances, the key *b* being allowed to rise by the finger through one-third of its play, and then again being depressed, another stroke of the hammer on the string will be produced; for in this case the hammer will be brought back to the level of the head of the sticker *g*, by which means it will be driven upwards upon the depression of the key.

In the combination of levers used in other pianofortes, the note cannot be repeated without allowing the key to rise to the position it has before it is depressed; consequently in this case, a repetition of the note is produced with one-third of the motion of the finger which is necessary in other pianofortes.

It must be understood that this mechanism affects only the *touch* of these instruments. The quality of *tone* for which they have been so long remarkable depends on other and different mechanical arrangements.

441. *Power of a machine, how expressed.*—That number which expresses the proportion of the weight to the equilibrating power in any machine, we shall call the *power of the machine*. Thus if, in a lever, a power of one pound support a weight of ten pounds, the power of the machine is ten. If a power of 2 lbs. support a weight of 11 lbs., the power of the machine is 5½, 2 being contained in 11 5½ times.

442. *Equivalent lever.*—As the distances of the power and weight from the fulcrum of a lever may be varied at pleasure, and any assigned proportion given to them, a lever may always be conceived

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having a power equal to that of any given machine. Such a lever may be called, in relation to that machine, the *equivalent lever*.

443. *Complex machine may be represented by an equivalent compound lever.*—As every complex machine consists of a number of simple machines acting one upon another, and as each simple machine may be represented by an equivalent lever, the complex machine will be represented by a compound system of equivalent levers. From what has been proved, it therefore follows, that the power of a complex machine may be calculated by multiplying together the powers of the several simple machines of which it is composed.

CHAP. IV.

WHEEL-WORK.

444. *The wheel and axle.* — The form of simple machine denominated the wheel and axle, consists of a cylinder which rests in pivots at its extremities, or is supported in gudgeons, and is capable of revolving between those pivots, or in those gudgeons. Attached to this cylinder, and supported on the same pivots or gudgeons, a wheel is fixed, so that the two revolve together with a common motion. The weight is supported by a rope or chain, which winds round the axle, and the power by another rope or chain which winds round the wheel.

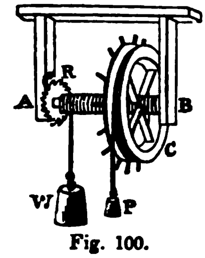

Fig. 100.

Such an arrangement is represented in *fig.* 100, where W is the weight, A and B the pivots or gudgeons, C the wheel, and P the power.

445. *Condition of equilibrium.* — The condition of equilibrium is, according to what has been already proved, the inverse proportion of the power and weight to the diameters of the wheel and axle; that is to say, the power is to the weight as the diameter of the axle is to the diameter of the wheel.

The weight is generally applied, as represented in the figure, by means of a rope coiled upon the axle.

446. *Various ways of applying power.*—The manner of applying the power is very various. Sometimes the circumference of the wheel is furnished with projecting points, as represented in *fig.* 100, to which the hand is applied when human force is the power.

Examples of this are numerous: a familiar one is presented in the steerage-wheel of a ship.

447. *Windlass.* — In the common windlass, the power is applied by means of a winch D C, as represented in *fig.* 101. The arm B C of the winch represents the radius of the wheel, and the power is applied to D C at right angles to B C.

In some cases no wheel is attached to the axle, but it is pierced with holes, directed towards its centre, in which long levers are incessantly inserted, and a continuous action produced by several men working at the same time, so that whilst some are transferring the levers from hole to hole, others are working the windlass, *fig.* 102.

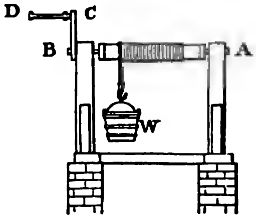

Fig. 101.

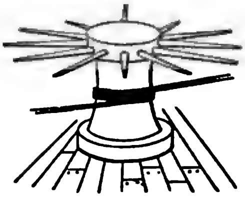

Fig. 102.

448. *The capstan.* — The axle is sometimes placed in a vertical position, the wheel or levers being moved horizontally.

The capstan is an example of this. A vertical axis is fixed in the deck of the ship, the circumference being pierced with holes presented towards its centre.

These holes receive long levers, as represented in *fig.* 102. The men who work the capstan walk continually round the axle, pressing forward the levers near their extremities.

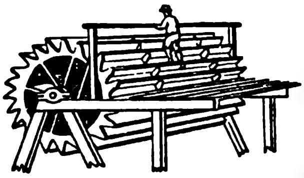

Fig. 103.

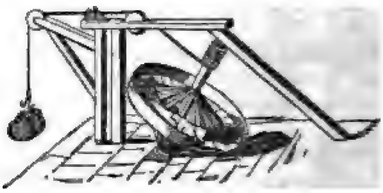

Fig. 104.

449. *The tread-mill, &c.* — In some cases the wheel is turned by the weight of animals placed at its circumference, who move forward as fast as the wheel descends, so as to maintain their position continually at the extremity of the horizontal diameter. The tread-mill, *fig.* 103., and certain cranes, such as *fig.* 104., are examples of this.

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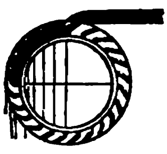

Fig. 105.

450. *Water-wheels, overshot, undershot, and breast-wheels.* — In water-wheels the power is the weight of water contained in buckets at the circumference, as in *fig.* 105., which is called an overshot wheel; and sometimes the impulse of water against float-boards at the circumference, as on the undershot wheel, *fig.* 106. Both these principles act in the breast-wheel, *fig.* 107.

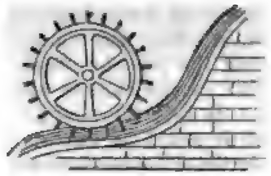

Fig. 106.

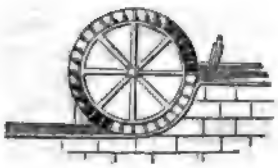

Fig. 107.

It often happens, in the practical application of the wheel and axle, that the power acts not continually like a descending weight, but with intermitting efforts. In such case, the machinery would be liable to react during the suspension of the power. An expedient called a ratchet-wheel is used to prevent this. Such a wheel is represented at B in *fig.* 100. It is a wheel furnished with teeth, placed in a direction contrary to that in which it moves. A click or bent bar falls between these teeth, and the combined effect of this click and the teeth is such, that the wheel is at liberty to move in one direction, the click falling successively between the teeth, but its motion in the other direction is checked by the pressure of the click against the teeth.

From what has been already explained, it is evident that the effect of the power upon the weight would be augmented by diminishing the thickness of the axle, and diminished by increasing that thickness.

451. *Case in which the power or resistance is variable.* — It sometimes happens that an invariable power has to act against a variable resistance, or a variable power against a constant resistance. In such a case, the effect of the wheel and axle as just described would vary: an augmentation of the power or diminution of the resistance would throw the power and weight out of equilibrium.

If, however, the axle were made to increase in thickness in the same proportion as the ratio of the power to the weight is augmented, then the change of such ratio would be compensated by a corresponding change in the leverage, and an equilibrium would be maintained between the power and weight notwithstanding their variation. Numerous instances of this are presented in the arts, some of which will be noticed hereafter.

452. *Method of augmenting the ratio of the power without complicating the machine.* — When a weight or resistance of comparatively great amount is to be raised by a very small power by means of the simple wheel and axle, either of two inconveniences would ensue; either the diameter of the axle would become too small to support the weight, or the diameter of the wheel would become so great as to be unwieldy in its operation. This has been remedied, without having recourse to a complex machine, by a simple expedient represented in *fig.* 108. The axle of the windlass here consists of two parts, one thicker than the other, and the rope by which the weight is raised rolls on the thicker while it rolls off the thinner. In each revolution, therefore, the part which is rolled on exceeds that which is rolled off by the difference between the circumferences of the two parts of the axle. That is, the part of the rope, by which the weight is suspended, is shortened in each revolution by this difference. But the height through which the weight is raised, is only half the shortening of the rope. The effect, accordingly, is the same as if an axle had been used, whose diameter is equal to half the difference between the diameters of the thicker and thinner part. Hence, *the power is to the weight as half the difference between the diameters of the axle is to the diameter of the wheel.* Since, then, without diminishing the thickness of the axle, we may diminish without limit the difference between the thicker and thinner parts, the ratio of the weight to the power may be augmented indefinitely without diminishing the strength of the axle. The apparatus just described is called the differential wheel and axle.

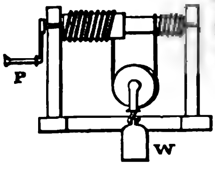

Fig. 108.

453. *Compound wheels and axles analogous to compound lever.* — When great power is required, wheels and axles may be combined in a manner analogous to the compound lever already explained. The power being supposed to act on the circumference of the first wheel, its effect is transmitted to the circumference of the first axle; this circumference acts on the circumference of the second wheel, and transmits motion thereby to the circumference of the second axle, which, in its turn, acts on the circumference of the second wheel, transmitting motion to the circumference of the third axle, and so on.

There is nothing different in the mechanical effect of such a combination from that of a system of compound levers, except that it admits more conveniently of a continuous action, and produces continued and regular motion. The relation between the power and the weight or resistance, when in equilibrium, is determined in exactly the same manner as in the case of the compound lever.

If the diameters of all the wheels be multiplied together, and the diameters of all the axles be also multiplied together, then the power will be to the weight as the product of the diameters of all the axles to the product of the diameters of all the wheels. Thus, if the diameters of all the axles be expressed by the numbers 2, 3, 4, and the diameters of all the wheels be expressed by the numbers 20, 25, and 30, then the ratio of the power to the weight will be as $2 \times 3 \times 4 = 24$ to $20 \times 25 \times 30 = 15{,}000$: that is, as 1 to 625.

454. *Various methods of communicating force between wheels and axles.* — The manner in which the wheels and axles act one upon another is very various. Sometimes a strap or cord is placed in a groove in the circumference of the axle, and carried round a similar groove in the circumference of the wheel. This, which is called an endless band, is represented in *figs.* 109. and 110.

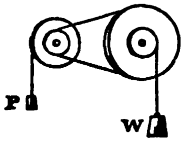

Fig. 109.

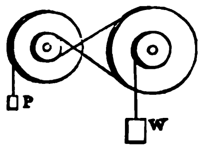

Fig. 110.

455. *By endless bands or cords.* — In the case represented in *fig.* 109. the wheels are driven in the same direction; in that represented in *fig.* 110., they are driven in opposite directions. Examples of this method of transmitting the motion from wheel to wheel are presented in every department of the arts and manufactures. In the turning-lathe and the grinding-wheel a cat-gut cord carried round the treadle-wheel imparts motion to the maundrell or the grindstone. In the great factories shafts are carried along the ceilings of the rooms, round which, at certain points, endless straps are carried which are conducted round the wheels, thus giving motion to the lathes or other machines. One of the chief advantages of this method of transmitting motion by wheels and axles is, that the bands by which the motion is conveyed may be placed at any distance from each other, and even in any position with respect to each other, and may, by a slight adjustment, receive motion in either one direction or the other.

456. *By rough surfaces in contact.* — When the circumference of the axle acts immediately on that of the wheel, which it moves without the intervention of a strap or cord, means must be adopted to prevent them from moving in contact, without transmitting motion, which they would do if both surfaces were perfectly smooth and free from friction.

This is accomplished by different expedients. In cases where great power is not required, motion is communicated through a series of wheels and axles by rendering their surfaces rough, either by facing them with rough leather, or making them of wood cut across the grain. This method is used in spinning machinery, where a large buffed wheel, placed in a horizontal position, is surrounded by a series of small buffed rollers pressed close against it, each roller communicating motion to a spindle. As the wheel revolves, revolution is imparted to the rollers, the velocity of which exceeds that of the wheel in the same proportion as the diameter of the wheel exceeds that of the roller.

This method is very convenient in cases where the motion of the rollers requires to be occasionally suspended, each roller being provided with a means by which it can be thrown out of contact with the wheel, and thus stopped.

457. *By teeth.* — The most frequent method of transmitting motion through a train of wheel-work is by the construction of teeth upon their circumference, so that the teeth of each falling into those of the other, the one wheel necessarily pushes forward the other.

When teeth are used, the axles are usually called pinions, and the teeth raised upon them are called leaves.

458. *Formation of teeth.* — In the formation of the teeth of wheels and pinions, expedients are adopted to prevent them from rubbing one upon another, when they move in contact with each other. A particular form is adopted for the teeth, in virtue of which the surfaces are applied one to the other with a rolling motion like that of a large wheel upon the road. By this expedient the rapid wear of the teeth, which would be produced by constant friction accompanied by pressure, is prevented.

459. *Methods of computing the condition of equilibrium in wheel-work.* — In computing the mechanical effects of toothed wheels and pinions, the number of teeth may be substituted for their circumferences and diameters.

The condition of equilibrium will therefore be obtained by multiplying together the number of teeth in all the wheels, and the number of teeth in all the pinions, the power being to the weight, when in equilibrium, as the latter product to the former.

460. *Spur, crown, and bevelled wheels.* — Toothed wheels are of three kinds, distinguished by the position which the teeth bear with respect to the axis of the wheel. When they are raised upon the edge of the wheel, as in *fig.* 111., they are called spur-wheels, or spur-gear. When they are raised parallel to the axis, as in *fig.* 112., they are called crown wheels. When the teeth are raised on a surface inclined to the plane of the wheel, as in *fig.* 113., they are called bevelled wheels.

If a motion round one axis is to be communicated to another axis

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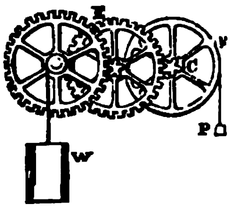

Fig. 111.

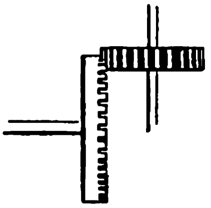

Fig. 112.

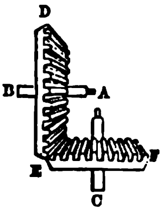

Fig. 113.

parallel to it, spur-gear is generally used: thus, in *fig.* 111., the three axes are parallel to each other. If a motion round one axis is to be communicated to another at right angles to it, a crown-wheel working in a spur pinion, as in *fig.* 112., will serve: or the same object may be attained by two bevelled wheels, as in *fig.* 113.

If a motion round one axis is required to be communicated to another inclined to it at any proposed angle, two bevelled wheels can always be used. In *fig.* 113., let A B and A C be the two axles; two bevelled wheels, such as D E and E F, on these axles will transmit the motion or rotation from one to the other, and the relative velocity may, as usual, be regulated by the proportionate magnitude of the wheels.

CHAP. V.

PULLEYS.

461. *Ropes not perfectly flexible, nor perfectly smooth.* — Although the term pulley implies the combination of a rope and a wheel on which it runs, the practical effects of the simple machine so denominated depend altogether on the rope; the wheel being introduced for the mere purpose of diminishing the effects of friction and imperfect flexibility, the consideration of both of which are omitted in theory. If a rope were perfectly flexible, and were capable of being bent over a sharp edge, and of moving upon it without friction, we should be enabled by its means to make a force in any one direction overcome a resistance or communicate a motion in any other direction.

Thus, if a perfectly flexible rope, F S, *fig.* 114., pass over a sharp edge P, and be connected with a weight R vertically, a force acting

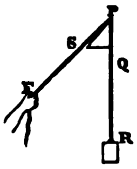

Fig. 114.

obliquely in the direction P F will raise the weight vertically in the direction R Q; but as no materials of which ropes can be made can render them perfectly flexible, and as in proportion to the strength by which they are enabled to transmit force their rigidity increases, it is necessary in practice to adopt means to remedy or mitigate those effects which attend the absence of perfect flexibility, and which would otherwise render cords practically inapplicable as machines.

But, besides the want of perfect flexibility, the surface of the rope is always rough, and often considerably so. This surface, in passing over an edge, would produce a degree of friction which would altogether stop its movement.

If a rope were used in the manner represented in the figure, to transmit a force in one direction to a resistance in another, some force would be necessary to bend it over the angle P, which the two directions form one with the other; and, if the angle were sharp, the effect of such a force might be the rupture of the rope.

462. *Hence the necessity of the sheave in the pulley.* — But if, instead of bending the rope at one point over a single acute angle, the change of direction were produced by successively deflecting it over several angles, each of which would be less sharp, the force necessary for the deflection and the liability of breaking the cord would be diminished. But such object will be still more effectually attained if the cord be deflected over the surface of a curve.

If the rope were applied merely to sustain a weight without moving it, a curved surface would therefore be sufficient to remove the inconvenience arising from imperfect flexibility; but when motion is required, the rope in passing over such a surface would be subject to great friction and rapid wear. This inconvenience is removed by causing the surface on which the rope runs to move with it, so that no more friction is produced than would arise from the curved surface itself rolling upon the rope. These objects are attained by the common pulley, which consists of a wheel called a sheave, fixed in a block turning on a pivot. A groove is formed in the edge of the wheel, in which the rope runs, the wheel revolving with it.

This apparatus is represented in *fig.* 115.

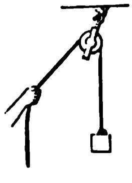

Fig. 115.

Notwithstanding, however, that this expedient removes the effects of friction and rigidity to so great a degree as to render the use of the cord practically available, it must not be supposed that these effects are altogether overcome; they still produce some impediments to the full efficiency of the power, as will be explained more fully hereafter. For the present, however, we shall consider the rope

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is rendered by this expedient perfectly flexible and free from friction.

463. ***Fixed pulley useful to change the direction of the power.*** — By means of a single sheave, a power acting in any one direction may be made to transmit its effects to a resistance in any other direction, provided the two lines of direction be in the same plane, and not parallel. Thus, let A B, *fig.* 116., be the direction in which the power acts, and let C D be the direction in which the weight or resistance acts.

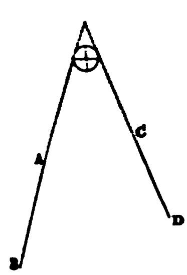

Fig. 116.

To transmit in this case the power to the weight, let the two directions B A and D C be prolonged until they meet, which they will do at O. In the angle O, formed by the two directions, let a sheave be placed, and let the power be connected with a rope in the direction B A. This rope being carried from A over the sheave at O, must be brought down in the direction C D, and connected with the resistance.

464. ***Case in which the power and resistance are parallel.*** — But if the direction of the power and resistance be parallel, as in *fig.* 117., then the effect of the power might be transmitted to the weight by means of a cord and two fixed pulleys, one placed over the direction of the force, and the other over the direction of the weight, as represented in the figure.

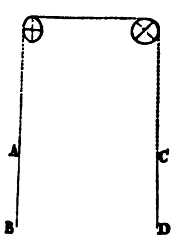

Fig. 117.

465. *Case in which they are in different planes.* — In fine, if the direction of the power and the weight be not placed in the same plane, then the effect of the power may still be transmitted to the weight by means of two pulleys.

Let us suppose, for example, that a power acting in a given horizontal line is required to be transmitted to a weight acting at some distance from it, in a certain vertical line, which is not in the same plane with the direction of the power.

Let two fixed pulleys be placed at two points in any convenient positions on the lines of direction of the power and weight, and let a line be supposed to join these points. Let the axis of one of the pulleys be placed at right angles to the plane formed by the line of direction of the power and the line joining the two pulleys, and let the other be placed with the axis at right angles to the plane passing through the line of direction of the weight and the line joining the two pulleys. By this arrangement, the cord being passed successively over both pulleys, the effect of the power will be transmitted to the weight.

466. *Power and weight equal when a single rope is used with fixed pulleys.* — In all these cases, the same cord by which the weight is suspended being directly connected with the power, and its tension throughout its entire length being the same, the weight and the power must be equal when they are in equilibrium. This condition is also rendered manifest by the fact, that from the motion of the mechanism connecting it, the weight and power will move with the same velocity.

It appears, therefore, that no mechanical advantage is gained by a single rope acting over one or more fixed pulleys; nevertheless, there is scarcely any engine, simple or complex, which is attended with more conveniencе.

467. *Mechanical convenience of changing the direction of the power.* — In the applications of power, whether of man or animals, or arising from other natural forces, there are always some directions in which it may be exerted to greater convenience and advantage than others, and in many cases the power is capable of acting only in one particular direction. Any expedient, therefore, which enables it to give the most advantageous direction to the moving power whatever be the direction of the resistance opposed to it, contributes as much practical convenience as one which enables a small power to balance or overcome a great weight.

468. *Fire escapes.* — By means of the fixed pulley a man may raise himself to a considerable height, or descend to any proposed depth. If he be placed in a chair or a basket attached to one end of a rope which is carried over a fixed pulley, by laying hold of this rope on the other side, he may, at will, descend to a depth equal to half of the entire length of the rope, by continually yielding rope on the one side, and depressing the basket or chair by his weight on the other. Fire escapes have been constructed on this principle, the fixed pulley being attached to some part of the building.

469. *The single moveable pulley.* — A single moveable pulley is represented in *fig.* 118.; a cord is carried from a fixed point F, and passing through a block B attached to a weight W passes over a fixed pulley C, the power being applied at P. We shall first suppose the parts of the cord on each side the wheel B to be parallel: in this case the whole weight W being sustained by the parts of the cords B C and B F and these parts being equally stretched, each must sustain half the weight, which is therefore the tension of the cord. This tension is resisted by the power at P, which must therefore be equal to half the weight.

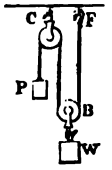

Fig. 118.

In this machine, therefore, the weight is twice the power.

470. *Case in which the cords are not parallel.* —If the parts of the cord B C and B F be not parallel, as in *fig.* 119., a greater power

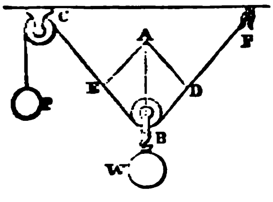

Fig. 119.

than half the weight is therefore necessary to sustain it. To determine the power necessary to support a given weight in this case, take the line B A in the vertical direction, consisting of as many inches as the weight consists of ounces; from A draw A D parallel to B C, and A E parallel to B F: the force of the weight represented by A B will be equivalent to two forces represented by B D and B E. The number of inches in these lines respectively will represent the number of ounces which are equivalent to the tensions of the parts B F and B C of the cord. But as these tensions are equal, B D and B E must be equal, and each will express the amount of the power P, which stretches the cord at P C.

It is evident that the four lines A E, E B, B D, and D A are equal; and as each of them represents the power, the weight which is represented by A B must be less than twice the power which is represented by A E and E B taken together. It follows, therefore, that as the parts of the rope which support the weight depart from parallelism, the machine becomes less and less efficacious, and there are certain obliquities at which the equilibrating power would be much greater than the weight.

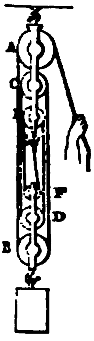

Fig. 120.

471. *Moveable block with several sheaves.* — If several sheaves be constructed in the same moveable block, the mechanical advantage may be proportionally augmented. In *fig.* 120., a system is represented in which three sheaves are inserted in the moveable block bearing the weight, the same number being inserted in the fixed block. The cord is carried from the power first over the fixed sheave A, then over the moveable sheave B, then over the fixed sheave C, the moveable sheave D, the fixed sheave E, and the moveable sheave F, and finally attached to the block at G.

Now, since the cord throughout its whole length is stretched with the same force, and since it is evident that at the part where the power is applied, this force of tension must be equal to the power, it follows that the six parts of the cord which support the weight will each be stretched by a force equal to the power, and that consequently the weight, when in equilibrium, must be equal to six times the power.

This condition may also be inferred from the fact, that if the power move the cord, its velocity will be six times that of the weight; for if six feet of the rope be drawn over the fixed pulley, these six feet must be equally distributed between the six parts of the rope which sustain the weight, *and consequently each part* must be raised through one foot, which is therefore the height through which the weight would be raised for every six feet through which the power passes.

472. *Condition of equilibrium of such a block.* — In general it may therefore be inferred, that, in a pulley which consists of a single moveable block containing one or more sheaves, the weight, when in equilibrium, will be just as many times the power as is represented by the number of cords, or the number of parts of the cord, which sustain the weight.

473. *Effect of attaching the end of the rope to the moveable block.* — In the form of moveable block represented in *fig.* 121., the cord, after passing successively over the sheaves, is finally attached to the lower block. This, by increasing the parts of the cords supporting the lower block by one, augments the efficiency of the instrument without increasing the number of sheaves.

474. *Smeaton's and White's pulleys.* — Two of the most powerful forms of pulley, consisting of a single moveable block, are represented in *figs.* 122 and 123.

The combination represented in *fig.* 122. is called Smeaton's pulley, having been invented by that celebrated engineer. The fixed and moveable block contain each ten sheaves, and the order in which the rope is carried over them is represented in the figure by the numbers 1, 2, 3, 4, &c. The total number of parts of the cord supporting the lower block is in this case twenty, and consequently the power is to the weight as 1 to 20.

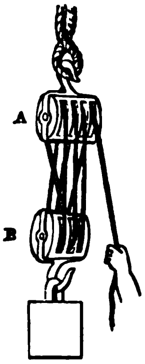

Fig. 121.

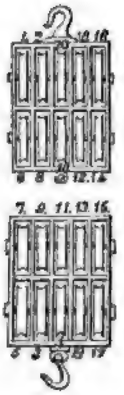

Fig. 122.

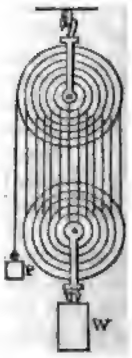

Fig. 123.

The form of pulley represented in *fig.* 123. is called White's pulley.

475. *Systems of pulleys consisting of several ropes and moveable blocks.* — If instead of one moveable block and a single rope, two or more moveable blocks with independent ropes be used, the power of

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the pulley may be augmented on the same principle as in the case of compound levers or compound wheel-work.

Different combinations of this kind are represented in *figs*. 124. 125. 126. and 127.

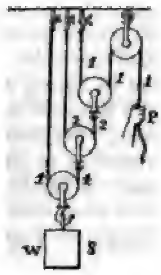

Fig. 124.

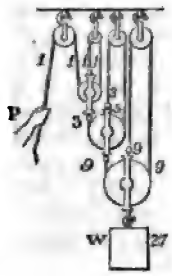

Fig. 125.

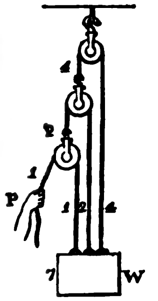

Fig. 126.

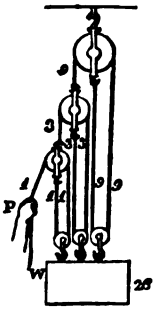

Fig. 127.

The figures which are annexed to the ropes in each case represent the power they respectively exert.

The first rope in *fig*. 124. is stretched by the force of the power only. The first moveable block being supported by two parts of this first rope will exert a force equal to double the power on the second rope; and the second moveable block being supported by two parts of this rope will exert a force upon the third rope equal to four times the power; and in the same way it follows that the third block will exert a force in supporting the weight equal to eight times the power.

In such a system, the addition of each moveable block doubles the mechanical effect.

But without augmenting the number of moveable blocks, but only adding fixed blocks, the effects may be augmented in a three-fold instead of a two-fold proportion, as represented in *fig*. 125., where each successive moveable block is supported by three parts of the same cord.

In *fig*. 126. the ends of the cords, instead of being attached to fixed points, are attached to the weight.

In this case, the weight is supported by each of the several cords, these cords being stretched by different forces. The first is stretched with a force equal to the power, the second with a force equal to double the power, and the third with a force equal to four times the power, and so on. In such a system, the mechanical effect for the same number of blocks is greater than in that represented in *fig*. 124., where three blocks only support a weight four times the power; whereas in the system represented in *fig*. 126, three blocks support seven times the power.

The effects of this system may be still further increased by attaching blocks to the weight, as represented in *fig.* 127., and carrying the ropes to the pulleys above.

The effect produced is indicated by the numbers fixed to the cords.

476. ***The practical effect of pulleys varies considerably from their theoretical effect.*** — From its portable form, cheapness of construction, and the facility with which it may be applied in almost every situation, the pulley is one of the most useful of the simple machines. The mechanical advantage, however, which it appears in theory to possess, is considerably diminished in practice, owing to the stiffness of the cordage and the friction of the wheels and blocks. By these means it is computed that in most cases so great a proportion as two thirds of the power is lost. The pulley is much used in building when weights are to be elevated to great heights; but its most extensive application is found in the rigging of ships, where almost every motion is accomplished by its means.

In all these examples of pulleys, we have supposed the parts of the rope sustaining the weight, and each of the moveable pulleys, to be parallel to each other. If they be subject to considerable obliquity, the relative tensions of the different ropes must be estimated according to the principle applied in 470.

CHAP. VI.

INCLINED PLANE — WEDGE AND SCREW.

477. *Effect of an inclined surface on a weight.*—A hard surface pressed against a weight or resistance in a direction at right angles to it, would support it, and the whole amount of such weight or resistance would in this case press upon the surface. But if, instead of being at right angles to it, it were placed in an oblique direction, then the weight or resistance would be resolved by the parallelogram of forces into two, one of which would act perpendicularly to the plane and produce pressure upon it, and the other would be parallel to the plane, and be free to produce motion.

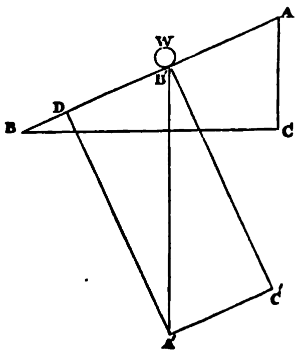

Fig. 128.

Let A B, *fig.* 128, be such a surface, and let W be a body producing

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resistance, or having a tendency to move in the direction B′ A′ ie to B A. Let the whole force with which W would move if ipported by the plane be expressed by B′ A′. Then, taking B′ A′ i diagonal of a parellelogram one of whose sides is B′ C′ at angles to B A, and the other B′ D in the direction of the plane, rce B′ A′ will be equivalent to two forces, one represented by B′ d the other by B′ D. The former being perpendicular to the will be resisted by its reaction, and the other only will take

To support the weight, therefore, in this case, would require e acting parallel to the plane, and opposite to the force repre- l by W D.

we take on the plane the length B A equal to B′ A′, and draw A llel to B′ A′, and B C perpendicular to it, then the triangle A B C e in all respects equal and similar to A′ B′ C′; in fact, it may be lered as the same triangle, but in a different position.

ice, therefore, A′ B′, B′ C′, and A′ C′, represent respectively the force of the body W, its pressure on the plane, and its tendency ove in the direction of the plane, these three forces will be ly represented in their effects by the lines A B, B C, and A C.

8. *The inclined plane — condition of equilibrium.*—We have taken the general case, and supposed the body W to exercise a in any direction whatever; but if we apply the principle to the of a heavy body resting upon a plane inclined to the vertical tion, then the machine becomes what is commonly called the in- l plane.

B is called the length of the plane, A C its height, and B C its

om what has been just proved, then, it follows, that if a weight laced upon an inclined plane, the weight consisting of as many ds as there are inches in the length of the plane, the pressure on lane will consist of as many pounds as there are inches in the and the tendency to move down the plane will be balanced by any pounds as there are inches in the height.

9. *Apparatus to illustrate experimentally the inclined plane.*— apparatus represented in *fig.* 129 is intended to represent this experimentally. The weight placed upon the plane is a roller so formed as to move freely upon it. A string is attached to it, which being carried parallel to the plane is conducted over a fixed pulley, and supports a dish bearing a weight. On comparing this weight, including the weight of the dish, with the weight of the roller i the plane, and by varying the angle of elevation of the plane, find that in every case the weight necessary to produce equili-

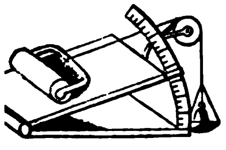

Fig. 129.

brium will be expressed by the height of the plane, the entire weight of the roller being expressed by its length.

It is evident from what has been just explained that the less the elevation of the plane is, the less will be the power requisite to sustain a given weight upon it, and the greater will be the pressure upon it; for the less the elevation of the plane is, the less will be its height and the greater will be its base.

480. *Inclined roads.* — Roads which are not level may be considered as inclined planes, and loads drawn upon them in carriages, regarded in reference to the powers which impel them, are subject to all the conditions which have been established for inclined planes.

The inclination of the road is estimated by the height corresponding to some proposed length: thus we say, a road rises one foot in twenty-five, or one foot in thirty; meaning that if twenty-five or thirty feet of the road be taken as the length of an inclined plane, the corresponding height of such plane would be one foot; and if twenty-five or thirty feet be measured upon the road, the difference of the level of the two extremities will be one foot. According to this method of estimating the inclination of roads, the power required to sustain a load upon them, friction apart, is always proportional to this rate of elevation. If a road rise one foot in twenty, then a power of one ton will be sufficient to sustain twenty tons; and so on.

481. *Inclined planes on railways.* — When a power is employed in moving a load upon a road thus inclined, the action of the power may also be regarded in another point of view.

Let us suppose a railway train weighing 200 tons moving up an inclined plane, which rises at the rate of one in two hundred; what mechanical effect does the moving power produce in moving up 200 feet of such a plane?

First, it acts against the friction, the atmospheric and other resistances to which it would be exposed if the plane had been level.

Secondly, it is employed in raising the entire weight of the train through the elevation which corresponds to 200 feet in length, that is, through one perpendicular foot.

The mechanical effect, therefore, is precisely the same as if the load of 200 tons had been first moved along a level plane 200 feet long, and then elevated up a step one foot high; but instead of being called upon to make this great exertion of raising two hundred tons directly through one perpendicular foot, the moving power is enabled gradually to accomplish the same object by a longer continuance of a more feeble exertion of force, such exertion being spread over 200 feet instead of being condensed into a single foot.

482. *Case in which the power acts in a direction inclined to the plane.* — In all that precedes, we have assumed that the power acts parallel to the plane; in some cases, however, it acts obliquely to it.

Let W P, *fig.* 130., be the direction of the power. Taking that as

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the diagonal of a parallelogram, it will be equivalent to W D and W E, the former perpendicular, and the latter parallel to the plane.

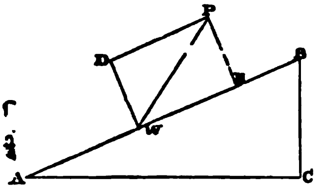

Fig. 130.

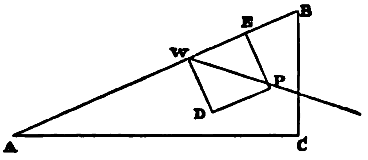

Fig. 131.

W D will have the effect of diminishing the pressure on the plane, and W E will be efficient in drawing the weight up the plane.

In some cases, the direction of the power is below the plane, as in *fig.* 131. In this case, as before, the power W P is resolved into two forces, W E parallel to the plane, and W D perpendicular to it.

The latter augments the pressure of the weight on the plane, and the former is efficient in drawing it up the plane.

483. *Case of double inclined plane.* — It sometimes happens that a weight upon one inclined plane is raised or supported by another weight upon another inclined plane. Thus, if A B and A B′, *fig.* 132., be two inclined planes, forming an angle at A, and W W′ be two weights placed upon these planes, and connected by a cord passing over a pulley at A, the one weight will either sustain the other, or one will descend, drawing the other up. To determine the circumstances under which these effects will ensue, draw the lines W D and W′ D′ in the vertical direction, and take upon them as many inches as there are ounces in the weights respectively. W D and W′ D′ being the lengths thus taken, and therefore representing the weights, the lines W E and W′ E′ will represent the effects of these weights respectively down the planes. If W E and W′ E′ be equal, the weights will sustain each other without motion; but if W E be greater than W′ E′, the weight W will descend, drawing the weight W′ up; and if W′ E′ be greater than W E, the weight W′ will descend, drawing the weight W up. In every case W F and W′ F′ will represent the pressures upon the planes respectively.

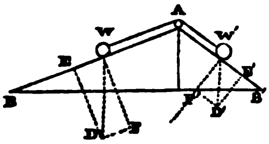

Fig. 132.

484. *Case of self-acting planes on railways.* — It is not necessary, for the effect just described, that the inclined planes should, as represented in the figure, form an angle with each other. They may be parallel, or in any other position, the rope being carried over a sufficient number of wheels, placed so as to give it the necessary deflection. This method of moving loads is frequently applied in great public works where railroads are used. Loaded waggons descend one

inclined plane, while other waggons either empty or loaded, so as to permit the descent of those with which they are connected, are drawn up the other.

485. *The wedge.* — When the weight is not moved upon the plane, but is stationary, the plane being itself moved under the weight, the machine is called a wedge.

Let D E, *fig.* 133., be a heavy beam, secured in a vertical position between guides, F G and H I, so that it is free to move upwards and downwards, but not laterally. Let A B C be an inclined plane, the extremity of which is placed beneath the end of the beam. A force applied to the back of this plane A C, in the direction C B, will urge the plane under the beam so as to raise the beam to the position represented in *fig.* 134. Thus, while the inclined plane is moved through the distance C B, the beam is raised through the height C A.

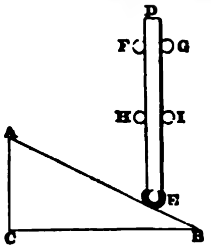

Fig. 133.

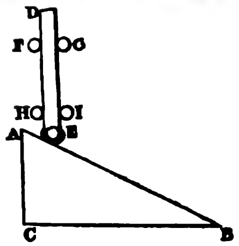

Fig. 134.

It follows, therefore, that in the case of the wedge the velocity of the resistance is to the velocity of the weight as the base of the inclined plane, which forms the wedge, is to its height.

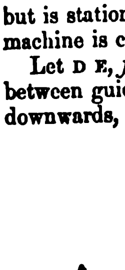

Fig. 135.

486. ***Wedges consist of two inclined planes.*** — Wedges, however, are more generally formed of two inclined planes, connected base to base, as represented in *fig.* 135. In this case, the back of the wedge is the sum of the heights of the two inclined planes, and the length of the wedge is their common base.

The force, therefore, which drives the wedge is to the resistance with which it equilibrates, as half the back of the wedge is to its length.

It follows from this, that wedges become more powerful as they become sharper.

487. *Theory of the wedge practically inapplicable.* — This theory of the wedge, however, is not applicable in practice with any degree of accuracy. This is owing chiefly to the enormous disproportion which friction in these machines bears to the power; but independently of this there is another difficulty in the theory of this machine.

[redacted]

3. *Power applied to wedge usually percussion.* — The power ionly used in the case of a wedge is not pressure, but percus- The force of a blow is of a nature so different from continued , such as the pressure of weights, that it admits of no numerical arison with the resistance offered by cohesion, to overcome which generally applied. We cannot properly state the proportion h a blow bears to a weight. The wedge is almost invariably l by percussion, while the resistances which it has to overcom s constantly forces of another kind.

lthough, however, no exact numerical computation can be made, it may be stated in general that the wedge is more and more rful as the angle is more acute.

9. *Practical use of the wedge.* — The cases in which wedges nost generally used in the arts and manufactures, are those in h an intense force is required to be exerted through a very small . This instrument is therefore used for splitting masses of tim- r stone; for raising vessels in docks, when they are about to be ched, by being driven under their keels; in presses where the juice eds, fruits, or other substances is required to be extracted, as, xample, in the oil-mill, in which the seeds from which the oil is cted are introduced into hair bags, which being placed between s of hard wood are pressed by wedges. The pressure exerted e wedges is so intense that the dry seeds are converted into solid es as hard and compact as the most dense woods. Wedges have used occasionally to restore to the perpendicular edifices which been inclined owing to the sinking of their foundations.

0. *Practical examples—cutting and piercing instruments, &c.* ll cutting and piercing instruments, such as knives, razors, shears, rs, chisels, nails, pins, needles, &c., are wedges. The angle of edge in all these cases is more or less acute, according to the ose to which it is applied. Chisels intended to cut wood have edge at an angle of about 30°; for cutting iron from 50° to 60°, for brass about 80° to 90°. In general, tools which are urged ressure admit of being sharper than those which are driven by ssion. The softer and more yielding the substance to be divided e more acute the wedge may be constructed.

1. *Utility of friction in the application of the wedge.* — In y cases the efficiency of the wedge depends on that which is en- y omitted in its theory, viz., the friction which arises between its ce and the substance which it divides. This is the case when , bolts, or nails are used for binding the parts of structures to- er, in which case, were it not for the friction, they would recoil their places and fail to produce the desired effect. Even when wedge is used as a mechanical engine the presence of friction is lutely indispensable to its *practical utility.*

The power, as has already been stated, generally acts by successive blows, and is therefore subject to constant intermission, and but for the friction the wedge would recoil between the intervals of the blows with as much force as it had been driven forward. Thus the object of the labour would be continually frustrated. The friction in this case is of the same use as a ratchet-wheel, but is much more necessary, as the power applied to the wedge is much more liable to intermission than in the cases where ratchet-wheels are generally used.

492. *The screw.* — In ascending a steep hill it has been the practice of road engineers, instead of making an inclined plane directly from the base to the summit, to carry the road round the hill, gradually rising as it proceeds. If we desire to ascend with ease to the top of a high column, we could do so if a path or ledge were formed on the outer surface, gradually winding round and round the column from the bottom to the top. Such a path would be, in fact, an inclined plane carried round the column. But it will be evident that such an arrangement would constitute *a screw.* This will be rendered still more apparent by the following contrivance.

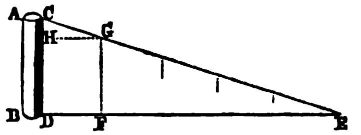

Fig. 136.

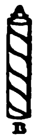

Fig. 137.

Let A B, *fig.* 136., be a cylindrical roller, and let C D E be an inclined plane cut in paper, the height of which C D is equal to the length of the roller.

Let the edge C D be pasted on the roller, and then let the roller be turned so that the paper shall be wrapped round it. When it makes one revolution of the roller, the portion of the edge C G will have made one spiral coil; the next revolution will make an equal spiral coil, and so on until all the paper has been rolled upon the roller, when the edge of the paper so coiled will show a regular spiral line round the roller, as represented in *fig.* 137.

Taking C H G, *fig.* 136., as the inclined plane thus rolled round the roller, it is evident that C H is its height, and H G its base. But C H is the distance between two successive coils of the spiral, and H G is the circumference of the roller. The coils of the spiral are called the *threads* of the screw, and the distance C H between the successive coils is called the *distance between the threads.*

493. *Power applied to screw by means of a lever.* — In the application of the screw, the weight or resistance is not, as in the

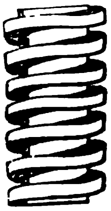

Fig. 138.

inclined plane and wedge, placed upon the surface of the plane or thread. The power is usually transmitted by causing the screw to move in a concave cylinder, on the interior surface of which a spiral cavity is cut, corresponding exactly to the thread of the screw, and in which the thread will move by turning round the screw continually in the same direction. This hollow cylinder is usually called the nut or concave screw. The screw surrounded by its spiral thread is represented in *fig.* 138.

494. *Methods of transmitting the power to the resistance.*—There are several ways in which the power is transmitted to the resistance by means of a screw; but by whatever means it may be so transmitted, it is evident that the screw will move the resistance in a single revolution through a space equal to the distance between two contiguous threads. The comparative velocities, therefore, of the power and weight will always be found in this class of simple machines by comparing the space described by the power, in imparting one revolution to the screw with the distance between two contiguous threads.

495. *Condition of equilibrium.*—The most common manner of urging the screw is by a lever attached to its head, as represented in *fig.* 139. at E F. Supposing the power to be applied at F, it will in producing one revolution of the screw, and therefore in moving the resistance through a space equal to the distance between two contiguous threads, make one complete revolution in a circle whose radius is the length of the lever on which it acts. The velocity, therefore, of the power will be to the velocity of the weight as the circumference of the circle described by the power is to the distance between two contiguous threads; and consequently, the condition of equilibrium between the power and weight will be this, that the power is to the weight as the distance between the contiguous threads is to the circumference described by the power.

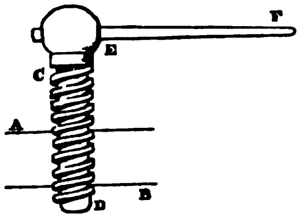

Fig. 139.

496. *Great mechanical force of screw explained.*—The great mechanical force exerted by the screw will hence be easily understood. There is no limit to the smallness of the distance between the threads, except the strength which is necessary to be given to them, and there is no limit to the magnitude of the circumference to be described by the power, except the necessary facility of moving in it. We can, therefore, conceive the power acting by a lever, and therefore moving through a great circumference while the screw moves through a com-

paratively minute space; and consequently, in such case, the power will be so much less than the resistance, as the distance between the threads is less than the circumference described by the power.

497. *Various methods of connecting the screw and nut.*—The manner of acting upon the resistance by means of the screw is very various. Sometimes the nut is fixed and the screw moveable; sometimes the screw is fixed and the nut moveable; sometimes the nut, though incapable of revolving, can be moved progressively; and sometimes the screw is incapable of revolving, but is moved progressively. These conditions admit of various combinations, which are severally adopted in practice.

In *fig.* 139., the nut A B being supposed to be fixed, if the lever F be turned, the end D of the screw will descend or ascend, according to the direction in which E F is turned, and will act upon the resistance accordingly.

If the screw be fixed, the nut may be moved upon it, either by turning the nut or the screw. In either case the nut will ascend or descend, according to the direction of the motion. In each revolution it will move through a space equal to the distance between two contiguous threads.

If we suppose the nut A B, *fig.* 139., to be incapable of ascending or descending, but to be capable of revolving, then by turning it round the screw which plays in it, the screw will ascend or descend through a space equal to the distance between two contiguous threads for every revolution made by the nut.

On the other hand, the apparatus may be so arranged that the screw, though capable of revolving, is incapable of a progressive motion, and the nut, though capable of a progressive motion, is incapable of revolving. In this case, when the screw is made to revolve, the nut in which it plays will be moved upwards or downwards, through a space equal to the distance between two threads, by the revolution of the screw.

The screw is generally used in cases where severe pressure is to be exercised through small spaces; it is, therefore, the agent in most presses.

In *fig.* 140., the nut is fixed, and by turning the lever which passes through the head of the screw a pressure is exercised upon any substance placed upon the plate immediately under the end of the screw. In *fig.* 141., the screw is incapable of revolving, but is capable of advancing in the direction of its length. On the other hand, the nut is capable of revolving, but does not advance in the direction of the screw. When the nut is turned by means of the lever inserted in it, the screw advances in the direction of its length, and urges the board which is attached to it downwards, so as to press any substance placed between it and the fixed board below.

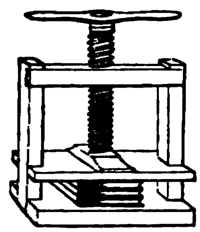

Fig. 140.

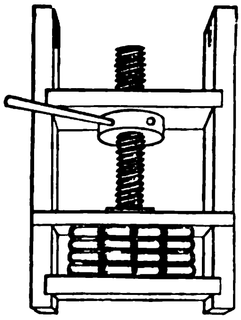

Fig. 141.

498. *Various examples of the application of the screw.* — In cases where liquids or juices are to be expressed from solid bodies, the screw is the agent generally employed. It is also used in coining, where the impression of a die is to be made upon a piece of metal, and in the same way in producing the impression of a seal upon wax or other substance adapted to receive it. When soft and light materials, such as cotton, are to be reduced to a convenient bulk for transportation, the screw is used to compress them, and they are thus reduced into hard dense masses. In printing, the paper is sometimes urged by a severe and sudden pressure upon the types by means of a screw.

499. *Manner of cutting a screw.* — A screw may be cut upon a cylinder by placing the cylinder in a turning-lathe, and giving it a rotary motion upon its axis. The cutting point is then presented to the cylinder and moved in the direction of its length at such a rate as to be carried through the distance between the intended threads while the cylinder revolves once. The relative motions of the cutting point and the cylinder being preserved with perfect uniformity, the thread will be cut from one end to the other. The shape of the threads may be either square, as in *fig.* 138., or triangular, as in *fig.* 140.

500. *Method of augmenting the force of the screw.* — If the lever by which the power acts on the screw were capable of indefinite increase, or the thread of indefinite fineness, there would be no limit to the mechanical effect of the instrument; but to both of them there are practical limits. The lever cannot be increased so as to render the operation of the power unwieldly and impracticable, and the thread cannot be diminished beyond that limit which will give suf-

ficient strength; and the cases in which the greatest mechanical efficacy is needed, are precisely those in which the thread of the screw requires to be strongest.

501. *Hunter's screw.* — To obtain an indefinite augmentation of the power of the screw, without diminishing the strength of the thread, Mr. Hunter proposed an arrangement which is known by his name, as the Hunterian screw. This is represented in *fig.* 142.

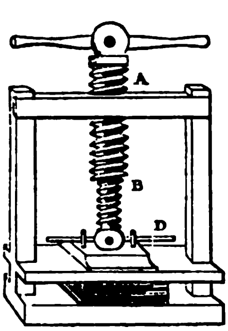

Fig. 142.

This contrivance consists in the use of two screws, the threads of which may have any strength and magnitude, but which have a very small difference of breadth.

While the working point is urged forward by that which has the greater thread, it is drawn back by that which has the less; so that during each revolution of the screw, instead of being advanced through a space equal to the magnitude of either of the threads, it moves through a space equal to their difference. The mechanical power of such a machine will be the same as that of a single screw having a thread whose magnitude is equal to the difference of the magnitudes of the two threads just mentioned.

Thus, without inconveniently increasing the sweep of the power on the one hand, or on the other diminishing the thread until the necessary strength is lost, the machine will acquire an efficacy limited by nothing but the smallness of the difference between the two threads.

502. *Micrometer screw.* — The very slow motion which may be imparted to the end of a fine screw by a very considerable motion of the power, renders it an instrument peculiarly well adapted to the measurement of very minute motions and spaces, the magnitude of which could scarcely be ascertained by any other means. To explain the manner in which it is applied: suppose a screw to be so cut as to have fifty threads in an inch, each revolution of the screw will advance its point through the fiftieth part of an inch. Now, suppose the head of the screw to be a circle whose diameter is an inch, the circumference of the head will be something more than three inches: this may be easily divided into a hundred equal parts, distinctly visible. If a fixed index be presented to this graduated circumference, the hundredth part of a revolution of the screw may be observed by noting the passage of one division of the head under the index. Since one entire revolution of the head moves the point through the fiftieth of an inch, one division will correspond to the

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five-thousandth of an inch. In order to observe the motion of the point of the screw in this case, a fine wire is attached to it, which is carried across the field of view of a powerful microscope, by which the motion is so magnified as to be distinctly perceptible.

CHAP. VII.

REGULATION OF FORCE.

503. *Regulation of motion necessary in machines.* — Regularity and uniformity are two of the conditions most universally indispensable in machinery. Sudden changes of motion are always injurious, and sometimes destructive to the apparatus, and never fail to produce inconvenience and imperfection in the articles fabricated.

Much attention, therefore, has been directed to, and much mechanical ingenuity expended on contrivances for insuring these conditions of regularity and uniformity in the movement of machinery, by removing those causes of inequality which can be avoided, and by compensating those which cannot.

504. *Causes of irregular motion.* — Irregularity in the motion of machinery will result in any one of the following cases,—

1°. When a varying power is opposed to a uniform resistance.

2°. When a uniform power is opposed to a varying resistance.

3°. When the power and resistance both vary, but not proportionally to each other.

4°. When the power is not transmitted with uniform effect to the working point in the successive positions assumed by the machine.

505. *When a varying power is opposed to a uniform resistance.* — The force of the prime mover is seldom regular. The force of water varies with the copiousness of the stream; the force of wind is proverbially capricious; the power of steam varies with the intensity of combustion in the furnace; and the force of animal power, depending on the temper and health, is difficult of control, human labour being the most unmanageable of all. No machine works so irregularly as one that is manipulated.

In some cases, the prime mover is subject, by the very conditions of its existence, to constant variation, as, for example, where it is a main spring, which gradually loses its energy as it is relaxed. In some cases, the prime mover is liable to intermission, and is totally suspended during certain intervals. An example of this is presented in the single acting steam-engine, where the force of the steam acts only during the descent of the piston, but is suspended during its ascent.

506. *Where a uniform power is opposed to a varying resistance.*— In almost all the applications of machinery, the load or resistance is subject to continual fluctuation.

In mills, a multiplicity of parts are liable to be occasionally and irregularly disengaged, and to have their operations suspended. In large factories for spinning, printing, dyeing, &c., a great number of separate spinning machines, looms, presses, and other engines, are usually driven by a common power, such as a water-wheel or a steam-engine. In such cases, the number of machines worked at the same time must necessarily vary according to the employment supplied to the factory, and to the fluctuating demand for the articles produced. Under such circumstances, the velocity with which the machinery is moved would suffer corresponding changes, increasing with each diminution, and being retarded with each increase of the resistance.

507. *When the variation of the power is not proportional to the variation of the resistance.*—In many cases, the variation of the power and the variation of the resistance are both from their conditions inevitable, and yet a uniform effect is indispensable. It is evident that this can only be insured by a class of contrivances which have for their object to proportion the power to the resistance, by either causing a diminution or increase of resistance to diminish or augment the supply of power; or, on the other hand, by causing the variation of the power to act in a corresponding manner upon the resistance or load.

In a word, uniformity of action in machinery can only be insured by providing means by which the power and the resistance, no matter what be their respective variations, shall always be proportional to each other.

Whenever the power is less than that which is in equilibrium with the resistance, the motion will be retarded, and if this condition continue it will ultimately stop; and whenever the power is greater than that determined by the condition of equilibrium, the motion will be accelerated, and if this condition should continue, the acceleration would continue until the machine would be destroyed by its own momentum.

508. *When the effect of the power is unequally transmitted to the resistance.*—There is scarcely any machine in which the energy of the power is transmitted uniformly to the resistance in all the phases of the mechanism. In all machines the moving parts assume in succession a variety of positions, in each of which their effect to transmit the power to the resistance is different; and thus the effective energy of the machine in acting against the resistance is subject to continual fluctuation. It is not easy to convey, without numerous examples, a general idea of those causes of inequality to those who are not familiar with machinery. It will, however, be more clearly

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l when we come to explain the methods of equalizing the the power and the resistance.

'egulators, the general principle of their action.*—The class ances which have for their object to render the power and proportionate to each other are called *regulators.* They act upon that point of the machine which commands the ' the power by means of some mechanical contrivances, ›ck the quantity of the moving principle conveyed to the whenever the motion becomes accelerated, and increase the ›enever it becomes retarded.

ıter-wheel, for example, this is accomplished by acting upon le, in a wind-mill by the adjustment of the sails, and in a ;ine by acting on a valve called the throttle valve placed in pipe, through which steam flows from the boiler to the

'The governor.—One of the most interesting and instructive examples of this class of contrivances is called the *governor.* This expedient, which was long used to regulate mill work and other machinery, owes its beautiful adaption to the steam-engine to the ingenuity of Watt. It consists of two heavy balls, B B, *fig.* 143., attached to the extremities of rods B F jointed at E, and passing through a mortice in the vertical stem D D'.

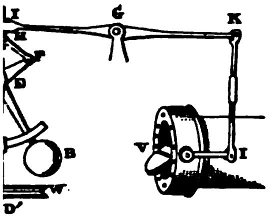

Fig. 143.

the balls B are driven from the axis by the centrifugal force om their rotation, their upper arms E F are caused to in- :ir divergence in the same manner as the blades of a scissors ›d by separating the handles. These acting upon the ring ans of the short limbs F H draw it down the vertical axis wards E. A contrary effect is produced when the balls B ;ht closer to the axis, and the divergence of the rods B E d. A horizontal wheel W is attached to the vertical axis ıng a groove to receive a rope or strap upon its rim. This ›es round the wheel or axis, by which motion is transmitted achinery, to be regulated so that the spindle or shaft D D' lways made to revolve with a speed proportionate to that of inery.

: shaft D D' revolves, the balls B are carried round it with a notion, and consequently acquire a centrifugal force which em to recede from the axle, and therefore to depress the On the edge or rim of this ring is formed a groove, which ›ed by the prongs of a fork I at the extremity of one arm of a lever whose fulcrum is at G. The extremity K of the other arm is connected by some means with the part of the machine which supplies the power. In the present instance we shall suppose it a steam-engine; in which case the rod K I communicates with a flat circular valve V placed in the principal steam pipe, and so arranged that when K is elevated as far as by their divergence the balls B have power over it, the passage of the pipe will be closed by the valve V, and the passage of steam entirely stopped; and, on the other hand, when the balls subside to their lowest position, the valve will be presented with its edge in the direction of the tube, so as to interrupt no part of the steam.

The property which renders this instrument so well adapted to its purpose is, that there is but one velocity at which the balls can remain in equilibrium.

When the instrument is constructed, and adapted to the machinery to which it is intended to be applied, the weight and proportion of its parts are adapted to particular velocities, with which the machine is to be moved. Whenever the velocity becomes greater than that, the balls recede from the axis, the arm I is drawn down, the valve V is closed, and the supply of the moving power diminished. The increase of velocity is thus stopped, and the movement is reduced to its normal rate, at which it continues, the balls maintaining their increased distance from the axis. Whenever, on the other hand, the velocity is diminished, the centrifugal force being also diminished, the balls fall nearer the axis; the arm I is raised, and the valve V is opened, so that an increased supply of the moving power is produced, and the tendency to retard the movement is checked.

When the governor is applied to a water-mill, it is made to act upon the shuttle through which the water flows, and to limit its quantity, in the same manner as above described. When it is applied to a wind-mill, it regulates the sails through the same principle.

The governor may also act upon the resistance, so as to accommodate it to the varying energy of the power.

This is sometimes done in corn-mills, where it acts upon the shuttle that metes out the corn to the mill-stones.

511. *The pendulum of a clock and balance-wheel of a watch.*— Of all the regulators used in machinery, those which are most familiar are the pendulum and balance, used in clock and watch-work. These contrivances have the property of always making their vibrations in the same time, no matter what be the length of the arcs through which they swing, limited, however, as these arcs are in practice.

The moving power, in a common clock, is that of a descending weight, which keeps in revolution a wheel, the motion of which is imparted through a series of other wheels, working one in another, *to the hands* which move on the dial plate. Now, if no regulating power intervene, the movement imparted to the hands would, in this case, be a uniformly accelerated motion, that being the motion which gravity would impart to the descending weight; but this is prevented by the regulating power of the pendulum acting on a wheel called the escapement. Connected with the upper end of the pendulum rod, is a small apparatus, furnished with two teeth, or projecting pieces, which fall alternately between the teeth of a wheel called the *escapement wheel*, the intervals of their action corresponding with the vibrations of the pendulum. This escapement wheel is itself impelled by the moving power of the descending weight; but it is continually stopped by the action of the two pins moved by the pendulum just mentioned.

The balance-wheel of a watch renders uniform the effect of the in-spring, in precisely the same manner.

he main-spring, which is the moving power in this case, consists strong spiral spring, which is coiled up to its highest tension the watch is wound up. It drives a wheel, the motion of which parted through a succession of other wheels to the hands. The ment thus imparted to the hands would be subject to all the ion of the relaxing force of the spring, were it not for the interon of the regulating effect of the balance-wheel, which acts in a er precisely similar to the pendulum just described.

2. *The water regulator.* — Another expedient sometimes used e regulation of the movement of machinery, is the water regu- This consists of a small pump, worked by the machine which be regulated, and which throws water into a reservoir, from it flows by a pipe of given magnitude. When the water is ed up with the same velocity as that with which it is discharged e pump, the level of the water in the reservoir will remain staу; if more is pumped up than is discharged, the level will rise; f less, it will fall.

is evident, therefore, that every fluctuation in the velocity of the ine will be indicated by the rising or falling of the level of the in the cistern; and that this level never can remain stationary, t when the machine has that exact velocity which supplies the ity of water discharged by the pipe. This pipe may be so ad- d as to discharge the water at any required rate, and thus the n may be adapted to indicate a constant velocity of any pro- l amount.

the cistern be constantly watched by an attendant, the velocity e machine may be abated when the level of the water is ob- d to rise, or increased when it falls, by regulating the power; his is done much more effectually and regularly by causing the e of the water itself to perform the duty. A float, or large w metal ball, is placed upon the surface of the water in the n. *This ball is connected with a lever* acting upon some part of the machinery which controls the power or regulates the amount of resistance, as already explained in the case of the governor. When the level of the water rises, the buoyancy of the ball causes it also to rise with a force equal to the difference between its own weight and the weight of as much water as it displaces. By enlarging the floating ball a force may be obtained sufficiently great to move those parts of the machinery which act upon the power or resistance, and thus either to diminish the supply of the moving principle, or to increase the amount of the resistance, and thereby retard the motion and reduce the velocity to its proper limit.

When the level of the water in the cistern falls, the floating ball, being no longer supported on the liquid surface, descends with the force of its own weight, and producing an effect upon the power or resistance contrary to the former, increasing the effective energy of the one or diminishing that of the other, until the velocity proper to the machine be restored.

The sensibility of these regulators is increased by making the surface of water in the cistern as small as possible, for then a small change in the rate at which the water is supplied by the pump will produce a considerable change in the level of the water in the cistern.

Instead of using a float, the cistern itself may be suspended from the lever which controls the supply of the power, and in this case a sliding weight may be placed on the other arm so that it will balance the cistern when it contains that quantity of water which corresponds to the fixed level already explained. If the quantity of the water in the cistern be increased by an undue velocity of the machine, the weight of the cistern will preponderate, draw down the arm of the lever, and check the supply of the power. If, on the other hand, the supply of water be too small, the cistern will no longer balance the counterpoise, the arm by which it is suspended will be raised, and the energy of the power will be increased.

513. *Fusee in watchwork.*—The effect of a power of variable energy may be rendered uniform by transmitting it to the working point, through the agency of a leverage of corresponding variation so regulated, that, in the same proportion as the power diminishes in energy, the leverage shall increase, and *vice versâ*. A well-known example of this occurs in the construction of certain watches, where the moving power, being a mainspring inclosed in a barrel, has a gradually diminished energy as the spring is relaxed. The chain as it is discharged from the barrel is coiled upon a conical spiral, called a *fusee*, represented in *fig.* 144. The leverage by which the force

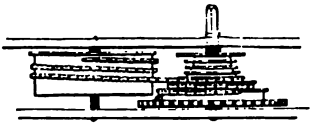

Fig. 144.

[redacted]

of the spring is transmitted, being the semi-diameter of the fusee, and the motion commencing from the top, or narrowest end, it follows that when the energy of the spring is greatest the leverage is least; and as the chain coils upon the barrel containing the spring, and is discharged from the fusee, the radius of each part of the fusee which discharges the chain gradually increases. The form of the fusee is ch that this increase of leverage is in the exact proportion of the minished force of the spring.

14. *When the efficacy of the machine to transmit the power to working point varies.* — The several parts of a machine have in periods of motion, in which they pass through a variety of ons, returning constantly to similar positions after equal intervals. he different positions assumed by the moving parts during these s, the effect of the power transmitted to the working point is nt; and cases even occur in which for a moment this effect is her interrupted, and the machinery is then in a predicament in the power loses all effect upon the resistance. The consequence would be, that, supposing the power and resistance to be both n, the action of the former upon the latter would be subject to cal variation, being at one time more and at another time less ould be necessary to keep the whole in equilibrium.

er these circumstances, it is possible to suppose that the movef the machine may continue, and even that its average rate uniform; but its motion would be subject to periodical variabeing alternately accelerated and retarded. This would be d not only with an injurious effect upon the work produced by chine, but would be also detrimental to the machine itself, moving parts would be subject to continual starts and strains from the alternate reception and destruction of momentum.

Effect of a crank. — To render these general observations learly intelligible, we shall take, as an example, the action of non crank, used in steam-engines and many other machines. rank is nothing more than a double winch. It is represented complete with both its arms in *fig.* 145. Attached to the middle of C D, by a joint, is a rod, which is the means of imparting the effect of the power to the crank. This rod is driven by an alternate motion like the brake of a pump. The bar C D is carried with a circular motion round the axis A F. Let the machine viewed in the direction A B E F of the axis be conceived to be represented in *fig.* 146., where A represents the centre round which the is to be produced, and G the point where the connecting rod attached to the arm *of the crank*. The circle through which *be urged by the rod is represented* by the dotted line. In the

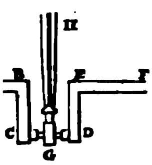

Fig. 145.

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position represented in *fig.* 146., the rod acting in the direction H G has its full power to turn the crank G A round the centre A. As the crank comes into the position represented in *fig.* 147., this power is diminished, and when the point G comes immediately below A, as in *fig.* 148., the force in the direction H G has no effect in turning the crank round A, but, on the contrary, is entirely expended in pulling the crank in the direction G A, and therefore only acts on the pivots or gudgeons which support the axle.

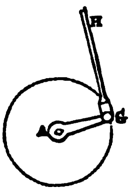

Fig. 146.

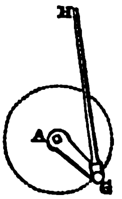

Fig. 147.

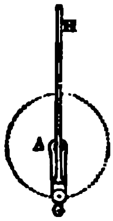

Fig. 148.

At this crisis of the motion, therefore, the whole effective energy of the power is annihilated.

After the crank has passed to the position represented in *fig.* 149., the direction of the force which acts upon the connecting rod is changed, and now the crank is drawn upward in the direction G H. In this position the moving force has some efficacy to produce rotation round A, which efficacy continually increases until the crank attains the position shown in *fig.* 150., when its power is greatest. Passing from this position its efficacy is continually diminished until the point G comes immediately above the axis A, *fig.* 151. Here again the power loses all its efficacy to turn the axle. The force in the direction G H or H G can obviously produce no other effect than a strain upon the pivots or gudgeons.

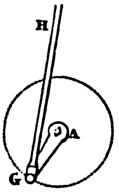

Fig. 149.

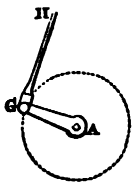

Fig. 150.

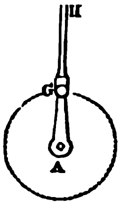

Fig. 151.

It will be evident from this that the action of the power transmitted to the working point G is very variable. At the dead points represented in *figs.* 148. and 151., the machine, if depending solely upon the moving power, must come to rest, for at both points the whole effect of the power would be exerted in producing pressure on the axle and gudgeons of the crank. Through a small space at either side of those dead points, the effect transmitted to G, though not absolutely nothing, is almost evanescent, so that it may be considered that through a small arc at either side of each of the dead points the machine is still inert.

It must, however, be considered, that, in virtue of its inertia, the motion which the machinery had previously to its arrival at its dead points has a tendency to continue; and if the resistance of the load and the effects of friction be not too great, this disposition to preserve its state of motion will extricate the machinery from the mechanical dilemma in which it is involved in these cases by the particular disposition of its parts. Although, however, the motion will not therefore, be actually suspended, on the arrival of the crank at the dead points, it will be greatly retarded; and, on the other hand, when the power acquires its greatest activity, as it does in the position represented in *figs.* 146., 150., it will be unduly accelerated.

516. *Regulating effect of a fly-wheel.* — These irregularities are equalized by fixing upon the axis of the crank, or at any other convenient part of the machine, a fly-wheel, which is a massive ring of metal, connected with a central box or nave, by comparatively light spokes, and turning on an axis with but little friction. If any force be applied to it, with that force, making some slight deduction for friction, it will move and will continue to move until some obstacle retard it, which obstacle will receive from it as much force as the fly-wheel loses.

The effect of such a wheel applied to the parts moved by the crank will equalize the inequality which has just been described. When the crank assumes the position represented in *figs.* 146., 150., where the power has full play upon it, the effect of the power is partly transmitted to the machine, and partly received by the moveable rim of the fly-wheel, to which it imparts increased momentum. There is here, it is true, an acceleration of the motion, but one which is comparatively small, inasmuch as the great mass of the fly-wheel receives the momentum without sensible increase of speed. When the crank gets into the predicament represented at the dead points *figs.* 148., 151., the momentum of the fly-wheel, received when the crank acted with most advantage, immediately conveys its force to the working-point G, extricates the machine, and carrying the crank out of the neighbourhood of the dead point, brings the power again to bear upon it.

517. *Methods of augmenting or mitigating the energy of the moving power.* — It happens frequently in the practical application of machinery, that the moving power is much too intense, or much too

feeble, for the resistance; and in the one case contrivances are required by which it may be greatly attenuated, and in the other by which it may be greatly augmented.

518. *Case of watchwork moved by a main-spring.*—In the case of watch-work, the resistance to be overcome is nothing more than that presented by the hands which move upon the dial plate. In this case the moving power is the force of the fingers, by which once in twenty-four hours the main-spring is wound up. The main-spring itself must be regarded, in this case, as a mere depository for the power exerted in winding up the watch, and not as a prime mover. The force which is thus deposited once in twenty-four hours in the main spring is delivered gradually and regularly, by such spring, to the fusee, and transmitted, through the system of wheels, to the hands.

In the case of clocks moved by main-springs instead of a weight, this attenuation of the moving power is still more extensive. These, being wound up, will frequently go for fifteen days. In this case, therefore, the mechanical force exerted by the hand in winding them up, and which is developed in less than a minute, is spread over fifteen times twenty-four hours by the mechanism of the clock.

519. *Case of clockwork moved by a weight.*—In the case of clocks which go by weight, the original moving force is also the application of the human hand in winding up the weight. The weight being lifted a height of three or four feet, descends slowly through the same height, imparting its descending force gradually and regularly to the clockwork. In this case, therefore, the descending force of a weight through a small height is so attenuated as to impart a motion to the hands which will continue sometimes for a month or longer.

520. *Cases in which the energy of the power is augmented.*—It is frequently required, on the contrary, to impart to the resistance a force vastly greater in intensity than the moving power. Numerous examples of this have been already given in illustrating the simple machines. In all cases where the leverage of the power is greater than the leverage of the resistance, there will be an augmented intensity of mechanical action in the same proportion; and this intensity, by combining levers or other simple machines, may be augmented without any practical limit.

521. *Analysis of the action of hammers, sledges, &c.*—But in some cases a force is required more intense than can be obtained even by these means. In such cases, it becomes necessary to convert the continued agency of the moving power into one which acts instantly and by intermission. If, for example, it be required to cause a nail to penetrate a beam of wood, we should attempt in vain to accom-

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ush this, by producing any pressure, however great, on the head of he nail. A few blows of a hammer, nevertheless, easily effect this. In this case, the moving power is the hand, or other force which raises the hammer. The mass of the hammer, in falling on the head of the nail, imparts instantly to the nail the entire force which was exerted in lifting it, but with this difference, that such force, in raising the hammer, was developed in a certain definite time, whereas it is discharged upon the head of the nail in an instant.

The same observations apply to all cases in which percussion is used. In all these cases the force is developed in a definite time, but is discharged upon the resistance in an instant.

522. *The pile engine.* — The pile engine, in which heavy weights are raised to a certain height by the moving power, as represented in *fig.* 152., and let fall upon the heads of the piles to be driven, presents an example of this. The entire force exerted in raising the weight is discharged instantly on the head of the pile the moment the descending weight strikes it.

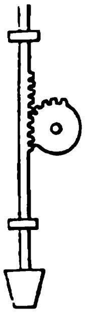

Fig. 152.

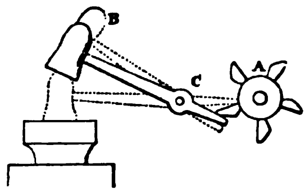

Fig. 153.

523. *The sledge-hammer.* — The sledge-hammer used in forging iron presents another example of this. The hammer is raised by machinery through the intervention of various mechanical expedients, such, for example, as that represented in *fig.* 153., where it is elevated by teeth A, called cams or wipers, attached to a wheel, which press down one arm of the lever C to which the sledge is attached, and raise the other. When the wiper passes it, the sledge falls upon the anvil with the full weight.

524. *Inertia supplies means of accumulating force.* — In some cases where a severe instantaneous action is required, the moving power is accumulated by means of the inertia of matter. A mass of matter retains by virtue of its inertia the whole amount of any force which may be given to it, except that part of which friction and the atmospheric resistance deprives it.

To render this method of accumulating force intelligible, let us first imagine a polished level plane, on which a heavy globe of metal, also polished, is placed. It is evident that the globe will remain at rest on any part of the plane without a tendency to move. Suppose, then, a slight impulse be given to it, which will cause it to move with any given velocity, for example, three feet per second.

It will then continue to move with this velocity for any length of time, except so far as it may be impeded by the resistance already mentioned.

Let us then imagine a second impulse given to it equal in force to the former: this will increase its velocity to six feet per second; a third impulse will augment it to nine feet per second, and so on. Now there is no limit to the number of impulses which may be successively given to the moving body, provided only space were given for its motion. Thus, ten thousand repetitions of the impulse would make the body move at the rate of thirty thousand feet a second. If the body to which these impulses were transmitted were a cannon-ball, it might, by the constant application of a feebly impelling force, be made to move at length with as much force as if it were impelled from a powerful piece of ordnance. The force with which such a ball would strike a buttress might be sufficient to reduce it to ruins; and yet such force may be nothing more than the accumulation of a number of feeble forces, not beyond the power of a child to exert, which are stored up and preserved in the moving mass, and then brought to bear at the same moment on the resistance against which the force is directed. It is the same for any number of actions exerted successively and during a long interval, brought into operation at one and the same moment.

But the case here supposed cannot actually occur, because we have not in general practical means of moving a body for a considerable time in the same direction, without much friction, and without encountering other obstacles which would impede its progress. If, however, a leaden ball be attached to the end of a string and whirled rapidly round, a great force would be given to it, and it will strike a board with such intensity as to penetrate it.

525. *Effect of weapon called life-preserver, flails, &c.*—A weapon called a life-preserver consists of a piece of lead sometimes attached to the end of a piece of cane or whalebone, with which a blow may be given with great force.

Innumerable examples of the application of this principle will present themselves to every mind. Flails used in threshing, clubs, canes, whips, and all instruments used for striking, axes, hatchets, cleavers, and all instruments which act by a blow, present examples of this principle.

526. *Loaded lever of screw-press.*—Where very intense force is

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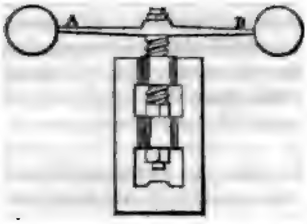

Fig. 154.

required, as, for example, in certain presses, two heavy balls are attached to the ends of a horizontal lever A B, with equal arms, *fig.* 154. This lever works a screw, at the lower end of which is the working point. A rapid motion is imparted to the balls by the hand, and the working point is driven against the resistance by the accumulated momentum acquired by the balls, augmented by the leverage of the arms to which they are attached, and the mechanical force of the screw.

527. ***This accumulation of force involves no paradox.***—The surprising effects produced by the accumulation of force are apt to lead to erroneous suppositions, that instruments thus acting by inertia have the effect of actually augmenting the amount of moving power. When the quality of inertia, however, is rightly understood, such an error cannot occur. The instruments by which force is thus accumulated, so far from augmenting the effect of the moving power, must to some extent diminish it; inasmuch as they are liable to friction and atmospheric resistance, by which more or less force is intercepted. An accumulator of force, whatever be its form, can never have more force than has been applied to put it in motion. Whether it be a falling weight, a revolving mass, a string which is coiled up, or air which is condensed, it cannot develop a greater amount of force than that which is imparted in raising it if it be a weight, in putting it in motion if it be a moving mass, in winding it up if it be a string, or in compressing it if it be air. The only difference between the power which is imparted to these agents, and the effects which they produce respectively upon the resistance, is in the time during which the effects are developed. The power is in general imparted slowly, while the effects are produced instantaneously.

528. ***Rolling and punching mills—effect of fly-wheels.***—Mills for rolling metals, or for punching boiler plates, supply striking examples of this. The water-wheel, or steam-engine, or whatever other power be used, is allowed for some time to act upon the fly-wheel alone, no load being placed upon the machine. When a sufficient momentum has been imparted to the mass of metal forming the fly-wheel, the metal to be rolled or pierced is submitted to the machine, and is immediately flattened or perforated by it, depriving at the same time the fly-wheel of a corresponding quantity of its momentum.

In the same manner, a force may be obtained by the arm of a man acting on a fly for a few seconds, sufficient to impress an image on a piece of metal by an instantaneous stroke. The fly is therefore the principal agent in coining-presses.

Some presses used in coining have flies with arms four feet long, bearing a hundred weight at each of their extremities. If such a velocity be imparted to such an arm that it shall make one revolution per second, the die will be driven against the metal with the same force as that with which 3¾ tons would fall from the height of 16 feet, which is an enormous power if the simplicity and compactness of the machine be considered.

529. *Position of the fly-wheel.* — The place to be assigned to a fly-wheel relatively to the other parts of the machinery is determined by the purpose for which it is used. If it be intended to equalize the action, it should be near the working point. Thus, in a steam-engine, it is placed near the crank which turns the axle, by which the power of the engine is transmitted to the object it is finally designed to affect. On the contrary, in hand-mills, such as those commonly used for grinding coffee, &c., it is placed upon the axis of the winch by which the machine is worked.

530. *Method of cutting open-work in metal.* — The open-work of fenders, fire-grates, and similar ornamental articles constructed in metal, is produced by the action of a fly in the manner already described.

The cutting tool, shaped according to the pattern to be executed, is attached to the end of the screw, and the metal being held in a proper position beneath it, the fly is made to urge the tool downwards with such force as to stamp out pieces of the required figure. When the pattern is complicated, and it is necessary to preserve with exactness the relative situation of its different parts, a number of punches are impelled together, so as to strike the entire piece of metal at the same instant, and in this manner the most elaborate open-work executed by a single stroke of the hand.

CHAP. VIII.

THE PENDULUM.

531. *Oscillation of a pendulous mass.* — Of the various mechanical contrivances comprised in the class of expedients called regulators, by far the most important is the pendulum. A pendulum consists of a heavy mass attached to a rod, the upper extremity of which rests upon a point of support in such a manner as to have as little friction as possible. Such an instrument will remain at rest when its centre of gravity is in the vertical line immediately under the point of suspension or support. But if the centre of gravity be drawn from this position on either side, and then disengaged, the instrument will

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swing horizontally from the one side to the other of the position in which it would remain at rest, the centre of gravity describing alternately a circular arc on the one side or the other of its position of rest. If there were neither friction nor atmospheric resistance, this motion of vibration or oscillation on either side of the position of equilibrium would continue for ever; but in consequence of the combined effects of these resistances, the distances to which the pendulum swings on the one side and on the other are continually diminished, until, after the lapse of an interval, more or less protracted, it comes to rest.

532. *Isochronism of the pendulum.* — It is related that Galileo, when a youth, happening to walk through the aisles of a church in Pisa, observed a chandelier suspended from the roof, whose position had been accidentally disturbed, and which was consequently in a state of oscillation. The young philosopher, contemplating the motion, was struck with the fact, that although the ranges of its vibration were continually diminished as it approached a state of rest, the times of the vibration were sensibly equal, the motion becoming slower as the number of the oscillations became more limited. This led Galileo to infer that property of the pendulum which is expressed by the word *isochronism*, in virtue of which the vibrations, whether in longer or shorter arcs, are performed in the same time.

Although, however, as we shall presently show, pendulums possess this property when the arcs of vibration are very small, they do not continue to manifest it when the range of vibration becomes more considerable.

533. *Analysis of the vibration of a pendulum.* — To simplify the exposition of the important theory of the pendulum, it will be convenient, in the first instance, to consider it as composed of a heavy mass of small magnitude, suspended by a wire or a string, the weight of which may be neglected. Thus, let us suppose a small ball of lead suspended by a fine silken string, the length of which is incomparably greater than the diameter of the leaden ball. Such an arrangement is called the *simple pendulum*.

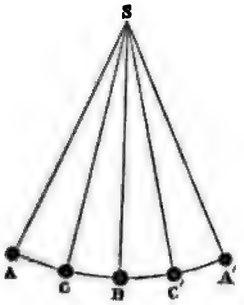

Fig. 155.

Let S, *fig.* 155., be the point of suspension; let S B be the fine silken thread by which the ball B is suspended, and the weight of which, in the present case, is neglected. Let B be the position of the ball when in the vertical under the point of suspension S. In that position the ball would remain at rest; but if we suppose the ball drawn aside to the position A, it will, if disengaged, fall down the arc A B, of which the centre is S, and the radius the length of

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the string. Arriving at B, it will have acquired a certain velocity, which, in virtue of its inertia, it will have a tendency to retain, and with this velocity it will commence to move through the arc B A'. Supposing neither the resistance of the atmosphere nor friction to act, the ball will rise through an arc B A' equal to B A; but it will lose the velocity which it had acquired at B, for it is evident that it will take the same space, and the same time to destroy, the velocity which has been acquired, as to produce it. Thus, the velocity at B, being acquired in falling through the arc A B, will be destroyed in rising through the equal arc B A'.

Having arrived at A', the ball, being brought to rest, will again fall from A' to B, and at B will have again acquired the same velocity which it had obtained in falling from A to B, but in the contrary direction; and in the same manner it may be explained that this velocity will carry it from B to A. Having arrived at A, the ball, being again brought to rest, will fall once more from A to B, and so the motion will be continued alternately between A and A'.

The motion of the pendulum from A to A', or from A' to A, is called an *oscillation*, and its motion between either of those points and B is called a semi-oscillation, the motion from B to A or from B to A' being called the ascending semi-oscillation, and the motion from A or A' to B, the descending semi-oscillation.

The time which elapses during the motion of the ball between A and A' is called the *time of one oscillation*.

It is evident, from what has been stated, that the time of moving from either of the extremities A A' of the arc of oscillation to the point B, is half the time of an oscillation.

If, instead of falling from the point A, the ball had fallen from the point C, intermediate between A and B, it would have then oscillated between C and C'; two points equally distant from B, and the arc of oscillation would have been C C', more limited than A A'.

But in commencing its motion from C, the declivity of the arc down which it falls towards B would be evidently less than the declivity at A; consequently, the force which would accelerate it, commencing its motion at C, would be less than that which would accelerate it, commencing its motion at A. The ball, therefore, commencing its motion at A, would be more rapidly accelerated than when it commences its motion at C.

The result of this is, that, although the arc A B may be twice as long as the arc C B, the *time* which the ball takes to fall from A to B will not be sensibly different from the time it takes to fall from C to B, provided that the arc of oscillation A B A' is not considerable.

It was at first supposed, as we have just stated, that, whether the oscillations were longer or shorter, the times would be absolutely the same. Accurately speaking, however, this is not the case; but if the *total extent* of the oscillation A A' do not exceed 5° or 6°, then the

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oscillation in it may be considered, practically, the same as ser arcs.

Experimental verification of isochronism. — This important may be easily experimentally verified. Let two small leaden uspended from the same point of support, but one being in f the other, so that in oscillating the two balls shall not h other. This being done, let one of the balls be drawn int of rest through an angle less than 3°, and let it be dis- It will oscillate as described above. Let the other ball be from its point of rest through a much less angle, and let ngaged that it shall commence its oscillation at the same th the commencement of one of the oscillations of the

short, be so managed, that when the one ball is at A, the e at C; and that both shall commence their descending rds B at the same moment. It will be then found that ions will be synchronous for a considerable length of s to say, the balls will arrive at A′ and C′, respectively, instant; and returning, will simultaneously arrive at A tively.

case, the oscillation of the ball A were made through an great as 10°; that is to say, 5° each side of the vertical, n of the ball C being made through an arc of 2°, it nd that 10,001 oscillations of the latter would be equal cillations of the former, so that the actual difference be- times of oscillation would not exceed the ten thousandth time.

ntaining power. — In the practical application of the owever, this departure from absolute isochronism, small omes unimportant; for a maintaining power is always which the loss of motion which would be produced by atmospheric resistance is repaired, and the magnitude of ns is maintained uniform.

e of oscillation independent of the weight of the pendu- ght be expected that the time of oscillation of different rould depend, more or less, upon the weight of the mat- g them, and that a heavy body would oscillate more a lighter one. Both theory and experience, however, sult to be otherwise. The force of gravity which causes m to oscillate acts separately on all the particles com- ass; and if the mass be doubled, the effect of this force lso doubled; and, in short, in whatever proportion the pendulum be increased or diminished, the action of the ity upon it will be increased or diminished in exactly the tion, and *consequently the* velocity imparted by gravity *lous mass at each instant* will be the same.

It is easy to verify this by experiment. Let different balls of small magnitude, of metal, ivory, and other materials, be suspended by light silken strings of the same length, and made to oscillate; their oscillations will be found to be equal.

537. *How the time of oscillation is affected by the length.*—If pendulums of different lengths have similar arcs of oscillation, the times of oscillation of those which are shorter will be less than the times of oscillation of those which are longer. Let *a*, *b*, *c*, *d*, and *e*, *fig*. 156, be five small leaden balls, suspended by light silken strings to the point of suspension S, and let all of those be supposed to form pendulums, having the same angle of oscillation. The arc of oscillation of the ball *a* will be $a\,a''''$, that of *b* will be $b\,b''''$, that of *c*, $c\,c''''$, and so on. In commencing to fall from the points *a*, *b*, *c*, *d*, *e*, towards the vertical line, these five balls are equally accelerated by the said fall, inasmuch as the circular arcs down which they fall are all equally inclined at this point to the vertical line. The same will be true if we take them at any corresponding points, such as a', b', c', d', e'. It may therefore be concluded, that throughout the entire range of oscillation of each of these five pendulums, they will be impelled by equal accelerating forces.

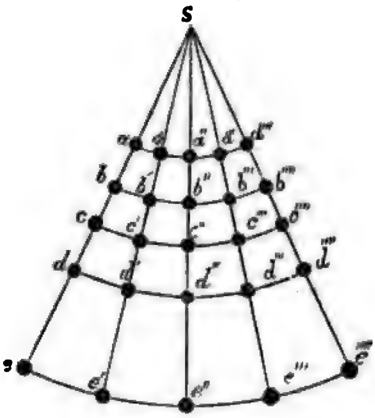

Fig. 156.

Now it has been shown that when bodies are impelled by the same or equal accelerating forces, the spaces through which they move are proportional to the squares of the times of their motion; therefore it follows, that the lengths of these arcs of oscillation are proportional to the squares of the times. But the lengths of these arcs are evidently in the same proportion as the lengths of the pendulums; that is to say, the arc $a\,a''''$ is to $b\,b''''$ as S *a* is to S *b*, and the arc $b\,b''''$ is to $c\,c''''$ as S *b* is to S *c*, and so on.

It follows, therefore, that the squares of the times of oscillation of pendulums are as their lengths, or, what is the same, the times of oscillation are as the square roots of their lengths. This principle is easily verified experimentally.

538. *Experimental illustration.*—Let three small leaden balls be suspended vertically under each other by means of loops of silken threads, as represented in *fig*. 157, and in such a manner that they can all oscillate in the same plane at right angles to the plane of the

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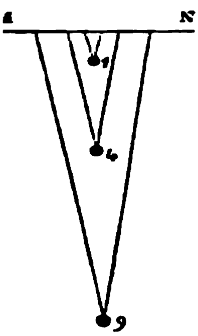

Fig. 157.

diagram, the suspending loops not interfering with each other.

Let the loops be so adjusted that the distance of the ball 1 below the line M N shall be 1 foot, the distance of the ball 4, 4 feet, and the distance of the ball 9, 9 feet.

Let the ball 9 be put in a state of oscillation through small arcs, and let the ball 4 be then drawn from its vertical position, and disengaged so as to commence one of its oscillations with an oscillation of the ball 9; and in the same manner let the ball 1 be started simultaneously with one of the oscillations of the ball 9.

It will be found that two oscillations of the one-foot pendulum are made in exactly the same ime as a single oscillation of the four-foot pendulum; consequently, he time of each oscillation of the latter will be double that of the ormer, while its length is fourfold that of the former.

In the same manner, while the one-foot pendulum makes three osillations, the nine-foot pendulum will make one; and, consequently, he time of oscillation of the latter will be three times that of the ormer, while its length is nine times that of the former.

539. *The time of vibration being given, to find the length of the pendulum, and vice versâ.* By this principle, the length of a pendulum which would oscillate in any proposed time, or the time of oscillation of a pendulum of any proposed length, can be ascertained, provided we know the length of a pendulum which oscillates in any given time. Thus, suppose L to be the length of a pendulum which oscillates in the time T. Let it be required to determine the length of a pendulum L′, which would oscillate in any other time T′. We shall have the following proportions:

$$L : L' :: T^2 : T'^2.$$

From this proportion, if L and T be both given, we can find the time T′ of oscillation of the other pendulum if L′ be given; or we can find the length L′ if T′ be given.

In the first case we have

$$T'^2 = T \times \frac{L'}{L};$$

in the second we have

$$L' = L \times \frac{T'^2}{T^2}.$$

540. *Time of oscillation varies with the attraction of gravity.*— Since the force which produces the oscillation of a pendulum is the accelerating force of gravity urging the pendulous body alternately from the extremities of the arc of oscillation to the middle point of that arc, it is evident that if this force were increased in its intensity, the velocity with which the pendulous body would be precipitated to its lowest position would be increased, and consequently the time of oscillation diminished; and if, on the other hand, the impelling force of gravity were diminished, the force urging the pendulous body being enfeebled, it would be moved with a diminished velocity, and, consequently, the time of oscillation would be increased.

It follows, therefore, that the same pendulum will oscillate more slowly or more rapidly, according as the force of gravity which acts upon it is diminished or increased.

541. *Law of this variation.* — But it is not enough to state that a variation in the force of gravity will change the time of oscillation of the pendulum. It is required to ascertain in what proportion it will produce this change; that is to say, if the force of gravity acting on the pendulum be augmented in any given ratio, in what corresponding ratio will the time of oscillation of such pendulum be diminished.

It is proved in the theory of accelerating forces, that under such circumstances, the squares of the times of oscillation will vary in the inverse proportion of the force; that is to say, in whatever ratio the force of gravity be augmented, the squares of the times of oscillation of the pendulums will be diminished in the same ratio.

542. *The pendulum indicates the variation of gravity in different latitudes.* — But as the squares of the times of oscillation are proportional to the lengths of the pendulums, it follows from this, that the lengths of the pendulums which oscillate in the same time under the influence of different accelerating forces, will be proportional to these forces; and that, consequently, if in any two places it be found that the pendulums which oscillate in the same time hang different lengths, it must be inferred that the forces of gravity in these two places are in the exact proportion of these lengths.

It is in virtue of this principle that the pendulum supplies means of determining the variation of the forces of gravity upon different parts of the earth's surface.

543. *Analysis of the motion of a pendulous mass of definite magnitude.* — We have hitherto supposed that the pendulous body is a heavy mass of indefinitely small magnitude, suspended by a wire or string having no weight. These are conditions which cannot be fulfilled in practice. Every real pendulous body has a definite magnitude, its component parts being at different distances from the point of suspension; the rod which sustains it is of considerable weight, and all the points of this rod, as well as those of the pendulous mass itself, are at different distances from the point of suspension. In es-

timating, therefore, the effect of pendulums, it is necessary to take into account this circumstance.

Let us suppose *a, b, c, d, e, f, g, fig.* 158., to be as many small heavy balls connected by independent strings, the weight of which may be neglected, with a point of suspension S, and let these seven balls be supposed to vibrate between the positions S M and S M′. Now if these balls were totally independent of each other, and connected with the point of suspension by independent strings, they would all vibrate in different times, those which are nearer the point S vibrating more rapidly than those which are more distant from it. If, therefore, they be all disengaged at the same moment from the line S M, those which are nearest to S will get the start of those which are more distant, and at any intermediate position between the extremes of their vibration they will assume the positions *a′, b′, c′, d′, e′, f′, g′*. That which is nearest to the point S, and which is the shortest pendulum, will be foremost, since it has the most rapid vibration. The next in length, *b′*, will follow it, and so on; the most remote from S being the longest pendulum, *g′* being the last in order.

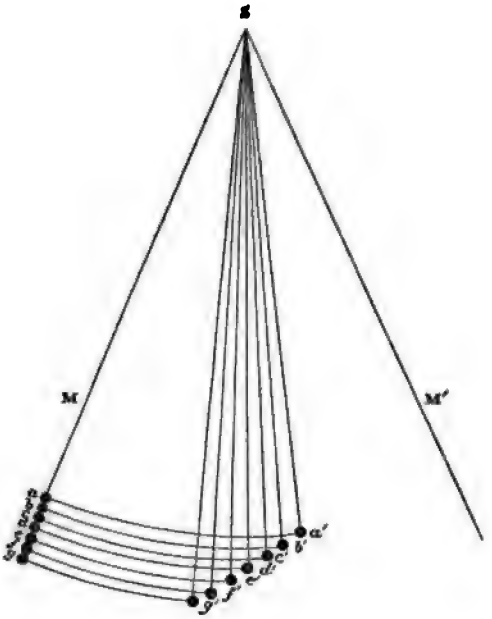

Fig. 158.

Now if, instead of supposing these seven balls to be suspended by independent strings, we imagine them to be fixed upon the same wire, so as to be rendered incapable of having any independent motion, and compelled to keep in the same straight line, then it is evident, that while the whole series vibrates with a common motion, those which are nearest to the point of suspension will have a tendency to accelerate the motion of those which are more distant, while those which are more distant will have a tendency to retard the motion of those which are nearer.

These effects will produce a mutual compensation; *b* and *c* will vibrate slower than they would if they were moving freely, while *e* and *f* will evidently move more rapidly than if they were moving freely. Among the series, there will be found a certain point, which will separate those which are moving slower than their natural rate, from those which are moving faster than their natural rate; and a ball placed at this point would vibrate exactly as it would do if no other balls were placed either above or below it. Such a ball would, as it were, be the centre which would divide those which are accelerated from those which are retarded.

544. *Centre of oscillation.* — Such a point has, therefore, been denominated the *centre of oscillation.*

It is evident, therefore, that a pendulous mass, of magnitude more or less considerable, will vibrate in the same time as it would do if the entire mass were concentrated at its centre of oscillation, and formed there a material point of insensible magnitude.

By the length of a pendulum, no matter what be its form, therefore, is always to be understood the distance of its centre of oscillation from its point of suspension.

545. *Centres of oscillation and suspension interchangeable.* — The centre of oscillation has the following remarkable and important quality, which is established by the higher mathematics, and verified by experiment.

If a pendulum be inverted and suspended by its centre of oscillation, its former point of suspension will become its new centre of oscillation, and the time of vibration will remain the same as before. This property is usually expressed by stating that the "centres of suspension and oscillation are interchangeable."

This property can be verified by experiment. If the centre of oscillation of any pendulous body be ascertained by mathematical calculation, and that it be taken as a point of suspension, it will be found that the time of oscillation of the pendulum will be the same as it was with the first point of suspension.

Since the length of a pendulum, and, therefore, the time of its oscillation in a given place and subject to a given intensity of the force of gravity, depends upon the distance between its point of suspension and its centre of oscillation, it is evident that any variation which may take place in this distance will cause a corresponding

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nge in the time of oscillation; and if the pendulum be applied to
hronometer, it will cause a variation in the rate of that instrument,
l a corresponding error in its indications.

46. *Variations of a pendulum consequent on change of temper-*
re. — Now, it is found in practice, that all pendulums are subject
a change, more or less, in their form and magnitude, in conse-
nce of the change of temperature of the atmosphere to which
y are exposed. With this change they expand and contract, and
h every expansion and contraction the distance between their cen-
of oscillation and suspension will be varied, unless expedients be
pted to counteract such an effect. Although the variations pro-
ed by these causes are not sufficiently great to render it necessary
provide a correction for them in common time-pieces, yet, in cases
ere extreme accuracy is required, expedients have been adopted to
vent the consequent error.

547. *Compensation pendulums.* — These expedients are called
mpensation pendulums.

The principle upon which all these depend is the combination of
o substances in the structure of the pendulum, which expand in
equal degrees for the same change of temperature; and they are so
ranged, that while the expansion of the one increases the distance
the centre of oscillation from the point of suspension, the expan-
on of the other has the contrary effect, and the dimensions of these
ro substances are so adjusted that the increase of distance produced
y the one shall be exactly equal to the diminution of distance pro-
uced by the other, so that the result is that the centre of oscillation
mains at the same distance from the point of suspension, and there-
re the time of oscillation of the pendulum remains unaltered.

548. *Pendulum a measure of time.* — The first, the most impor-
nt, and the most universal use of the pendulum, is as a measure of
ime. The uniformity of the rate of its vibration is the property
hich renders it so eminently qualified for this purpose.

A pendulum vibrating alone, independently of any mechanism,
ould measure the time which elapses during its motion. It would
e only necessary for an observer to sit by it and count the number
f its oscillations. If the time of one oscillation were previously
nown, then the number of oscillations performed in any interval
ould at once give the length of such interval.

But, in order to supersede the attention and vigilance of such an
bserver, a train of wheel-work is placed in connexion with the pen-
ulum, the movement of which it regulates; and in connexion with
his train of wheel-work are fixed the dial-plate and the hands of the
lock, by which the number of oscillations of the pendulum which take
lace in a day, or in any part of a day, are indicated and registered.

549. *Pendulum a measure of the force of gravity.* — When the
me *pendulum is transported to different* parts of the earth's surface,

it is found that the rate of its vibration varies, and this variation is proved to take place even after precautions have been taken to keep the centres of oscillation and suspension at the same distance from each other. Now this change in the rate of vibration under such circumstances can only be explained by a change in the intensity of the force of gravity by which the pendulum is moved. It is found, that when the pendulum is carried towards the terrestrial equator, the time of its vibrations is longer; and that when it is carried towards the pole, the time of its vibrations is shorter: the inference deduced from which is, that the force of gravity diminishes as we approach the equator, and increases as we approach the pole.

If the earth had the form of an exact sphere, did not revolve on its axis, and was of uniform density, the force of gravity at all parts of its surface would be the same, and no such variation in the rate of a pendulum could take place when transported from one point of the surface of the earth to another. But if the earth be an exact sphere, revolving upon its own axis in 23h. 56 min., then the effect of such motion of rotation would be to produce a certain small diminution of the intensity of the force of gravity in approaching the equator, and an increase in such intensity in approaching the pole The amount of such diminution or increase produced by such rotation is capable of calculation, and, being computed and compared with such change of intensity of the force of gravity indicated by the variations of a pendulum, is found not to correspond exactly with it. This absence of complete correspondence indicates another cause affecting the force of gravity besides the rotation of the earth.

If the earth be not an exact sphere, but have a form of which a turnip and an orange are exaggerated representations, called in geometry an oblate spheroid, such a form, combined with the rotation of the earth, would produce a further effect in varying the force of gravity in proceeding towards the equator or towards the pole. Now it is found by calculation, that a certain degree of this form, combined with the diurnal rotation of the earth, would produce exactly that variation in the force of gravity going towards the equator and going towards the pole, which is indicated by the variation in the time of vibration of the same pendulum.

550. *Pendulum indicates the form and diurnal rotation of the earth.* — Hence it appears that the pendulum becomes an instrument by which not only the doctrine of the diurnal rotation of the earth is verified and corroborated, but by which the departure of the earth from an exact globular form is also established.

From what has been stated, it will appear that the length of a pendulum which vibrates seconds in different parts of the earth will be different; the force of gravity in lower latitudes being less than in higher, the length of the pendulum which vibrates seconds will be *proportionally* less.

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551. ***Pendulum measures the velocity of falling bodies.*** — As the ndulum thus supplies a measure of the intensity of the force of avity, it necessarily also affords the means of calculating the height om which a body falling freely would descend in a second if it oved in a vacuum.

The method of determining this by the pendulum is susceptible much greater accuracy than that which has been already indicated y Attwood's machine.

To find the space through which a body will fall at any place in a econd of time, let the length of a pendulum which vibrates seconds n that place, expressed in inches, be multiplied by 4·9347, and the roduct will express in inches the height through which a body would fall in a second in that place independently of the resistance of the air.

552. ***Table showing the lengths of a seconds pendulum, and the force of gravity in different latitudes.*** — In the following table is given the length of the pendulum vibrating seconds, and the heights through which a body would fall in a vacuum in a second at the places severally named in the first column. In the second column is given the latitudes of the places of observation. In the third column is given the lengths of the pendulum as determined by the observers themselves respectively, and expressed in the measures which they adopted; which, as it will appear, differed according to the different epochs of the experiments, and the different countries whose measures the observers used. Thus Borda expressed the length of the pendulum in lines, twelve of which composed an old French inch. The observations of Biot and his associates were made according to the system of measures of time and length adopted after the French Revolution; the lengths are accordingly expressed in millimetres, and the seconds measured by the pendulum were centesimal seconds, corresponding to the decimal division of the quadrant. The lengths of the pendulums given by Kater and Sabine are in English inches.

The lengths of the pendulums given by Freycinet and Duperry, have for their unit the length of a pendulum beating seconds at Paris. In the fourth column of the table is given the height of the stations at which the observations are respectively made above the level of the sea.

Since these heights produce an effect, more or less, on the intensity of gravity, it is necessary, in order to compare the observations one with another, to reduce them all to the level of the sea at their respective places. This is accordingly done in the fifth column, in which the length of pendulums vibrating seconds at the level of the sea, in the several places of observation, are given in English inches.

But as the force of gravity is more directly expressed by the height through which a body would fall freely in a vacuum in a second, these heights are given in the sixth column of the table, the names of the observers being inserted in the last column. The numbers in the third and fourth column of the table correspond with those given by Pouillet in his "Élémens de Physique."

TABLE OF OBSERVATIONS ON THE LENGTHS OF THE PENDULUM IN DIFFERENT LATITUDES.

Names of Stations.	Latitude.	Length of Pendulum at the Station.	Height of Station above the Level of the Sea.	Length of Secs. Pendulum reduced to the Level of the Sea.	Height through which Body will fall freely in 1 Second.	Names of Observers.
I.	II. ° ′ ″	III.	IV. *Feet.*	V. *Inches.*	VI. *Inches.*	VII.
Paris	48 50 14 N	440·5593	229·66	39·12	193·038	Borda.
Unst	60 45 25 N	742·721034	29·53	39·16	193·235	Biot.
Leith	55 58 37 N	742·408533	68·90	39·15	193·185	Biot.
Dunkirk	51 02 10 N	742·07610	13·12	39·13	193·087	Biot, Mathieu.
Paris	48 50 14 N	741·90112	229·66	39·12	193·038	Biot, Bouvard, Math.
Clermont	45 46 48 N	741·61059	1332·00	39·11	192·988	Biot, Mathieu.
Bordeaux	44 50 26 N	741·60464	55·78	39·11	192·988	Biot, Mathieu.
Figeac	44 36 45 N	741·56033	731·64	39·11	192·988	Biot, Mathieu.
Formentera	38 39 56 N	741·20540	666·00	39·09	192·890	Biot, Arago, Chaix.
Unst	60 45 28 N	39·17145	29·53	39·16	193·235	Kater.
Portsoy	57 40 59 N	39·16140	95·15	39·15	193·185	Kater.
Leith	55 58 41 N	39·15540	68·90	39·15	193·185	Kater.
Clifton	53 27 43 N	39·14517	337·93	39·14	193·136	Kater.
Arbury Hill	52 12 55 N	39·14077	738·20	39·14	193·136	Kater.
London	51 31 08 N	39·13908	91·87	39·13	193·087	Kater.
Shanklin Farm	50 37 24 N	39·13551	242·79	39·13	193·087	Kater.
Paris	48 50 14 N	1·00000000	229·68	39·12	193·038	Freycinet.
Mowi (Island) (Sandwich)	20 52 07 N	0·99792769	4·92	39·04	192·643	Freycinet.
Guam (Island)	13 27 51 N	0·99750268	6·56	39·03	192·593	Freycinet.
Rawak (Island) . . .	0 01 34 S	0·99709528	4·92	39·01	192·493	Freycinet.
Isle of France	20 09 56 S	0·99793739	60·86	39·04	192·643	Freycinet.
Rio Janeiro	22 55 13 S	0·99784413	16·40	39·04	192·643	Freycinet.
Port Jackson	33 51 34 S	0·99876087	106·29	39·07	192·791	Freycinet.
Cape of Good Hope .	33 55 15 S	0·99871111	49·21	39·07	192·791	Freycinet.
Falkland Islands . . .	51 35 18 S	1·00022130	19·69	39·13	193·087	Freycinet.
St. Thomas	0 24 41 N	39·02069	19·69	39·01	192·493	Sabine.
Maranham	2 31 43 S	39·01197	75·46	39·01	192·493	Sabine.
Ascension	7 55 48 S	39·02406	16·40	39·02	192·143	Sabine.
Sierra Leone	8 29 28 N	39·01954	190·15	39·01	192·493	Sabine.
Trinidad	10 38 56 N	39·01879	19·69	39·01	192·493	Sabine.
Bahia	12 59 21 S	39·02478	213·26	39·02	192·143	Sabine.
Jamaica	17 56 07 N	39·03508	9·84	39·03	192·593	Sabine.
New York	40 42 43 N	39·10153	65·62	39·10	192·939	Sabine.
London	51 31 08 N	39·13908	91·87	39·13	193·087	Sabine.
Drontheim	63 25 54 N	39·17428	121·39	39·17	193·284	Sabine.
Hammerfest	70 40 05 N	39·19512	29·53	39·19	193·383	Sabine.
Greenland	74 32 19 N	39·20328	29·53	39·19	193·383	Sabine.
Spitzbergen	79 49 58 N	39·21464	19·69	39·21	193·482	Sabine.
Paris	48 50 14 N	1·00000000	229·66	39·12	193·038	Duperrey.
Toulon	43 07 09 N	0·99903745	9·84	39·10	192·939	Duperrey.
Ascension	07 55 08 S	0·99731962	16·40	39·02	192·143	Duperrey.
Isle of France	20 09 19 S	0·99790540	15·40	39·04	192·643	Duperrey.
Port Jackson	33 51 39 S	0·99873338	19·69	39·07	192·791	Duperrey.
Falkland Islands . . .	51 31 44 S	1·00025461	19·69	39·13	193·087	Duperrey.

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CHAP. IX.

RESISTING FORCES.

553. *Forces which destroy but cannot produce motion.* — A physical agent capable of imparting motion to a quiescent body is called a force. It is evident that such an agent would also be capable of increasing the velocity of a body already in motion if it were applied in the direction of the motion, or diminishing the velocity, or even altogether destroying the motion and bringing the body to rest if it were applied in an opposite direction.

This principle is not, however, convertible: although it follows that an agent capable of imparting motion is also capable of diminishing or destroying it, it does not follow that an agent capable of diminishing or destroying motion is also capable of imparting or increasing it.

The class of forces to which we now refer are capable of diminishing the velocity of a body in motion, and of bringing it to rest, but they are incapable of imparting motion to a body at rest, or of augmenting any motion it may have. The former class of forces, which are capable of producing or increasing motion, may be described for distinction *active forces*, and the latter *passive forces.*

The force of gravity, for example, comes under the former class. A body freely suspended, being disengaged and submitted to the action of gravity, is put in motion, and its motion is continually accelerated as it moves downwards, until it encounters some obstacle which brings it to rest.

If the same body be projected upwards with the velocity with which it strikes the ground, the force of gravity will then gradually diminish its motion until it rises to the height from which it fell, where its motion will altogether be destroyed.

554. *Resisting forces.* — Of the passive or resisting forces, the most important are friction and the resistance of fluid media, such as air or water.

All bodies moving at or near the surface of the earth are subject to some, or all, of these forces, and, consequently, all terrestrial motions whatever are liable to constant retardation; and, to be maintained, require the constant agency of some impelling force to repair the loss produced by the resisting forces to which they are exposed.

The smallest attention to the phenomena which form the subject of mechanical inquiries, will render manifest the great importance of investigating and comprehending the effects of resisting forces.

In the preceding part of this volume, the construction and properties of machinery have been explained, on the supposition that the moving force of the power is transmitted to the working point with undiminished effect. In order to disembarrass the questions of their complexity, and present them in the most simple and intelligible form, machines have been considered as absolutely free from the effects of all resisting forces; surfaces moving in contact have been considered to be perfectly free from friction; axles were regarded as mathematical lines: pivots as mathematical points; ropes as perfectly flexible: and, in a word, the effect of the moving power has been considered as absolutely undiminished by any resistance whatever, in its transmission through the machinery to the working point.

It is scarcely needful to observe, that none of these suppositions are perfectly true. The surfaces of the machinery which move in contact are never perfectly smooth; axles have always definite thickness, and move in sockets never perfectly polished; ropes have considerable rigidity, and this rigidity is necessarily greater in proportion to their strength. Much has been accomplished, it is true, by a variety of expedients, to diminish these resistances; highly polished surfaces and effective lubricants have been applied, to obtain additional smoothness: but, still, the surfaces in contact continue to be studded with small asperities, which, coming constantly in opposition to each other in their motion, produce considerable resistance, and robbing the moving power of a great part of its efficacy, transmit it with proportionally diminished intensity to the working point.

To estimate therefore, correctly, the practical effects of any machinery, it is essential that we should calculate the effect of this resistance, and subduct it from that effect of the power which has been computed on the theoretical principles established in the preceding chapters; the overplus of effect after this deduction is all that part of the power which can be regarded as practically available.

555. *Effects of friction.* — The effect of friction on a power supporting a weight or resistance at rest is different from its effect when the weight or resistance is moved. In the one case, friction assists; in the other, it opposes the power.

Let us suppose, for example, a power P supporting a weight W, by means of a single moveable pulley. From what has been already proved (469.) it is evident, that if the power P be half the weight of W, they would be in equilibrium, and the power would keep the weight at rest, if the pulley were subject to no friction; and in that case, the slightest diminution of the power would cause the weight to descend, and draw the power upwards. But if the pulley be subject, s it always is in practice, to friction, then a small diminution of the wer will be resisted by this friction, and the weight will not descend d overcome the power, until the diminution of the power shall be- me so great as to enable the weight to overcome the friction.

556. *Friction aids the power in supporting a weight, but opposes the power in moving it.* — It follows, therefore, that when a pulley,

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ıy other machine, is subject to friction, a less power is sufficient ıpport a weight at rest, than would be necessary if there were no ion; and the greater the friction is, the less will be the power ssary to support the weight. It is in this sense that friction is to aid the power, when the weight or resistance is supported at

But if the power be required, not merely to support the weight, to raise it, then we shall find that the friction, instead of aiding, ses the power.

et us suppose, for example, the power P acting on the weight, ugh the intervention of a single moveable pulley, the power being the weight. In the absence of friction, the slightest addition to power would cause it to descend, and to raise the weight; but n the machine is subject to friction, then the power will not ded, until it shall receive such an addition as will be sufficient to come the friction.

57. *How this modifies the conditions of equilibrium.* — This cirstance modifies materially the conditions of equilibrium. Repreing by P that amount of the power applied to any machinery tever which would keep the weight W in equilibrium in the ab- e of friction, let f express the addition which must be made to P rder to enable it to overcome the friction and put the weight in ion; then f will also express the amount by which the power t be diminished, in order to enable the weight to prevail over it to descend. It is evident, therefore, that any power which is less ı $P + f$, and greater than $P - f$, would keep the weight in equiium and at rest.

58. *Cases in which friction is the whole power.* — It may therebe inferred, generally, that when a machine of any kind is used ply to sustain a weight or to balance a resistance, the friction, actin common with the power, becomes a mechanical advantage.

n many instances, this resisting force constitutes the entire effiıcy of the instrument. Thus, when screws, nails, or pegs are used oind together the parts of any structure, their friction with the ace with which they are in contact prevents their recoil, and gives m their entire binding power. In the ordinary use of the wedge lf, we have another striking example of the mechanical advantage friction. When the wedge is used for any purpose, as, for exple, to split timber, it is urged forward by percussion, the action the moving power being only instantaneous, and being totally susded between each successive blow.

But for the resisting force of the friction which takes place between surface of the wedge and the surface of the timber, the wedge ıld react after each blow, and render abortive the action of the ving power. The friction, therefore, in this case, plays the part a ratchet-wheel, preventing the reaction of the wedge, and making d the action of the power.

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559. *Great use of friction in the economy of nature and art.*—Notwithstanding the disadvantages which attend the presence of friction in machines, it is an agent eminently useful in giving stability to structures, and in giving efficiency to the movements of almost all bodies, natural and artificial. Without friction, most structures, natural and artificial, would fall to pieces. The stones and bricks used in building owe to the mutual friction of their surfaces a great part of their solidity. Manual exertion would become impracticable, if no friction existed between the limbs and the objects upon which they act. The friction between the foot and the ground gives a purchase to the muscular force, so as to enable it to produce progressive motion. Without friction, every effort of the foot to propel the body forward would be attended with a backward action, so that no progressive motion would ensue. The difficulty of moving upon ice, or upon ground covered with greasy or unctuous matter, illustrates this. Without friction we could not hold any body in the hand. The difficulty of holding a lump of ice is an example of this. Without friction, a locomotive engine could not propel its load, for if the rail and the tire of the driving-wheels were both absolutely smooth, one would slip upon the other, without affording the necessary purchase to the steam power.

560. *Friction of sliding and rolling.* — Friction is manifested in different ways, according to the kind of motion which one surface has upon the other.

When one surface slides upon the other in the manner of a sledge, the friction is called sliding or rubbing friction.

When one body rolls upon another, so that different points of such bodies come into successive contact with each other, it is called rolling friction.

561. *Laws of friction empirical, but still useful.*—The laws which regulate friction are derived exclusively from experiments, independently of theory. There are no simple or general principles from which they can be deduced by mathematical reasoning.

It is a matter of regret, that even amongst the best conducted experiments that have been made, considerable discrepancies ar observable, and that differences of opinion prevail between the mos respectable authorities respecting many particulars connected with th properties and laws of these resisting forces.

Although these laws, so far as they are known, depend thus wholl on experiment, yet the general principles of science, as applied t them, are far from being useless.

They serve as a guide in the selection of the experiments whic are best adapted to develop those laws which are the subject of inquiry, as well as to show the inconclusiveness of some experiment on which reliance might otherwise be placed, and thus enable us fu

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duce from the results of experimental inquiries numerous actical results.

Methods of measuring sliding friction. — There are two by which the quantity of friction produced when two surfaces are moved one upon another can be ascertained.

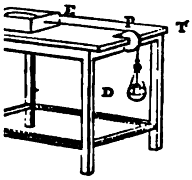

Fig. 159.

1st. The surfaces being rendered perfectly flat, let one be fixed in a horizontal position, on a table T T, *fig.* 159., and let the other be attached to the bottom of a box B E, adapted to receive weights, so as to vary the pressure.

Let a flexible cord be attached to this box, and being carried parallel to the table, let it pass over a fixed pulley at P, and have a dish suspended to it at D.

friction existed between the surfaces, the smallest weight ed from D would cause the box B E to move with a uniformly ted motion along the table towards the point; but the resist- friction renders it necessary, before motion can take place or ntained, that the weight D shall be equal to the amount of ction. If the weight D and the friction be equal, then, the nd the resistance being in equilibrium, the box B E, if put in will move towards the point with any velocity which may be d to it continued uniform. If the weight D be greater than tion, then the motion of the box towards P will be accelerated; the weight be less than the friction, then any motion which given to the box will be retarded, and will soon cease ner.

determination, therefore, of the weight acting at D, which nts the exact amount of the friction, will depend upon the given to the box in the direction B P being maintained n.

. Let one of the surfaces be attached, as before, to a flat plane A B, *fig.* 160., but instead of being horizontal, let it be inclined, and so arranged that the inclination may be varied at pleasure. The box W being constructed as before, and placed upon the plane, let such an elevation be given to the plane that the box shall be capable of moving

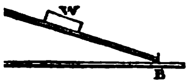

Fig. 160.

t with a uniform velocity, without acceleration or retarda- If the elevation be greater than this, the motion of the box he plane will be accelerated, and the gravity of the plane will ter than the friction; if it be less, the motion of the box will rded, and the gravity will be less than the friction.

563. *The angle of repose.* — That particular inclination of the plane, corresponding to the friction of any given surface, which renders the gravity of the plane equal to the friction, is called the *angle of repose* or the *angle of friction.*

According to the principles already explained, it follows that, in these cases, if the length of the plane A B represent the total weight W, the gravity down the plane, which is equal to friction, will be represented by the height A E, and the pressure upon the plane will be represented by the base B E, and, consequently, the ratio of the friction to the pressure will be that of the height A E to the base B E.

Experiments conducted according to both these methods have given nearly the same results, which may be summarily stated to be as follows.

564. *Friction proportional to pressure, other things being the same.* — The proportion of the friction to the pressure, when the *quality* of the surface is given, is always the same, no matter how the weight or the magnitude of the surface may be varied, except in extreme cases, when the proportion of the pressure to the surface is very great or very small. Thus it is found that in the mode of experiment represented in *fig.* 159., in proportion as we increase the weight contained in the box B E, we must increase the weight suspended at D. If the weight in B E be doubled, the weight suspended at D must be also doubled; if the weight in B E be increased in a three or four-fold proportion, the weight suspended at D must be increased also in a three or four-fold proportion. Or if the surface forming the bottom of the box B E be increased or diminished, the weight contained in it being the same, no difference will take place in the weight suspended from D, which we find necessary to overcome the friction.

This effect is what might have been expected; for when the surface is diminished, the total pressure remaining the same, the pressure on each square inch of the surface will be increased in exactly the same proportion as the surface has been diminished; so that although the amount of friction would be diminished by the diminution of the rubbing surface, the amount of the friction is increased in exactly the same proportion by the increase of the pressure per square inch upon it.

565. *This law fails in extreme cases.* — But if an extreme increase or an extreme diminution of surface take place, the pressure remaining the same, then it is found that the result varies from this condition, the ratio of friction to the pressure being somewhat increased by the extreme increase of rubbing surface, and somewhat diminished by its extreme diminution.

566. *Effect of continued contact.* — When surfaces are first placed in contact, the friction is less than when they are suffered to rest in contact for a certain time. This is proved by observing the force which in this case is necessary to move the one surface upon the

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r. **This force is found to be less if applied at the first moment ontact, than when the contact has been long continued.** This ease of force, however, due to continuance of contact, has a limit. re is a certain time, different in different substances, within which i resistance attains its greatest amount. With surfaces of wood it erally attains its maximum in two or three minutes; with surfaces metal, the maximum is attained almost immediately. But when urface of wood is placed in contact with one of metal, this resistce continues to increase for several days. In general, the duration the increase of resistance by continued contact increases as the rfaces of contact are increased, and is greater when the surfaces are different kinds than when they are of the same kind.

567. *Similar surfaces have greater friction than dissimilar.* — general it is found that similar surfaces have greater friction than similar surfaces. The harder the body is, the less, in general, ll be the friction produced by its surface.

568. *Friction diminished by wear.* — It is evident that the oother the surfaces are which move in contact, the less will be eir friction.

On this account, the friction of surfaces when first brought into ntact is greater than after their attrition has been continued for a rtain time, because such attrition has a tendency to remove and b off those minute asperities and projections on which the friction pends; but this has a limit, and after a certain quantity of attrition e friction ceases to decrease.

Newly planed surfaces of wood have at first a friction which is ual to about half their entire pressure; but after they are worn by trition, this is reduced to one third.

Owing to the cause already explained, of the increase of friction by e continuance of contact, it follows that the friction of surfaces at e commencement of motion from a state of rest must be greater an while actually moving, because, while in motion, the surfaces in y one position are only momentarily in contact, and consequently ive not time to acquire that increased friction due to the continuance contact.

569. *Effect of crossing the grains.* — If the surfaces in contact placed with their grains in the same direction, the friction will be eater than if their grains cross each other. Smearing the surfaces ith unctuous matter diminishes the friction, probably by filling the ities between those minute projections which produce the friction.

570. *Pivots of wheels.* — The pivots of pendulums or balances e usually made of steel, and rest upon hard polished stones, differt surfaces being used for the purpose of diminishing the amount of iction. Brass sockets are generally used for iron axles on the same inciple.

571. *Selection of lubricants.*—*In the* selection of lubricants, those

of a viscous nature are selected, in the case of the rough surfaces of softer bodies, and those which are more fluid are applied to the smoother surfaces of harder bodies.

Thus, when metal moves upon wood, tallow, tar, or some solid grease is generally used; but when metal moves upon metal, oil is preferred, and the harder and the smoother the metal, the finer the oil.

Finely pulverized plumbago is found to be a very efficient agent in diminishing friction, especially as applied to the axles of carriages and the shafts of machinery.

The *anti-attrition metal*, which is composed of 1 part copper, 2 parts antimony, and 3 parts tin, is now very generally used by machinists in the United States, for diminishing the friction and preventing the heating of gudgeons, pivots, boxes, &c.

572. *Rolling friction.* — The friction which attends a rolling motion is very much less than that which would attend a sliding or rubbing motion with the same surfaces and under the same pressure. Hence it is that rollers are used with so much success as an expedient for diminishing friction. A roughly chiselled block of stone weighing 1080 lbs. was drawn from the quarry on the surface of the rock, by a force of 758 lbs. It was then laid upon a wooden sledge and drawn upon a wooden floor, the tractive force being 606 lbs. When the wooden surfaces moving upon one another were smeared with tallow, the tractive force was reduced to 182 lbs.; but when the load was in fine placed upon wooden rollers, three feet in diameter, the tractive force was reduced to 28 lbs.

573. *Use of rollers.* — When heavy weights are to be moved through small spaces, the expedient of rollers is attended with great advantage; but when loads are to be transported to considerable distances, the process is inconvenient and slow, owing to the necessity of continually replacing the rollers in front of the load, as they are left behind by each progressive advancement.

574. *Use of carriage wheels.* — The wheels of carriages may be regarded as rollers which are continually carried forward with the load.

In addition to the friction of the rolling motion on the road, they have, it is true, the friction of the axle in the nave; but, on the other hand, they are free from the friction of the rollers with the under surface of the load or the carriage in which the load is transported. The advantage of wheel carriages in diminishing the effects of friction is sometimes attributed to the slowness with which the axle moves within the box, compared with the rate at which the wheel moves over the road; but this is erroneous. The quantity of friction does not in any case vary considerably with the velocity of the motion, but least of all does it in that particular kind of motion here considered.

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575. ***Friction wheels.*** — In certain cases where it is of great im-
rtance to remove the effects of friction, a contrivance called friction
ɪeels or friction rollers is used. The axle of a friction wheel, in-
ad of revolving within a hollow cylinder which is fixed, rests upon
e edges of wheels which revolve with it: the species of motion thus
comes that in which the friction is of least amount.

576. *Effect of the magnitude of wheels.* — In carriages, the rough-
as of the road is more easily overcome by large wheels than by
all ones; hence we see wheels of very great magnitude used for
rrying beams of timber of extraordinary weight. The animals
awing these, notwithstanding their weight, do not manifest any con-
lerable exertion. The cause of this arises, partly from the carriage-
ɪeels bridging over the cavities in the road, instead of sinking into
em, and partly because, in surmounting obstacles, the load is ele-
ted less abruptly.

577. ***Best line of draught.*** — If a carriage were capable of moving
a road absolutely free from friction, the most advantageous direc-
n in which the tractive force could be applied, would be parallel to
e road; but when the motion is impeded by friction, as in practice
always is, it is better that the line of draught should be inclined to
e road, so that the drawing force may be exerted partly in lessening
e pressure on the road, by in some degree elevating the carriages,
d partly in advancing the load.

It can be established by mathematical reasoning, that the best line
draught, in all cases, is determined by the angle of repose; that
to say, the traces should form an angle with the road equal to the
evation of a plane which would exactly overcome the friction.
ence it appears, that the smoother the road, and the more perfect
e carriage, and consequently the less the friction, the more nearly
rallel to the road the line of draught should be.

In wheel carriages, there exist two sources of friction: one which
evails between the tires of the wheel and the road on which they
n, the amount of which depends on the quality of the road; and
e other which prevails between the axle and the nave of the wheel
which it turns. This latter is sliding friction; but the rubbing
rface is small, being the line of contact of the axle with the nave
socket.

578. *Friction of wheel carriages on roads.*—From the structure
the axle and the nave, this source of friction admits of being al-
ost indefinitely diminished, by the application of lubricants and
her expedients.

The other resistance, depending on the action of the tires of the
heels, amounts, on well-paved roads, to about $\frac{1}{70}$th of the load. On
avelled roads it is but $\frac{1}{35}$th; and when a fresh layer of gravel has
en laid, it is increased to the $\frac{1}{18}$th of the load.

It is found, however, on a well macadamized road, when in good order, that the resistance does not exceed the $\frac{1}{35}$th or $\frac{1}{40}$th of the load.

The most perfect modern road is the iron railway, by which the resistance due to friction is reduced to an extremely small amount.

Various experiments have been made, with a view to determine this resistance; but much difficulty arises, owing to the effects of atmospheric resistance being combined with those of friction. An extensive series of experiments was made by the author of this volume, in the year 1838,* with a view to determine the amount of resistance to railway trains; the result of which showed, that this resistance was much more considerable than it had been previously supposed to be; but that it depends in a great degree upon the velocity, and probably arises more from the resistance of the air than from friction properly so called.

579. *Effects of imperfect flexibility of ropes.*—When ropes or cords form a part of machinery, the effects of their imperfect flexibility are in a certain degree counteracted by bending them over the grooves of wheels.

But although this so far diminishes these effects as to render ropes practically useful, yet still, in calculating the power of machinery, it is necessary to take into account some consequences of the rigidity of cordage, which even by these means are not yet removed.

To explain the way in which the stiffness of a rope modifies the operation of a machine, we shall suppose it bent over a wheel, and stretched by weights A and B, *fig.* 161, at its extremities. The weights A and B being equal, and acting at C and D in opposite ways, balance the wheel. If the weight A receive an addition, it will overcome the resistance of B, and turn the wheel in the direction D E C. Now, for the present, let us suppose that the rope is perfectly inflexible; the wheel and weights will be turned into the position represented in *fig.* 162. The leverage by which A acts will be diminished, and will be-

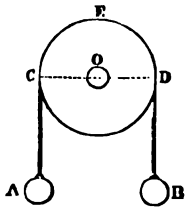

Fig. 161.

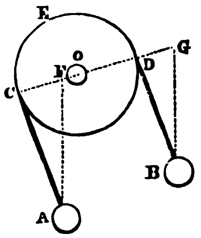

Fig. 162.

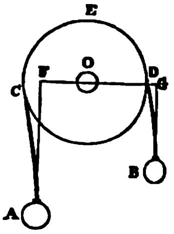

Fig. 163.

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* The details of these experiments will be found in the published reports of the meetings of the British Association in 1838 and 1841.

come O F, having been before O C; and the leverage by which B acts will be increased to O G, having been before O D.

But the rope, not being inflexible, will yield to the effect of the weights A and B, and the parts A C and B D will be bent into the forms represented in *fig.* 163. The preponderating weight A still has a less leverage than the weight B, and consequently, a proportionate part of the effect of the moving power is lost.

The extent to which the rigidity of cordage affects the motion of machinery has been ascertained by experiment in a still more imperfect manner than the results of friction. Many incidental circumstances vary the conditions, so as to throw great difficulties in the way of such an investigation. Different ropes, and the same ropes at different times, produce extremely different effects, influenced by the circumstances of their dryness or humidity, the quality of their material, the mode in which they are prepared and twisted, &c. These circumstances, it is evident, do not admit of being estimated or expressed with any degree of accuracy. It may, however, be stated generally that the resistance produced by the rigidity of a rope is directly proportional to the weight that acts upon it, and to its thickness. Other things being the same, it is also in the inverse proportion of the diameter of the wheel or axle upon which the rope is coiled; the greater the weights, therefore, which are moved, and the stronger the ropes, the greater will be the resistance proceeding from rigidity; and, on the other hand, the greater the diameter of the wheel or axle on which the rope runs, the less in proportion will be the force necessary to overcome the rigidity.

The resistance from new ropes is greater than from those of the same quality which have been some time in use. Ropes saturated with moisture offer increased resistance on that account.

580. *Empirical formula for determining the effects of the rigidity of ropes.* — The following rule for ascertaining the effects of rigidity is given in Peschel: —

"It has been experimentally proved, that a weight of 1 lb., hanging on an axis of 1 inch in diameter by a cord 1 line thick, requires a resistance of half an ounce, or $\frac{1}{32}$ of a lb., on account of the stiffness of the rope. Assuming this law as the basis of our calculations, and keeping in view the proportions named above, we may find the friction of the ropes in any machine. Suppose, for instance, that from an axle 8 inches in diameter, there is suspended a weight of 1600 lbs. by a rope 16 lines in thickness; then, by the above rule, if the rope were 1 line thick, and the roller 1 inch in diameter, the resistance would be $\frac{1600}{32}$, or 50 lbs.; but since the former is 16 lines thick, it would, if moved around a 1-inch axle, be 50 × 16 = 800 lbs.; the axle, however, is 8 inches in diameter, whence the exact resistance is $\frac{800}{8}$ = 100 lbs. Or, in general terms, if *d* be the diameter of the axis in inches, d' that of the rope in lines, and w the weight in lbs. to be moved, the resistance of the friction will be

$$R = \frac{d'\,w}{32\,d}\text{ lbs.}$$

581. *Resistance of fluids.* — Since all the motions which commonly take place on the surface of the earth are made in the atmosphere or in water, it is of great practical importance to ascertain the laws which govern the resistance offered by these fluids to the motion of bodies passing through them.

A body moving through a fluid must displace as it proceeds as much of that fluid as fills the space which it occupies; and in thus imparting motion to the fluid, it loses by reaction an equivalent quantity of its momentum.

If a body thus moving were not impelled by a motive force in constant action, it would be gradually deprived of its momentum, and at length brought to rest. Hence it is, that all motions which take place on the surface of the earth, and which are not sustained by the constant action of an impelling power, are observed gradually to diminish, and ultimately cease.

Since the resistance produced by a fluid to the motion of a solid through it is equivalent to the momentum imparted by the solid to the fluid, which it thrusts out of its way in its motion, it follows evidently that, other things being the same, this resistance will be proportional to the density or weight of the fluid. Thus the resistance produced by air is less than that produced by water, in the proportion of the weight of air to the weight of an equal bulk of water.

The resistances are proportioned to the quantity of the fluid which the moving body thrusts from the path; and this again depends upon the form and magnitude of the body, and more especially on its *frontage*.

582. *Resistance depends on the frontage of the moving body.* — The resistance which a body encounters in moving through a fluid is greater, therefore, with a broad end foremost, than with a narrow end foremost. A ship would evidently encounter a much greater resistance if it were driven sideways, than if it move in the direction of the keel. It would also encounter a greater resistance if it moved stern foremost than in the usual direction.

The blade of a sword would be wielded with difficulty if moved with its flat side against the air, whereas it is easily flourished when moved edge foremost.

583. *Also affected by the form of the front.*—Bodies whose foremost ends have the form of a wedge or a point, move through a fluid with less resistance than if the pointed ends were cut off and they presented a flat surface to the fluid medium, because in their motion act upon the principle of the wedge, and more easily cleave the

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fluid. Nature has formed birds and fishes in this manner to facilitate their passage through the air and through the water.

584. *Increases in a high ratio with the speed.*—The resistance which a body moving through a fluid encounters, increases in a high ratio with its velocity.

If the body move with the velocity of one foot per second, it will act in each second upon a column of the fluid, whose base is equal to its own transverse section, and whose length is one foot, and it will impel such a column with a velocity of one foot per second; but if the velocity of the moving body be doubled, it will not only drive before it in one second a column of fluid two feet long — that is, double the former length, but it will impel this column with double the former speed.

The resistance, therefore, will be doubled on account of the double quantity of the fluid, and again doubled on account of this quantity receiving double the velocity.

The moving force, therefore, imparted by the body to the fluid when the velocity is doubled, will be increased in a fourfold proportion.

In the same manner it may be shown that if the velocity of the body be increased in a threefold proportion, it will drive from its path three times as much of the fluid per second, and impart to it three times as great a velocity; consequently, the moving force which it will impart to the fluid will be nine times that which it imparted moving at the rate of one foot per second.

In general, therefore, it follows that the moving force which the body imparts to the fluid in moving through it, will be increased in proportion to the square of the velocity; but as the resistance which the body suffers must be equal to the momentum which it imparts to the fluid, it follows that the resistance to a body moving through a fluid will be proportional to the square of its velocity.

585. *Advantage of ponderous missiles.* — Hence it follows that missiles lose a less porportion of their moving force in passing through the air, as their weight is increased; for, according to what has been stated, the resistance which they suffer at a given velocity will be proportional to their transverse section, which in this case is in the ratio of the squares of their diameters; but as their weight increases as the cubes of their diameters, and is proportional to their moving force, it follows that in increasing their magnitude their moving force is increased in a higher ratio than the resistance they encounter. For example, if two cannon-balls have diameters in the proportion of 2 to 3, the resistances which they will encounter at the same velocity of projection will be in the ratio of 4 to 9, while their weights will be in the ratio of 8 to 27.

The resistance, therefore, *of the smaller* ball will bear to its weight *a greater ratio than that of the larger.*

586. *Resistance of the air to the motion of falling bodies.*—It has been shown that a body obedient to the action of gravity would descend in a vertical line with a uniformly accelerated motion. Its velocity would increase in proportion to the time of its fall, so that in ten seconds it would acquire ten times the velocity which it acquired in one second; but these conclusions have been obtained on the supposition that no mechanical agent acts upon the body, save gravity itself. If, however, the body fall through the atmosphere, which in practice it must always do, it encounters a resistance which augments with the square of the velocity. Now, as the accelerating force of gravity does not increase, while the resistance continually increases, this resistance, if the motion be continued, must at length become equal to the gravitation of the falling body; and, when it does, the velocity of the falling body will cease to increase. It follows, therefore, that when a body falls through the atmosphere, its rate of acceleration is continually diminished; and there is a limit beyond which the velocity of its fall cannot increase, this limit being determined by that velocity at which the resisting force of the air will become equivalent to the gravity of the body.

As the resisting force of the air, other things being the same, increases with the magnitude of the surface presented in the direction of the motion, it is evidently possible so to adapt the shape of the falling body that any required limit may be impressed upon the velocity of its descent. It is upon this principle that parachutes have been constructed.

When a body attached to a parachute is disengaged from a balloon, its descent is at first accelerated, but very soon becomes uniform, and as it approaches the earth, the air becoming more and more dense, the resistance on that account increases, and the fall becomes still more retarded.

The theory of projectiles, which is founded upon the supposition of bodies moving in a vacuum, is rendered almost inapplicable in practice, in consequence of the great effect produced by atmospheric resistance to bodies moving with such a velocity as that which is generally imparted to missiles. According to experiments and calculations, it has been found that a 4-lb. cannon-ball, the range of which in a vacuum would be 23,226 feet, was reduced to 6,437 feet by the resistance of the air. Hutton showed that a 6-lb. ball, projected with a velocity of 2000 feet per second, encountered a resistance a hundred times greater than its weight.

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CHAP. X.

STRENGTH OF MATERIALS.

587. *Strength of solid bodies.* — The solid materials of which structures, natural and artificial, are composed, are endued with certain powers, in virtue of which they are capable of resisting forces applied to bend or break them. These powers constitute an important class of resisting forces, and are technically called in mechanics the strength of materials.

Experimental inquiries into the conditions which determine the strength of solid bodies, and their power to resist forces tending to tear, break, or bend them, are obstructed by practical difficulties, the nature and extent of which have deterred many from encountering them.

588. *Difficulty of ascertaining its laws.* — These difficulties arise partly from the great force which must be employed in such experiments, but more from the peculiar nature of the bodies upon which such experiments are made.

The object of such an inquiry must necessarily be the establishment of a general law, or such a rule as would be strictly observed if the materials were perfectly uniform in their texture, and subject to no casual inequalities. In proportion, however, as such inequalities are frequent, experiments must be multiplied, so that they shall include cases varying in both extremes, so that the peculiar effects of each may be effaced from the general average result which shall be obtained. These inequalities of texture, however, are so great, that even when a general law has been established by a sufficiently extensive series of experiments, it can only be regarded as a mean result from which individual examples will be found to depart in so great a degree, that the greatest caution must be observed in its practical application.

Although the details of this subject belong more properly to engineering than to an elementary treatise like the present, it may nevertheless be useful to give a general view of the most important principles which have been established.

589. *Several ways in which strength may be manifested.* — A mass of solid matter may be submitted to the action of a force tending to separate its parts in several ways, of which the principal are—

I. A direct pull; as when a weight is suspended to a wire or a rope, or when a tie-beam resists the separation of the walls of a structure.

II. A direct pressure or thrust; as when a weight rests upon a pillar, or a roof upon walls.

III. When a force is applied to twist or wrench a body asunder by turning a part of it round a point within it.

IV. A transverse strain; as when a beam, being supported at its centre, weights are suspended from its ends; or, being supported at its ends, a weight is suspended from its centre.

590. *The strength to resist a direct pull.*—When a rod, rope, or wire is extended between forces applied to its ends, and tending directly to stretch it, its strength to resist such force is, other things being the same, in proportion to the magnitude of its section.

Thus, suppose an iron wire stretched by a weight which it is just able to support without breaking. It is evident that a wire having twice the quantity of iron of the same quality in its thickness would support double the weight, because such wire would be in effect equivalent to two wires like the former combined. In the same manner, a wire having three times the quantity of iron of the same quality in its thickness, being equivalent to three wires like the first, would support three times the weight, and so on.

Thus the power of bodies to resist a direct pull will be in general in proportion to the area of their transverse section.

In practice, it is found that when the length is much increased, the strength to resist a direct pull is diminished.

This departure, however, from the general law is explained by the increased probability of casual defects of structure in the increased length; and, subject to such qualification, it may be stated generally, that the strength of body to resist tension, or a direct pull drawing from end to end, is in the direct ratio of the area of its section made at right angles to the direction in which it is stretched.

591. *Method of experimentally measuring it.*—The strength of bodies to resist a direct pull is experimentally estimated by attaching the upper extremity securely to a point of support, and suspending weights to the lower extremity, which are increased gradually until the body under experiment is broken, the weight which breaks it is taken as the expression of its strength.

The bodies which have been subjected to experiments of this kind are chiefly metals, woods, and ropes, these being most generally used in structures, in which their capacity for resisting tension is of great importance, as in suspension bridges, iron roofs, cables, cordage, &c.

592. *Table showing the most recent results of such experiments.*—In the following table are collected the results of the most recent and extensive experiments on this subject.

The bodies subjected to experiment are supposed to be in the form of long rods, the cross section of which measures a square inch. In the second column is given the amount of the breaking weights, which are the measure of their strength.

This table is selected from various recent works. *(See next page.)*

[redacted]

TABLE SHOWING THE STRENGTH WITH WHICH PRISMS OF THE UNDERMENTIONED SUBSTANCES RESIST A DIRECT PULL, EXPRESSED IN LBS. PER SQUARE INCH OF THE AREA OF THEIR TRANSVERSE SECTION.

Name.		lbs.		lbs.
1st. Metals:—				
Steel, untempered	from	110,690	to	127,094
— tempered	"	114,794	—	158,741
— cast	"	184,256		
Iron, bar	"	53,182	—	84,611
— plate, rolled	"	58,920		
— wire	"	58,780	—	112,905
— Swedish malleable	"	72,064		
— English do.	"	55,872		
— cast	"	16,248	—	19,464
Silver, cast	"	40,997		
Copper, do.	"	20,820	—	87,880
— hammered	"	87,770	—	89,968
Brass, cast	"	17,947	—	19,472
— wire	"	47,114	—	58,981
— plate	"	52,240		
Gold	"	20,490	—	65,287
Tin	"	3,228	—	6,666
— cast	"	4,786		
Bismuth, cast	"	8,187		
Zinc	"	2,820		
Antimony, cast	"	1,062		
Lead, molten	"	887	—	1,824
— wire	"	2,548	—	8,828
2d. Woods:—				
Teak	"	12,915	to	15,405
Sycamore	"	9,680		
Beech	"	12,225		
Elm	"	9,720	—	15,040
Memel fir	"	9,540		
Christiana deal	"	12,846		
Larch	"	12,240		
Oak	"	10,867	—	25,851
Alder	"	11,458	—	21,780
Lime	"	6,991	—	20,796
Box	"	14,210	—	24,048
Pinus sylv.	"	17,056	—	20,895
Ash	"	18,480	—	28,455
Pine	"	10,088	—	14,965
Fir	"	6,991	—	12,876
3d. Cords:—				
Hemp twisted				
½ to 1 inch thick		8,746		
1 to 3 "		6,800		
3 to 5 "		5,845		
5 to 7 "		4,860		

593. *Iron the most tenacious: effect of alloys.* — From this table it appears that the strongest of the bodies for resisting tension is iron,

[redacted]

and that the strongest condition of iron is that of tempered steel. In general, metals when cast are less strong than when hammered. Thus, cast iron has not one-third of the tenacity of wrought iron.

It is also found that metals which are composed of two or more alloyed together are often stronger than any of their components. Thus, brass wire, which is composed of zinc and copper, has greater tenacity than copper wire, although the tenacity of zinc, as appears by the table, is extremely small.

594. *Effect of heat.* — It is also found that the strength of metals is affected by their temperature, being diminished in general as their temperature is raised. Sudden, frequent, and extreme changes of temperature impair tenacity.

595. *Strength of timber.* — The woods are subject to extreme variations, produced in general by the great inequalities which are incidental to them. Thus the strength of oak varies, as appears by the table, between the limits of 10,000 and 25,000 lbs.

It is found that trees which grow in mountainous places have greater strength than those which grow on plains, and also that different parts of the same tree, such as the root, trunk, and branches, vary in strength within wide limits.

596. *Strength of cordage.* — It is found that cords of equal thickness are strong in proportion to the fineness of their strands, and also to the fineness of the fibres of which these strands are composed. It is found also that their strength is diminished by being overtwisted. Ropes which are damp are stronger than ropes which are dry, those which are tarred than the untarred, the twisted than the spun, and the unbleached than the bleached. Other things being the same, silk ropes are three times stronger than those composed of flax.

The strength of many substances is increased by compressing them; this is the case with leather and paper, for example.

597. *Animal substances.* — Animal and vegetable substances which, being originally liquid, are rendered solid by evaporation, change of temperature, or exposure, often possess extraordinary strength. Examples of this are presented in the gums, glue, varnish, &c. Count Rumford found that a copper plate having the thickness of 1-20th of an inch, rolled into the form of a cylinder, had its strength doubled when coated with well-sized paper of double its own thickness; and that a cylinder composed of sheets of paper glued together, having a sectional area of one square inch, was capable of supporting a weight of 15 tons for every square inch in its sectional area; and, in fine, that a cylinder composed of hempen fibres glued together had a strength greater than that of the best iron, being capable of supporting 46 tons per square inch of sectional area.

598. *Strength to resist pressure or direct thrust.* — According to the theory of Euler, the strength of a column composed of any material and of any prismatic form to resist the crushing force of a

[redacted]

weight placed upon it, increases as the square of the number expressing its sectional area divided by the square of the number expressing its height.

This law, which is of great practical importance on account of its application to architectural purposes, is found to be in very near accordance, within practical limits, with the experiments of Musschenbrock and others; and more recently with the still more extensive, experiments of M. Eaton Hodgskinson made on pillars of wrought iron and timber.

It is necessary to observe here, that when the height is reduced to a very small magnitude, the pillar is more easily crushed than would be indicated by this law. Exceptions are also presented in the case of pillars formed of particular materials; such, for example, as cast iron, which M. Hodgskinson found to vary in rather a higher proportion in reference both to the sectional area and to the height.

According to Eytelwein's experiments, the strength of rectangular columns to resist compression is directly as the product of the larger side of their section multiplied by the cube of its shorter side, and inversely as the square of their height.

This will coincide with that of Euler when the pillar is square.

599. *Tables showing the strength of columns.* — Eytelwein gives the following table of the weights *necessary to crush pillars composed of the materials expressed in the first column, the numbers expressed in the second column being the total crushing weights in lbs. per square inch.*

Name.		lbs.	lbs.
1. **Metals**:—			
Cast iron	from	115,813	to 177,776
Brass, fine	"	164,864	
Copper, molten	"	117,088	
" hammered	"	103,040	
Tin, molten	"	15,456	
Lead, molten	"	7,728	
2. **Woods**:—			
Oak	from	3,860	to 5,147
Pine	"	1,928	
Pinus sylv.	"	1,606	
Elm	"	1,284	
3. **Stones**:—			
Gneiss	"	4,970	
Sandstone, Rothenburg	"	2,556	
Brick, well baked	"	1,092	

600. *Results of Hodgskinson's researches.* — Mr. Hodgskinson gives the following results of his experiments as to the strength of timber pillars. (*For Table, see next page.*)

TABLE.

Description of Wood.	Strength per Square Inch in Lbs.	
	Moderately dry.	After subjected to drying Process.
Alder.	6,831	6,960
Ash.	8,683	9,363
Bay	7,518	7,518
Beech	7,733	9,363
English Birch.	8,297	6,402
Cedar	5,674	5,863
Red Deal	5,748	6,586
White Deal	6,781	7,293
Elder	7,451	9,973
Elm.	—	10,331
Fir (spruce)	6,499	6,819
Mahogany	8,198	8,198
Oak, Quebec	4,231	5,982
— English.	6,484	10,058
Pine, Pitch	6,790	6,790
— Red.	5,395	7,518
Poplar	3,107	5,124
Plum, dry.	8,241	10,493
Teak.	—	12,101
Walnut	6,063	7,227
Willow	2,898	6,128

The numbers in the first column are the number of lbs. per square inch, deduced from experiments made on cylinders one inch in diameter and two inches in height, with flat sides, the wood being moderately dry.

The second column gives the strength of similar pillars of the same woods which have been subjected to a drying process in a warm place for two months and upwards.

The great difference in the strength of the two cases, shows in a striking manner the effect of dryness upon the strength of timber.

601. *Strength to resist torsion.* — Neither theory nor experiment has thrown much light on the laws which govern this mode of resistance to the separation of the constituent parts of bodies. The following results, showing the comparative strength of various metals, were obtained from a series of experiments made by Mr. Rennie.

[redacted]

Table showing the comparative Strength of various Metals to resist Torsion.

Lead	1,000
Tin	1,438
Copper	4,312
Brass	4,688
Gun metal	5,000
Swedish iron	9,500
English iron	10,125
Cast-iron	10,600
Blaster-steel	16,688
Shear-steel	17,063
Cast-steel	19,562

It is stated by M. Bankes, that a square bar of cast-iron which would measure an inch, is wrenched asunder by an average force of 631 lbs. applied at the extremity of a lever two feet long.

602. *Strength to resist transverse strain.*—Bodies submitted to a transverse strain are usually considered as having the form of prismatic beams, at right angles to which the strain is applied. A prismatic beam is one, all whose transverse sections are equal and similar figures, and the character of the beam is determined by the figure of this section. Thus, a cylindrical beam is one whose transverse section is a circle; a rectangular, one whose transverse section is a right-angled parallelogram; a square, whose transverse section is a square; and so on.

The force producing the strain may be resisted by one or by two points of support, in the latter case the weight being placed between these two points.

Case of a beam supported at one end. — In the first case, let B C, *fig.* 164., be a prismatic beam, fixed in a wall, or other vertical means of support, at B A, and let a weight W be supported from its end C. We shall, for the present, omit the consideration of the weight of the beam itself.

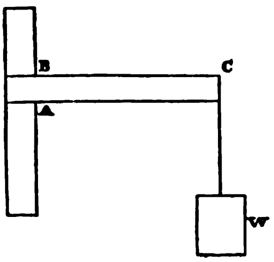

Fig. 164.

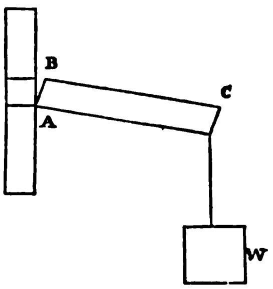

Fig. 165.

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The effect of the weight W produced at B A will be to turn the beam round the point A, as if it were a hinge, as represented in *fig.* 165, and thus to tear asunder the fibres which unite the parts of the body forming its transverse section at B A.

Let f be the force corresponding to the unit of surface of this section: let A be the area of the section; then $f \times A$ will express the total force of the body over the whole surface of the section. But if the beam tends to turn round the point A as a fulcrum, this force acts by a leverage at each point, corresponding to the distance of such point from A. The total force expressed by $f \times A$ may therefore be considered as having an average leverage, determined by the various distances of all the parts in the section from a horizontal line passing through A.

Now the point determined by these conditions is the *centre of gravity* of the section of the beam at A B. Let the distance of this centre of gravity from the horizontal line passing through the point A be c; we shall then have the effect of the forces of cohesion by which the body is united in the surface of the section expressed by $f \times A \times c$. Against this the weight W acts with the leverage O B; that is to say, the length of the beam.

Let this length be expressed by L, and we shall have, according to the properties of the lever,

$$W \times L = f \times A \times c,$$

and therefore

$$W = f \times \frac{A \times c}{L}.$$

It appears from this, therefore, that the weight necessary to fracture a beam placed in this manner, will be in the direct ratio of the area of the transverse section multiplied by the height of the centre of gravity above the lowest point of this section, and divided by the length of the beam.

For beams of the same length, their strength is proportional to the area of their section, multiplied by the distance of the centre of gravity above the lowest point; and for beams of the same section, the strength is inversely as their length.

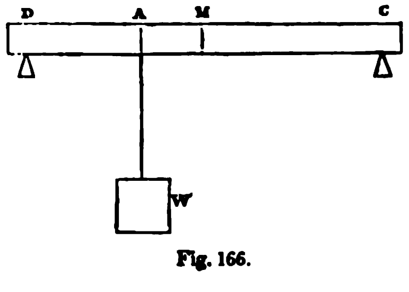

Fig. 166.

By the length of the beam in this case is to be understood the distance of the part of the beam at which the weight is suspended from the point of support.

[redacted]

603. ***Case of a beam supported at both ends.***—Let us now consider the case in which the body is supported at two points, the breaking weight being placed at some intermediate point. Let such a beam be represented in *fig.* 166, supported at the points D and C, the weight being placed at an intermediate point A. In this case, the tendency of the weight is to rupture the fibres in the vertical section of the beam passing through A, and in doing so, the beam would break as if a hinge were placed at A, on the upper surface of the section supporting the weight, as represented in *fig.* 167.

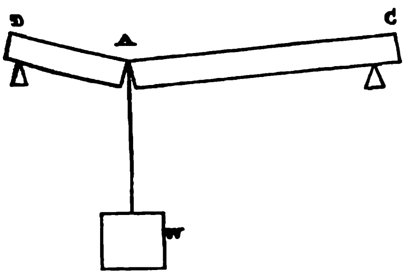

Fig. 167.

The fracture may in this case be considered to be produced by the reaction of the points of support D and C.

To determine the effects of this reaction, we are to consider that, by the properties of the lever, the part of the weight which acts at C is expressed by

$$W \times \frac{DA}{CD},$$

and the part of the weight which acts at D as expressed by

$$W \times \frac{CA}{CD}.$$

But the former of these acts upon the section at A with the leverage C A, and the latter with the leverage D A. If each be therefore multiplied by its respective leverage, we find that the effect of each of these actions in producing the fracture at A will be expressed by

$$W \times \frac{DA \times CA}{CD};$$

the total effect therefore will be

$$2\,W \times \frac{DA \times CA}{CD}.$$

Against this force the strength of the beam acts, and the strength of the beam at the section A is determined and expressed in exactly the same manner as in the former case, except that, in the present case, the average leverage by which the strength of the fibres resists rupture, is the distance of the centre of gravity below the highest point A of the section. *Expressing this* distance as before by C, we shall have the condition of equilibrium at the moment of fracture expressed by

$$2\,\text{W} \times \frac{\text{DA} \times \text{CA}}{\text{CD}} = f \times \text{A} \times c.$$

This may be simplified in the following manner:—

Let M, *fig.* 166., be the middle point of the beam between the two points of support, and let a express D M, half its length; let X express M A, the distance of the point where the breaking weight is placed from the middle point. We shall then have, by well-known principles of geometry,

$$\text{DA} \times \text{CA} = a^2 - \text{X}^2.$$

Hence we shall have

$$2\,\text{W} \times \frac{a^2 - \text{X}^2}{2\,a} = f \times \text{A} \times c,$$

and consequent y we shall have

$$\text{W} = \frac{f \times \text{A} \times c \times a}{a^2 - \text{X}^2}.$$

Hence it follows that if the weight be placed at the middle point between the two points of support, we shall have $\text{X} = 0$; and consequently,

$$\text{W} = \frac{f \times \text{A} \times c}{a}.$$

It appears from this, that the weight placed at the centre of a beam, between two points of support necessary to break it, is double that which would be sufficient to break the same beam if it were supported only at one point, the weight being placed at the other point.

Such are the general practical principles for determining the transverse strength of beams as established by Galileo; and although other practical formulæ have been proposed by later writers, the above have been found in such near accordance with the average results of experiments made upon a large scale, that they have been generally adopted by mechanical writers as the basis of their investigations upon the strength of materials.

Let us now see how the preceding formulæ are modified for beams of particular forms.

604. *Case of rectangular beams.* — If the beam be rectangular, let its breadth be b, its depth d, and its length $2\,a$. The distance of the centre of gravity of its section, therefore, from its upper or its lower surface will be $\frac{1}{2}\,d$. Now the area of its section will be $b \times d$: hence we have

[redacted]

$$A = b \times d,$$
$$c = \frac{1}{2} d,$$
$$L = 2a;$$

and consequently we have, for a beam supported at one end,

$$W = f \times \frac{b \times d^2}{4a}.$$

In like manner, if the beam be supported at both ends, the weight being placed at the middle, we shall have

$$W = f \times \frac{b \times d^2}{2a}.$$

If the beam be square, its breadth will be equal to its depth, and the breaking weight, where there is but one point of support, will be

$$W = f \times \frac{b^3}{4a};$$

and when there are two points of support,

$$W = f \times \frac{b^3}{2a}.$$

605. *How the strength of a beam is affected by the form of its transverse section.* — From the general principles here established it is evident, that, so long as the quantity of matter composing the beam, and therefore its sectional area, remains the same, its strength will be augmented by any modification of form which will carry its centre of gravity to a greater distance from its lower surface, if the beam have but one, and from its upper surface if it have two points of support.

Some curious and instructive consequences ensue from this.

Thus, the strength of a rectangular beam, when its narrow side is horizontal, is greater than when its broad side is horizontal, in the same proportion as the width of its broad side is greater than the width of its narrow side. Hence, in all parts of structures, where beams are subject to transverse strain, as in the rafters of floors, roofs, &c., they are placed with their narrow sides horizontal and their broad sides vertical.

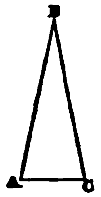

Fig. 168.

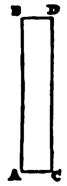

Fig. 169.

If a beam, supported at two points, bear a strain at any intermediate point, a given quantity of matter composing it will have greater strength if it be so formed that its area shall be less in the upper part than in the lower.

Thus, a beam of triangular section, such as A B C, *fig.* 168., will be stronger than a rectangular beam of equal section, such as A B D C, *fig.* 169, because the centre of gravity of the

[redacted]

triangle with its vertex upwards, will be further from B than that of the parallelogram from B D.

But if the beam be supported at one point only, then the position of the triangle must be inverted.

606. *Transverse strength of solid and hollow cylinders.* — If a solid and a hollow cylinder of equal lengths have the same quantity of matter, so that their sectional areas shall be equal, then their strength will be proportional to the distances of their centres of gravity from their external surface. But their centres of gravity being at their geometrical centres, it follows, that the strength of the solid cylinder will be less than the strength of the hollow cylinder, in the ratio of the diameter of the solid cylinder to the diameter of the external surface of the hollow cylinder.

It appears, therefore, that the strength of a tube is always greater than the strength of the same quantity of matter made into a solid rod; the practical limitation of the application of this principle being, that the thinness of the tube should not be so great as to cause a local derangement of its form by the application of a strong force.

607. *Examples in the structure of animals and plants.* — Innumerable striking and beautiful examples of this principle occur in the organized world. The bones of animals of every species are hollow cylinders, thereby combining strength with lightness. The stalks of numerous species of vegetables which have to bear a weight at their upper end are also tubes, whose lightness is remarkable when their strength is considered.

These are intended to resist not only the crushing weight of the ear which they bear at their summit, but also the lateral strain produced by the movement of the air.

The quills and the plumage of birds, and especially of their wings, present a still more striking example of the application of this principle in the animal structure. The surface of the extended wing acting on the air produces a strong transverse strain upon the quill, which has a single point of support near the joint of the wing.

The number expressed by f in the preceding formulæ is the strength of a beam, whose sectional area is the square unit, which in this case is generally taken as the square inch. This number is always the same for the same species of material, but different for different materials. In tabulating the strength of materials obtained from experiment, this is accordingly the number which is taken to designate the strength of each species of substance. When this number f is known, the strength of a beam of any proposed dimensions can be immediately calculated by the formulæ; for it is only necessary

[redacted]

to multiply this number f by the number of square inches in the sectional area, and the number of inches in the distance of the centre of gravity of this area from the lower or upper surface, as the case may be, and to divide the product by the length, or by half the length of the beam, according to circumstances; and the result of this arithmetical process will give the breaking weight: the first alternative being taken when the beam is supported only at one end.

It must be understood that this weight, and the force expressed by the number f, are to be expressed in the same unit.

608. *Tabular statements of the average transverse strength of beams.* — In ascertaining by experiment the average strength of each material, the number which it is important to determine is therefore that which is here expressed by f, and which may be considered the unit of strength, because, this being once determined, the strength of a beam of the proposed materials of any proposed denomination can be immediately computed.

This quantity f can be determined by direct experiment. If the amount of the breaking weight upon a beam of given dimensions be ascertained, we shall know the numbers severally expressed by W b d and a in the preceding formulæ; and in this case we shall have for a beam supported at both ends

$$f = \text{W} \times \frac{2\,a}{b \times d^2};$$

and for a beam supported at one end

$$f = \text{W} \times \frac{4\,a}{b \times d^2}.$$

In practice it will be found that the values obtained for the number f will be subject to considerable variations in different experiments, owing to casual inequalities and defects which affect each particular beam submitted to experiment.

The value of f must therefore, for each material, be obtained by taking an average of the results of numerous experiments; the casual inequalities being therein made to disappear from the result in proportion to the number and variety of trials.

609. *Tredgold's table of the transverse strength of metals and woods.* — In the following table is given the results of a series of experiments made by Tredgold upon the transverse strength of prismatic beams. The numbers in the second column represent the values of f in the preceding formulæ, while the numbers in the third column express the number of lbs. weight in a cubic foot of the material under experiment.

TABLE.

Name.	f.*	Lbs. Weight in Cubic Foot.
1. Metals:—		
Malleable iron	17,800	475
Hammered iron	—	487
Cast-iron	15,800	450
Brass	6,700	586·25
Zinc	5,700	439·25
Tin	2,880	455·7
Lead	1,500	709·5
2. Woods:—		
Oak	8,960	52
Fir (red or yellow) . . .	4,290	34·8
Pine (American yellow) . .	8,900	26·75
Fir (white)	8,680	29·3
Ash	8,540	47·5
Elm	2,840	34

610. *Barlow's table of the transverse strength of wood.*—Professor Barlow of Woolwich gives a table of the strength of beams to resist transverse strain, from which the following is extracted. The numbers in the second column in this case represent the same number *f* as those in the second column of the preceding table.

TABLE.

Name.	f.
Teak - - - - - - - - -	9,848
Poon - - - - - - - - - -	8,884
English oak, 1st specimen - - - -	4,724
" 2d specimen - - - - - -	6,688
Canadian oak - - - - - - - -	7,064
Ash - - - - - - - - -	8,104
Beech - - - - - - - - -	6,224
Elm - - - - - - - - -	4,052
Pitch pine - - - - - - - -	6,528
New England fir - - - - - - -	4,408
Riga fir - - - - - - - -	4,432
Mar Forest fir - - - - - - - -	5,048
Larch - - - - - - - - -	4,596
Norway spar - - - - - - - -	5,896

611. *Effect of the distribution of the weight upon the beam.*—If the weight producing the strain upon a beam, instead of being con-

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* The numbers given by Tredgold are the values of

$$W \times \frac{L}{4B \times D^2}.$$

We have obtained the numbers given above by multiplying this by 4.

[redacted]

ıted at a single point, as supposed in the cases here investigated, ually distributed over the whole beam, the power of suspension ıes twice as great as if it were applied at the middle point of eam.

has been also shown that each point of the beam has a greater rting power the nearer it is to either point of support. It is ntly, therefore, advantageous in all structures, in the distribution e weight upon them, to throw a less quantity at the centre, and crease the quantity towards the points of support. In this man- of loading, a far greater weight may be placed near the walls at the centre. In all cases, however, a concentration of weight single point is to be avoided.

sheet of ice which would break under the weight of a skater d sustain the same skater if his weight were equally distributed its surface.

allowing for the weight of the beam itself, this weight may be dered as uniformly spread over its surface.

2. *Strength of a beam increased by partially sawing it trans- ly and inserting a wedge.*—According to Peschel, the transverse gth of a beam of timber may be greatly increased by sawing from one third to one half of its depth, and driving in a wedge etal or hard wood until the beam is forced at the middle out of orizontal line, so as to form an angle presented upwards. It found by such an experiment that the transverse strength of a thus cut to one-third of its depth, was increased one-nineteenth; cut to one-half of its depth, it was increased one twenty-ninth; when cut to three-fourths of its depth, it was increased one y-seventh.

3. *Why the strength of a structure is diminished as its magni- is increased.*— It follows from the principles which have been ined, that if any structure be increased in magnitude, the pro- on of its dimensions being preserved, the strength will be aug- ted as the squares of the ratio in which it is increased. Thus, dimensions be increased in a two-fold proportion, its strength be increased in a fourfold proportion; if they be increased in a fold proportion, its strength will be increased in a ninefold pro- ion, and so on. But it is to be considered, that by increasing trength in a twofold proportion, its volume, and consequently its ht, will be increased in an eightfold proportion; and by increas- its dimensions in a threefold proportion, its volume and weight be increased twenty-seven times; and so on. Thus it is appa- that the weight increases in a vastly more rapid proportion than strength, and that, consequently, in such increase of dimensions, nit would speedily be attained at which the weight would become l to the strength, and beyond this limit the structure would be hed under its own weight. On the other hand, the more below

this limit the dimensions of the structure are kept, the greater will be the proportion by which the strength will exceed the weight.

614. *The strength on the large scale not to be judged by that of the model.*—The strength of a structure of any kind is therefore not to be determined by its model, which will always be much stronger relatively to its size. All works, natural and artificial, have limits of magnitude, which, while their materials remain the same, cannot be exceeded. Small animals are stronger in proportion than larger ones. We find insects and animalcula capable of bodily activity, exceeding almost in an infinite degree the agility and muscular exertion manifested by the larger class: the young plant has more available strength in proportion than the forest tree.

An admirable instance of beneficence in the consequences of this principle is, that children, who are so much more exposed to accidents, are less liable to injury from them than grown persons.

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PRACTICAL QUESTIONS FOR THE STUDENT.

1. Two parallel forces, acting in the same direction, have the magnitudes 5 and 13, and their points of application are 6 feet distant. What is the magnitude of their resultant, and its distance from each point of application? (158.)

2. If the forces in the previous question act in opposite directions, what is the magnitude and situation of their resultant? (159.)

3. On the supposition that the earth describes an orbit of 600 millions of miles in 365¼ days, with what velocity does it move per second?

4. A ship weighing 386,000 lbs., is dashed against the rocks in a storm, with a velocity of 16 miles per hour: with what momentum does it strike? (198.)

5. Suppose the battering-ram of Titus, which weighed 5,760 lbs., was found sufficient, when impelled with a velocity of 11 feet per second, to demolish the walls of Jerusalem; with what velocity must a cannon-ball, weighing 32 lbs., move in order to do the same execution?

6. A body has been falling 9 seconds; what space has it fallen through and what velocity has it acquired? (248.)

7. How far must a body fall to acquire a velocity of 150 feet per second?

8. What space was described in the last second by a body which has fallen 7 seconds?

9. With what velocity must a body be projected into a well 350 feet deep, in order that it may arrive at the bottom in 4 seconds?

10. A body is projected perpendicularly upwards with a velocity of 200 feet per second; how high will it ascend? (254.)

11. A cannon-ball, being fired perpendicularly upwards, returned in 20 seconds to the place from which it was fired; how high did it ascend, and what was the velocity of projection?

12. A body is projected downwards with a velocity of 25 per second; how far will it fall in 5 seconds?

13. Wishing to ascertain the difference in the depths of two wells, I dropped a stone into one of them, and heard it strike the water in 6 seconds, and then into the other and heard it strike in 8 seconds; what was the difference of their depths, assuming the instantaneous transmission of sound?

14. What velocity will be acquired by a body in falling for 15 seconds down an inclined plane, whose length is 2½ times its height? (255.)

NOTE.—The length of the inclined plane is to its height, as the velocity acquired by a body falling freely is to the velocity acquired by a body falling down the inclined plane.

15. How long would a body be in falling down an inclined plane whose height is to its length as 7 to 15, to acquire a velocity of 75 feet per second?

16. The length of an inclined plane is 400 feet, and its height 250 feet: through what space will a body descend in 3½ seconds? and what velocity will it acquire?

17. A stone weighing 4 lbs. is whirled around by a string 2 yards long, making three revolutions per second; what is the centrifugal force? (315.)

18. In a lever of the first kind, 4 feet in length, the power is 10 lbs. and the weight 14 lbs.; what must be their respective distances from the fulcrum? (426.)

19. A lever of the second kind is 25 feet long: at what distance from the fulcrum must a weight of 100 lbs. be placed, so that it may be sustained by a power of 80 lbs.?

20. Two persons, A and B, sustain upon their shoulders a weight of 200 lbs., by means of a pole 6 feet long, the point of suspension being 2½ feet from A: what portion of the weight does each sustain? (437.)

21. In the compound lever represented in *fig.* 97., let the arms P F, P′ F′, and P″ F″ be respectively 7, 13, and 11 feet; and the arms P′ F, P″ F′, and W F″ respectively 2. 5. and 6: what weight will a power of 16 lbs. sustain?

22. A weight of 100 lbs. is suspended by a rope going round an axle whose radius is 6 inches; what must be the diameter of the wheel, in order that the weight may be kept in equilibrium by a power of 12 lbs.? (445.)

23. A power of 14 lbs. acts on a wheel 9 feet in diameter: what weight, acting on an axle of 7 inches in diameter, will keep it in equilibrium?

24. There are two wheels on the same axle; the diameter of one is 5 feet, that of the other 4 feet, and the diameter of the axle is 20 inches: what weight on the axle would be supported by a power of 48 lbs. on the larger and of 50 lbs. on the smaller wheel?

25. In a differential wheel and axle, the diameter of the wheel is 3 feet, the diameter of the thicker part of the axle 7 inches, and that of the thinner part 6½ inches: what weight will a power of 60 lbs. sustain? (452.)

26. What power will be necessary to sustain a weight of 2,387 lbs., in a system of 10 pulleys, constructed according to *fig.* 126?

27. In a system of pulleys, represented in *fig.* 124., a power of 1 lb. sustains a weight of 128 lbs.: what is the number of pulleys?

28. A man can draw a weight of 125 lbs. up the side of a perpendicular wall, 20 feet high: what weight will he be able to raise along a smooth plank 44 feet long, laid sloping from the top of the wall? (478.)

29. With what force will a weight of 1,728 lbs. press on an inclined plane, whose length is 55 feet and base 44 feet?

[redacted]

30. What force must be exerted to sustain a ton weight on a screw, the thread of which makes 150 revolutions in the height of 1 foot, the power being applied to a lever 6 feet long? (495.)

31. What weight can be sustained on a screw by a power of 2 lbs., having a leverage of 3 yards, the distance between the threads being 1 inch?

32. A body is projected up an inclined plane whose length is 8 times its height, with a velocity of 60 feet per second: in what time will its velocity be destroyed? and how far will it ascend the plane?

33. What is the length of a pendulum, in Paris, which oscillates twice in 1 second? (540. and 552.)

34. What is the time of oscillation of a pendulum 10 feet long?

35. If the length of the seconds pendulum in New York is 39·10168 inches, through what space will a body fall there in one second?

36. What weight might be supported at the middle point of a bar of cast-iron 10 feet long, and whose transverse section is 3 inches square, its own weight not being considered? (604. and 609.)

37. In the previous example, determine what weight might be supported, if the weight of the bar were considered?

38. According to Tredgold's experiments (609.) what must be the length of a beam of oak, 3 inches square, and supported at both ends, which is just capable of bearing its own weight? (611.)

39. A boat weighing 350 lbs. is moving at the rate of 7 miles per hour: at what rate would it move if it were connected by a tow-line with a schooner weighing 120 tons and moving 2 miles per hour?

40. A cylindrical bar weighing 140 lbs. and 7 feet long, is employed as a lever to raise a weight of 2 tons. The fulcrum is 1½ ft. from the weight. Taking into consideration the weight of the lever itself, what must be the amount of the power?

NOTE.—The weight of the lever must be regarded as a power applied at the centre of gravity of the lever; which in this case is 2 ft. from the fulcrum.

41. A body weighs 11 pounds at one end of a false balance, and 17 lbs. 3 oz. at the other: what is the real weight?

NOTE.—A false balance is an ordinary pair of scales whose arms are unequal. Such a balance is used for fraud, the article to be sold being placed in the scale attached to the longer arm.

Let p be the arm to which the 11 lbs., and w that to which the 17 lbs. 3 oz. is attached. Call the article to be weighed $\mathbf{w}$.

Then $$11 \times p = \mathbf{w} \times w:$$

and $$17\tfrac{3}{16} \times w = \mathbf{w} \times p.$$

By multiplying these two equations, we get

$$11 \times 17\tfrac{3}{16} \times p \times w = \mathbf{w}^2 \times p \times w;$$

or, $$11 \times 17\tfrac{3}{16} = \mathbf{w}^2;$$

Consequently $$\sqrt{11 \times 17\tfrac{3}{16}} = \mathbf{w}.$$

Hence, to ascertain the true weight of a substance by means of a false balance, *weigh the substance in both scales, multiply these two false weights together, and take the square root of the product.*

42. A ball weighing 650 lbs. is moving at the uniform rate of 20 miles per hour, when it is overtaken by a ball of 50 lbs., moving 160 miles per hour: what will be the common velocity of both balls after collision?

43. What must be the length of a pendulum which shall oscillate 20 times in a second at New York? (see Question 35.)

44. What must be the length of an inclined plane, whose height is 15 feet, that the exertion of 42 pounds shall draw up 200 pounds?

45. If a cord, which runs over 3 moveable pulleys, be attached to an axle 4 inches in diameter, the wheel of the axle being 38 inches in diameter, and a power of 20 pounds be exerted at the circumference of the wheel, what weight would be raised under the pulleys?

46. Suppose a power of 48 pounds is to be employed to raise a weight of 5,000 pounds, by means of a screw whose threads are 1·3 inches apart: what must be the length of the lever, allowing ⅓ of the power to be lost in overcoming friction?

47. A body is projected perpendicularly upwards with a velocity of 250 feet per second: how far will it rise in 3 seconds? and what will be its velocity then?

48. In a press like that represented in *fig.* 154., the balls each weigh 250 pounds, and their centres are 5 feet 1 inch from the axis; the threads of the screw are 1⅝ inches apart; and the balls make two revolutions per second: what force will be exerted by the end of the screw?

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A

HAND-BOOK

OF

HYDROSTATICS,

HYDRAULICS, PNEUMATICS,

AND

SOUND.

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ILLUSTRATED BY NINETY-SEVEN ENGRAVINGS
ON WOOD.

PHILADELPHIA:
BLANCHARD AND LEA.
1859.

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CHAPTER I.

THE TRANSMISSION OF PRESSURE.

615. *The liquid state defined.*—The liquid state has already been defined to be that in which the constituent molecules of the body manifest neither cohesion nor repulsion. They have no tendency like those of a solid to cohere, and are separated by the least force applied to them; neither have they, on the other hand, any tendency like those of a gas to repel each other and fly asunder. The particles composing a mass of liquid lie in juxtaposition, each affected merely by its own gravity.

These observations would equally apply, however, to a collection of fine sand or dust, and it may therefore be asked in what respects such a mass differs from a liquid. The particles composing a liquid differ, in the first place, from those composing a mass of sand or dust in being infinitely small. A solid body can be reduced to no powder so impalpable, but that the separate grains of it may be individually contemplated and ascertained to possess all the characters of a solid body. This is not the case with the particles of a liquid which admit of unlimited subdivision, each part so divided still continuing to possess all the character of a liquid.

But independently of this, the particles of a liquid have the further quality, in which they are distinguished from a pulverized solid, of moving amongst each other without friction. There is no powder so fine or impalpable as to possess this property. The particles of a liquid, on the contrary, possess it in the most absolute manner.

616. *Liquids transmit pressure in all directions.*—From this absolute freedom of motion amongst each other, and total absence of friction, may be inferred the fact that liquids are capable of transmitting pressure equally in every direction, a quality which may be considered as the fundamental mechanical property of these bodies, and one from which all the circumstances attending the mechanical phenomena of liquids will follow.

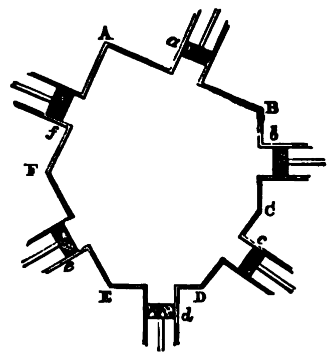

Fig. 170.

To explain this property of the free transmission of pressure, let us suppose a vessel A B C D E F, *fig.* 170., of any form whatever, to be filled with a liquid which we shall suppose for the present to be divested of weight.

Let a small circular aperture having the magnitude of a square inch, be made in each of the sides of this vessel at *a*, *b*, *c*, *d*, *e*, *f*, and let a small cylinder be imagined to be inserted in each of these apertures, in which a piston is fitted, so as to be in contact with the liquid, and so that the liquid shall not pass by the piston in the cylinder.

As we have supposed the liquid itself to be divested of weight, it will have no tendency to press any of the pistons outwards. Now, if any pressure whatever, as, for example, one pound weight pressure on the piston *a*, force it inwards, it will be found that all the other pistons, *b*, *c*, *d*, *e*, *f*, will immediately be forced outwards, and that the force necessary to resist this tendency would be one pound; so that when the piston *a* is pressed inwards with a force of one pound, it would be necessary to apply a like force pressing inwards to each of the other pistons to keep them in their places. It follows from this, that a pressure of one pound applied at *a* exerts a corresponding pressure of one pound against the inner surface of each of the pistons *b*, *c*, *d*, *e*, *f*. But the same would be true whatever positions the pistons *b*, *c*, *d*, *e*, *f*, might have; and it follows, therefore, that a pressure of one pound exerted upon the square inch of surface forming the base of the piston *a*, will produce a pressure of one pound upon every square inch of the interior of the surface of the vessel containing the liquid.

This property of transmitting pressure equally and freely in every direction, is one in virtue of which a liquid becomes, in the strictest sense of the term, a machine.

617. *The hydrostatic paradox no paradox.* — Some of the effects which are consequent upon it are so surprising and unexpected that they have acquired for it the generally known title of the hydrostatic paradox. There is, however, in this effect nothing more deserving of the title of paradox than is to be found in the effects of all other machinery.

One of the forms under which the hydrostatic paradox is commonly presented, is the following: Let A B C D, *fig.* 171., be a close vessel d with water.

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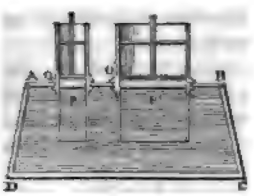

Fig. 171.

Let O be a cylinder, having in it a piston P, the area of whose base is one square inch; and let O' be another cylinder having in it a piston P', the area of whose base is 1000 square inches. According to what has been stated, a pressure of 1 lb. acting on the piston P, will produce an outward pressure of 1000 lbs. acting on the piston P', so that a weight of 1 lb. resting upon the piston P would support a weight of 1000 lbs. resting upon the piston P'. Now, if the piston P be moved downwards with a force of 1 lb., the piston P', loaded with 1000 lbs., would be raised.

There is, nevertheless, in these facts, nothing contrary to what might be inferred from the common principles already explained as governing the effects of mechanical force. The action of the forces here supposed differs in nothing from that of like forces acting on a lever having unequal arms in the proportion of 1 to 1000. A weight of 1 lb. acting on the longer arm of such a lever would support or raise a weight of 1000 lbs. acting on the shorter arm. The liquid, in the present case, performs the office of the lever, and the inner surface of the vessel containing it performs the office of the fulcrum.

Nor is there, in the fact that 1000 lbs. on the piston P' are raised by the descending force of 1 lb. on the piston P, anything which might not be naturally expected. If the piston P descend one inch, a quantity of water which occupies one inch of the cylinder O will be expelled from it, and as the vessel A B C D is filled in every part, the piston P' must be forced upwards until space is obtained for the water which has been expelled from the cylinder O. But as the sectional area of the cylinder O' is 1000 times greater than that of the cylinder O, the height which the piston P' must be raised to give this space will be 1000 times less than that through which the piston P has descended; therefore, while the weight of 1 lb. on P has been moved through one inch, the weight of 1000 lbs. on P' will be raised through only the $\frac{1}{1000}$th part of an inch. If this process were repeated a thousand times, the weight of 1000 lbs. on P' would be raised through one inch; but in accomplishing this, the weight of 1 lb. acting on P would be moved successively through 1000 inches. The mechanical action, therefore, of the power in this case is expressed by the force of 1 lb. acting successively through 1000 inches, while the mechanical effect produced upon the resistance is expressed by 1000 lbs. raised through one inch.

Now, it is evident that in this there is nothing really paradoxical or difficult.

If the power could act directly on the 1000 lbs. which rest upon the piston P', and could separately raise them pound by pound one

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inch, it would accomplish the same mechanical effect without the instrumentality of the hydrostatic apparatus here described.

618. ***Bramah's hydrostatic press.***—An engine founded on this principle is well known as the hydrostatic or hydraulic press, and is sometimes denominated, from the engineer who gave it its present form, and brought it into general use, a Bramah's press. This machine is represented in *fig.* 172.

A piston or plunger A, playing through a water-tight collar, moves in a small cylinder C. At the bottom of this cylinder there is a valve B, which opens upwards, and communicates with a pipe which descends into a vessel of water placed under the apparatus.

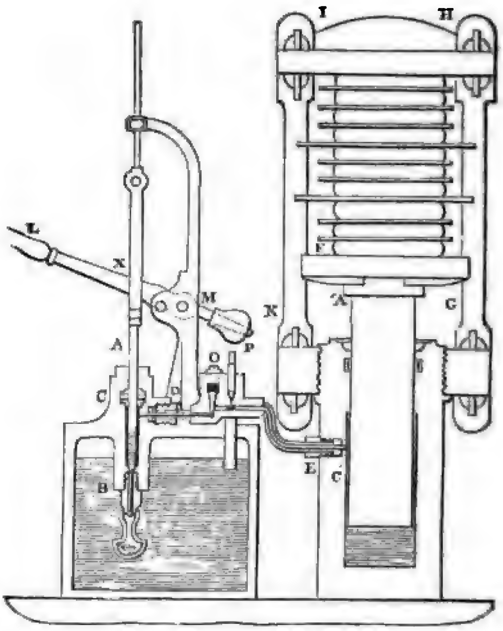

Fig. 172.

In the side of the cylinder C, there is a narrow tube, which communicates at E with another cylinder C', of much larger dimensions, in which there is likewise a plunger A', moving in a water-tight collar, and supporting a strong iron plate, surrounded by a strong framework K I H G. When the plunger A' is forced upwards, any object which may be placed between the plate which it supports and the top of the framework will be submitted to a pressure proportional to the force with which the plunger A' is urged. In the tube D E, there is

[redacted]

a valve O which opens towards the greater cylinder C'; and in the same tube there is also a cock P, by which a communication between the cylinder C' and the reservoir of water can be established at pleasure. The rod A of the small plunger is connected at X with a lever L M, which works on a fulcrum at M; and the press is worked by raising and depressing this lever, which operation is attended with a process which we shall now describe.

Suppose the water, if there be any in the great cylinder C', to be discharged into the reservoir by the cock P, a process which will necessarily take place on opening the cock P, because in this case the weight of the plunger A' will force the water, which is under and around it, through the passage E, and through the open cock P, into the reservoir. The water in that case could not pass back through the valve O, because, as has been already explained, that valve opens towards the cylinder C', and, consequently, the pressure produced by the plunger A' acting upon it would only hold it more firmly closed.

Let us, then, suppose both pistons to be at the bottom of their respective cylinders, the cock P being closed. If the lever L be now raised, the piston A will be elevated, and an action will take place similar to that of a common pump, in consequence of which water will be drawn up from the reservoir, and will fill the barrel of the cylinder C. This water cannot return to the reservoir, because the valve B, opening upwards, is only held more firmly closed by any pressure exerted upon the water in the cylinder.

Let the lever L be now pressed down; the water in the cylinder will be forced by the plunger through the valve O and through the tube D E into the great cylinder C'. By continuing this process, a quantity of water, equal to the volume of the plunger A, will be forced at each stroke from the cylinder C into the cylinder C'. In this action, the pressure upon the plunger A will be transmitted to the plunger A', augmented in the proportion of the magnitude of the sectional area of the one to that of the sectional area of the other.

Thus, if the sectional area of the plunger A' be 1000 times that of the plunger A, a pressure of 10 lbs. produced upon A will transmit a pressure of 10,000 lbs. to A'. During the operation of the machine in the intervals of the ascent of the plunger A, its action upon A' is suspended, and if the tube D E were open, A' would press the water back into the cylinder C, and render abortive the action of the machine, since every upward motion of the plunger A', produced by the descending stroke of the plunger A, would be neutralised by an equal descending motion during the upward stroke. This is prevented, however, by the valve O, which opens towards the cylinder C', and prevents the return of the water into the cylinder C, and its passage into the reservoir through the cock P is prevented by that cock being previously closed.

It will be perceived, therefore, that the valve O, in this case, dis-

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charges the function of a ratchet-wheel, allowing the motion of the fluid in one direction, but obstructing it in the other.

When it is required to release the object submitted to the action of the press, the cock P is opened, the weight of the plunger A′ then forces the water through the passage E, and through the open cock P, into the reservoir, and the plunger A′ descends.

It is evident that the power of the machine is augmented by the mechanical effect of the lever L M.

It has been shown that a force of 10 lbs. acting upon the small plunger will produce a force of 10,000 lbs. upon the great one; and if, in addition to this, the large arm of the lever L M be ten times the length of the short arm, a force of 1 lb. at the extremity of the former will produce a force of 10 lbs. on the plunger. Under such circumstances, therefore, a moving power of 1 lb. applied at the extremity of the working lever will produce a pressure of 10,000 lbs. upon the great plunger.

The advantage of the hydrostatic press over screw-presses, and those constructed upon mechanical principles, is that in the former, the force lost by friction is comparatively inconsiderable, being limited to the friction of the plungers in the cylinders.

619. *The hydrostatic bellows.*—The apparatus called the hydrostatic bellows, represented in *fig.* 173, acts upon the same principle.

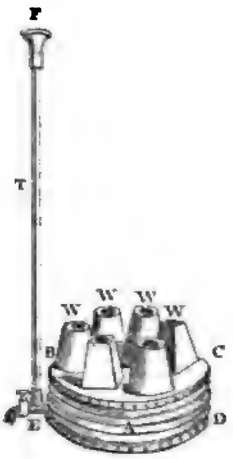

Fig. 173.

Two boards B C and D E are united by a flexible cloth, like a common bellows. The vertical tube T communicates with the interior, through the short pipe E, at right angles to it, which is also supplied with a stop-cock, by which the water contained in the bellows may be discharged. The apparatus being empty, let it be loaded with the weights W, and let water be then poured in at the funnel F; as the bellows fills, the weights will be raised, and the weight of a small column of water in the vertical pipe T will be capable of supporting a weight upon the board B C, greater than the weight of the water in the pipe, in the same proportion as the area of the board B C is greater than the sectional area of the bore of the pipe. Thus, if we suppose the area of the bore of the pipe to be a quarter of an inch, and the area of the board to be a square foot, then the proportion of the pipe to the area of the board will be that of 1 to 576; consequently, the weight capable of being supported by the board will be 576 times the weight of the water contained in the pipe.

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l20. *Transmission of pressure by water suggested as a tele- phic agent.*—This power of liquids to transmit pressure to a dis- ce was proposed as a means of telegraphic communication before invention of the electric telegraph.

It was proposed that small water-pipes should be buried along the es of road, and that a pressure produced upon the column of ter at any one place being transmitted to any other place, however tant, should be used as a signal. Thus, if a tube filled with ter extended from London to Edinburgh, a pressure exerted on the uid at the end of the tube at London would cause a corresponding essure or motion at the end of the column at Edinburgh, no matter at might be the course of the tube between the two places, whether aight, curved, or angular, and whether the pipes proceeded down- rds, obliquely, or horizontally, or whether they were carried rough the walls of a building, or the course of a river, or under, er, or around any obstruction or impediment whatsoever.

The same property of transmitting pressure has been proposed to e applied to several purposes, where it is required to produce a pres- ure on some internal part which cannot be approached except by a lexible tube, through which an instrument cannot safely or con- eniently be inserted.

621. *Example of hydrostatic pressure in the circulation of the blood.*—The animal economy supplies an example of the laws of hydrostatics, as striking as that which the skeleton exhibits of the laws of mechanics.

The heart from which the blood is supplied to all parts of the system, is an organ endued with great powers of expansion and contraction.

When it is contracted, a pressure is exerted upon the blood in immediate contact with the muscles of the heart, by which that fluid is driven through the arteries, pressing forward the blood which already fills those canals into the veins. These various pipes and conduits are formed of an elastic material, and supplied with valves, like the valve O in the hydrostatic press, which prevent the reflux of the blood. These valves, therefore, by their reaction, form so many fulcrums, from which the contractile force of the vessels containing the blood derive their effect; and the motion of this fluid is thus continued through the veins and returns to the heart. The muscular power of the heart to exert pressure on the blood, is illustrated by a striking experiment recorded by Dr. Hales in his statical essays. A vertical tube was put in communication with the arterial blood of an animal. The fluid, yielding to the pressure received from the heart, rushed into the tube, and rose to a certain height proportioned to the pressure. This height was found to vary in different animals, being greater in the larger than in the smaller classes. In the horse, a column of ten feet was supported; in the human body, the height of

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the column was but eight feet. The pressure of the venous blood was less than that of the arterial, as might have been anticipated; in the human species its pressure sustaining only a column of six inches.

CHAP. II.

THE PRESSURE OF LIQUIDS DUE TO THEIR WEIGHT.

622. *Effect of the weight of a liquid.* — The property by which liquids transmit pressure freely in every direction being understood, the manner in which this property modifies the effects of their own weight, and distinguishes them from those which attend the weights of solid bodies, remains to be explained.

If a solid body be placed in a vessel having a horizontal bottom and upright sides, and so corresponding in shape to the body that the latter shall exactly fill it as a plug would fill a tube, the effect of the weight of such a body, placed in such a vessel, would be to press with its whole force upon the horizontal bottom, no pressure whatever being exerted on the sides. If in such a case the sides were detached from the vessel, the body contained in it would remain undisturbed, pressing upon the bottom as before.

But if we suppose the body thus contained in the vessel to be liquefied, we shall then be unable to remove the sides without totally disarranging the state of the body; since, the moment the body becomes liquid, the sides will have to resist the tendency of its component particles to fall asunder, which tendency was before resisted by that mutual cohesion which constitutes the character of solid bodies.

The change from solid to liquid would in this case make no change in the pressure produced upon the horizontal base of the vessel; but the pressure on the sides will depend on conditions determined by the depth of the particles of fluid, which we shall now explain.

Let A B C D, *fig.* 174., be such a vessel as is above described, B C being the horizontal bottom, and A B and D C vertical sides. If it be filled with a solid body of its own form, the upper surface of which E F is level and parallel to the bottom B C, this body will press upon the bottom B C with the full amount of its weight, and no pressure whatever will be exerted on the perpendicular sides A B and D C. But if such body be liquefied, a pressure will immediately take place on all the sides, *the pressure* on the bottom remaining the same as before.

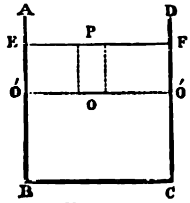

Fig. 174.

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Let O be a part of a level stratum of the liquid, which we shall suppose to have the magnitude of a square inch, and to be placed at any part within the dimensions of the liquid. It is evident that the surface O will sustain directly the pressure of the vertical column of liquid O P, which is immediately above it, extending from the surface of the liquid downwards to O. This pressure will be equivalent to the weight of the column — that is to say, of as many cubic inches of the liquid as there are inches in the depth of the stratum O below the level. It appears, therefore, evident, that every square inch of any stratum of the liquid must sustain a downward pressure equal to the weight of a column of the liquid whose base is a square inch, and whose height is equal to the depth of the proposed stratum.

But since this downward pressure is transmitted equally in every direction (616.), it is clear that it will be transmitted to the sides of the vessel, and will act upon them laterally with the same force as that with which it acts downwards; consequently it follows, that a square inch of the vessel at O′, in the same level stratum with O, will sustain a pressure perpendicular to the surface of the same amount.

623. ***Pressure of a liquid contained in a vessel proportional to the depth.*** — It follows, therefore, in general, that *each square inch of the surface of a vessel containing a liquid is pressed by a force perpendicular to such surface, equal to the weight of a column of the liquid whose base is a square inch, and whose height is equal to the depth of the point of the surface in question below the level of the fluid.*

In *fig.* 174. the sides of the vessel are represented to be perpendicular, but the same reasoning will be applicable, whatever be their position. Thus, if they diverge from the bottom, as in *fig.* 175., the pressure produced upon a square inch of the surface of the side at O′ will, for the same reason, be equal to the weight of a column of the liquid whose height is equal to the depth of the point O, and whose base is a square inch.

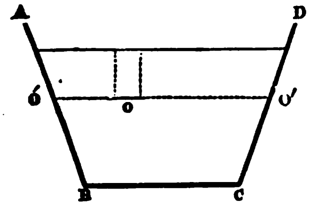

Fig. 175.

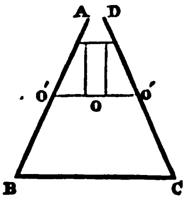

Fig. 176.

Again, if the sides converge, as in *fig.* 176., the same principle will obtain.

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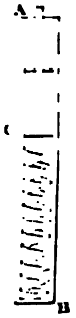


Fig. 177.

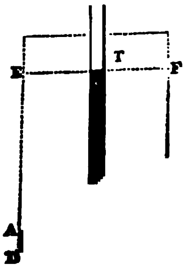
624. *Experimental proof of this.* — There are other expedients by which this pressure of liquids proportional to their depth can be verified experimentally.

Let A B, *fig.* 177., be a strong metallic cylinder, open at A and closed at B, in which a piston C moves water-tight, and rests upon a spiral spring extending to the bottom B, so that any force tending to press the piston downwards is resisted by the elasticity of the spring. If such an instrument be plunged to any depth in a liquid, the piston will be pressed inwards with a force corresponding to the pressure of the liquid.

Now it is found by experiments made with this instrument, that the pressure which urges the piston from C towards B is always equal to the weight of a column of the liquid in which it is immersed, whose base is equal to the magnitude of the piston C, and whose height is equal to the depth of the piston below the surface of the liquid. It is further proved, that this pressure is equally exerted in every direction, because the piston will act against the spring with the same force, in whatever position the instrument may be placed. If it be placed vertically with the open end A upwards, it will indicate the downward pressure of the liquid; if it be placed vertically with the end B upwards, it will indicate the upward pressure of the liquid; if it be placed with the length A B horizontal, it will indicate the lateral pressure; and in all intermediate positions it will indicate the pressure in every other possible direction.

In all these cases, the piston will compress the spring to the same point, whatever direction be given to the instrument, provided only the piston C be kept at the same depth.

625. *Pressure on the bottom of a vessel.* — From what has been just established, it follows that the pressure upon any part of the surface of a vessel containing a liquid which is horizontal will be uniform, and will be equal to the weight of a column of the liquid whose base is equal to the lower surface, and whose height is equal to the depth of such surface below the level of the liquid; and in this case it must be observed, that it is not necessary that a column of the liquid should be actually above the surface in question. This consequence leads to another form under hich the hydrostatic paradox presents itself.



626. *Another form of the hydrostatic para- r.* — Let A B C D, *fig.* 178., be a close vessel, th a small hole O on the top, in which a narrow ibe T O is screwed, water-tight. Let the vessel T O be filled with water. According to what the pressure on the bottom C D will be equal to

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the weight of a column of water whose base would be equal to the area of the bottom C D, and whose height would be T M; that is, it would be equal to the quantity of water which would fill a vessel whose base is C D, having perpendicular sides D E and C F, and whose height is D E. This will be true, however shallow the vessel A B C D, and however narrow the tube T O, may be; and hence an indefinitely small quantity of water may be made to produce a pressure on the bottom of the vessel which contains it, equal to the weight of any quantity of water, however great. As the pressure depends only on the depth of D C below the level of the water in the tube T O, it is not necessary that the tube T O should be straight; it may be bent or deflected in any direction or form whatever; but whatever be its shape, the depth of the fluid which determines the pressure is to be estimated by the perpendicular distance of its upper surface in the tube from the bottom of the vessel.

627. *Pressure on the side of a vessel.*—If any part of the surface of a vessel containing a liquid be not horizontal, the pressure against its different parts will vary according to their depth. Let A B C D, *fig.* 179., be a vessel with a flat bottom and perpendicular sides, and let it be supposed to be filled with water. The depth of the side A B, being divided into ten equal parts, let us suppose that the pressure produced against the first division of the side between 0 and 1 is one pound; then the pressure produced against the division between 1 and 2 will be two pounds, the pressure against the division between 2 and 3 will be three pounds, and so on; so that, in fine, the pressure produced against the last division, between 9 and 10, will be ten pounds.

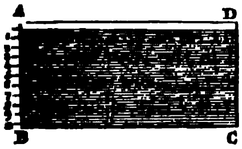

Fig. 179.

Since, therefore, the intensity of the pressure from A to B increases uniformly with the depth, the average pressure will be found at the fifth division, being the middle point of the depth, and the total pressure upon the side will be the same as if it sustained such average pressure upon the fifth division, distributed uniformly over the whole surface. Hence it follows that the total pressure upon the side of such a vessel will be equal to the weight of a column of the liquid whose base is equal to the area of such side, and whose height is equal to one half the depth of the liquid in the vessel, or, in other words, to the depth of the middle point of the side below the surface.

We have here supposed the side of the vessel to be vertical; but the same conclusion will follow if it be inclined either outwards, as in *fig.* 180., or inwards, as in *fig.* 181.

In all cases, therefore, where the surfaces which contain a liquid are either vertical, or inclined to the vertical line, the total pressure which they sustain can be found by multiplying the number of square

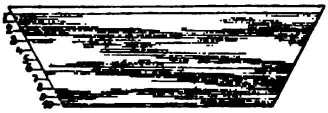

Fig. 180.

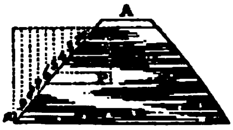

Fig. 181.

feet in the area of such surfaces by the number of feet in the depth of its middle point, or more generally by the number of feet in the perpendicular distance of its point of average depth below the surface.

628. *Total pressure on the bottom and sides much greater than the weight of the liquid.* — It follows, therefore, that the actual pressure produced upon the bottom and sides of a vessel which contains a liquid is always much greater than the weight of the liquid. If, for example, the vessel have a cubical form, the pressure on the bottom will be equal to the weight of the liquid, and the pressure on each of the four sides will be equal to one half the weight of the liquid; consequently, the total pressure on the bottom and sides will be exactly three times the weight of the liquid contained in the vessel.

In tall narrow vessels containing liquids, the pressure against the sides far exceeds the weight of the liquid: tall casks, cisterns, and tubes, which are carried in a vertical direction, require to have a lateral strength very far exceeding that which would be necessary merely to support the liquids they contain.

629. *Pressure on dam or embankment.*—The increase of pressure proportional with the depth, suggests the expediency of observing a corresponding variation in the strength of the several parts of embankments, dams, flood-gates, and other resistances opposed to the course of water.

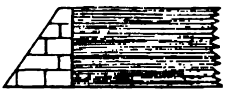

Fig. 182.

The pressure near the surface is inconsiderable, and therefore a small degree of strength is sufficient in the resisting surface; but as the depth increases, the pressure increases in the same proportion. If, therefore, as in the case of dams and embankments, the strength depends upon the thickness, the thickness must gradually increase from the top to the bottom, in proportion to the depth, so that while the interior surface presented to the liquid is perpendicular, the exterior surface will gradually slope, giving increased thickness to the wall or dam, as represented in *fig.* 182.

The pressure produced by a liquid on the horizontal bottom of the vessel containing it depends exclusively on the magnitude of such bottom and the depth of the liquid, and is altogether independent of

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the shape, magnitude, or position of the sides, and therefore of the quantity of liquid contained in the vessel.

630. *Experimental verification of these principles.* — It has been shown that in a vessel with perpendicular sides and horizontal bottom, such as that represented in *fig.* 179., the pressure on the bottom is equal to the total weight of the liquid contained in the vessel; but if the shape of the vessel were that represented in *fig.* 180., the pressure on the bottom would still remain the same, although the quantity of liquid would be considerably greater; and if the vessel were shaped as in *fig.* 181., the bottom being still the same, the pressure on the bottom would remain the same, although the quantity of liquid in the vessel would be considerably less. These results may be verified experimentally in various ways.

Thus, if three vessels be provided, the first with perpendicular, the second with diverging, and the third with converging sides, each having a moveable bottom fitting it water-tight, let the bottoms be pressed against each vessel with equal forces, which may be done by a lever, one arm of which is pressed upwards against the bottom by a weight suspended from the other arm. If water be then poured into the vessels severally until such a quantity has been introduced that its pressure on the bottom shall overcome the resistance of the lever, it will be found that the depth of the water in each case necessary to accomplish this is the same.

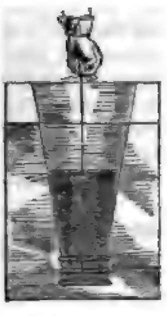

Fig. 183.

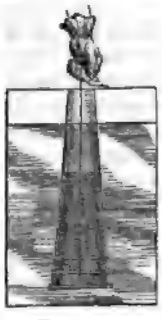

Fig. 184.

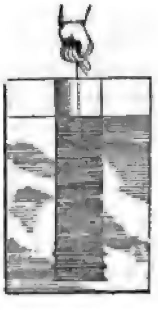

Fig. 185.

Another method of illustrating experimentally the same principle, is as follows: — Let the moveable bottom be pressed against each vessel by a string attached to it on the inside, and carried up through the vessel, and then let the vessels be plunged in a cistern of water, as represented in *figs.* 183., 184., and 185., until they attain such a depth, that the upward pressure of the water against the bottom will be sufficient to keep it attached to the vessel.

If the vessels be immersed to the same depth, the upward pressure thus acting upon the bottoms will be the same. Now let water be poured into each of the three vessels. It is evident that when such a quantity shall have been introduced as shall produce a downward pressure upon the bottom equal to the upward pressure, the bottom will no longer adhere, and it will fall. Now it is found by experiments conducted in this manner, that it requires the same depth of water in each of the three vessels to accomplish this.

631. *Pressure on the surfaces of vessels, whatever be their form.* — In the preceding examples, the surfaces confining the liquid have been considered to be flat. The surfaces, however, of vessels and reservoirs are subject to a variety of forms; and it is necessary in practical science to be in possession of rules which are applicable generally to all such surfaces.

The various parts of a surface containing a liquid will, according to the principles established, be subject to pressures depending only upon depths below the surface of the liquid in the vessel, all parts of the same depth being subject to the same pressure. If we imagine the entire surface of a reservoir below the level of the water to be divided into square inches, each square inch will sustain a pressure equal to the weight of a column of water whose base is a square inch, and whose height is equal to the depth of the square inch of the surface in question.

The total pressure, therefore, sustained by the surface of the reservoir, may be ascertained if the average depth of the surface below the level of the water could be determined, as in this case the total pressure exerted by the liquid on the surface of the vessel or reservoir would be equal to the weight of a column of the liquid, whose base would be equal in area to the entire surface of the vessel or reservoir, and whose height would be equal to the average depth of this surface.

632. *Method of ascertaining the point of average depth.* — Mathematical science supplies a method by which this point of average depth can be in all cases calculated.

If we suppose a thin sheet of any uniform substance, such as metal, to be spread over the surface of the reservoir, and in close contact with it, just as the inner surfaces of some vessels are lined with tinfoil, then the point of average depth will be identical with the centre of gravity of such a lining. The methods, therefore, by which the centre of gravity is determined, supply the means of ascertaining the points of average depth in all vessels and reservoirs; and these points being ascertained, the total pressure upon them can be computed.

633. *Examples of the application of this.* — Excepting in the case of certain surfaces of regular form, the determination of the centre of gravity, however, is a problem which cannot be solved without the application of mathematical principles of considerable difficulty. The theorem, however, just stated, may be illustrated by examples sufficiently simple to be generally understood.

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Let a hollow sphere be filled with a liquid through a small hole on the top. The centre of gravity of the surface of the sphere is evidently its centre, and therefore the depth of this point below the height at which the level of the liquid stands is one half the diameter of the sphere. The total pressure will therefore be found by multiplying the number of inches in half the diameter of the sphere by the number of square inches in its surface. By the principles of geometry, it is proved that the solid contents of a sphere are determined by multiplying the number of inches in half the diameter by a third part of the number of square inches in the surface. Hence it appears that the pressure produced upon the surface of the sphere by the liquid it contains, is three times the weight of its contents.

If a solid be immersed in a liquid, the pressure which its surface suffers from the surrounding liquid is determined by the same principles as those which determined the pressure on the surface of the vessel containing the liquid. Thus, if a sphere be plunged in a liquid, the total pressure upon its surface is found by multiplying the number of inches in the depth of its centre below the surface of the liquid by the number of square inches in its surface.

The two hydrostatical principles which have been established in the present and in the preceding chapters, first, that liquids transmit pressure equally in all directions; and, secondly, that the pressure they produce by their own weight is proportional to the depth, serve to explain many familiar and remarkable facts.

634. *Pressure of different liquids different.* — But to render them applicable to such exposition, it is not sufficient to know these laws determining the transmission and the variation of the pressure. It is necessary also to know the actual amount of this pressure for each particular liquid. Thus, for example, if two equal and similar vessels be filled to the same depth, the one with water and the other with quicksilver, it is evident that the pressure produced upon any part of the surface of the one will be greater than the pressure produced upon the corresponding part of the surface of the other, in exactly the same proportion as quicksilver is heavier than water.

To be enabled, therefore, to declare the actual intensity of the pressure produced in each case, it would be necessary to know the actual pressure which would be produced upon a surface of given magnitude by a column of water of given height; and further, to know the proportion which the weight of other liquids under inquiry would bear bulk for bulk to the weight of water.

635. *Actual pressure of water.* — We shall hereafter explain how the proportional weight of other liquids is ascertained and recorded For the present, we shall limit our observations to the most universal of all liquids, water.

It is ascertained that the weight of a cubic inch of water of the common temperature of 62° Fahr. is a portion of a pound expressed by the decimal

0·036065.

The pressure, therefore, of a column of water one foot high, having a square inch for its base, will be found by multiplying this by 12, and consequently will be

0·4328 lb.

The pressure produced upon a square foot by a column one foot high, will be found by multiplying this last number by 144; the number of square inches forming a square foot in which will therefore be

62·3232 lbs.

636. TABLE SHOWING THE PRESSURE IN LBS. PER SQUARE INCH AND SQUARE FOOT, PRODUCED BY WATER AT VARIOUS DEPTHS.

Depth in Feet.	Pressure per Square Inch.	Pressure per Square Foot.
	lbs.	lbs.
I.	0·4328	62·3232
II.	0·8656	124·6464
III.	1·2984	186·9696
IV.	1·7312	249·2928
V.	2·1640	311·6160
VI.	2·5968	373·9392
VII.	3·0296	436·2624
VIII.	3·4624	498·5856
IX.	3·8952	560·9088
X.	4·3280	623·2320

By the aid of the above table, the actual pressure of water on each part of the surface of a vessel containing it can always be determined, the depth of such part being given.

Thus, for example, if it be required to know the pressure upon a square foot of the bottom of a vessel where the depth of the water is 25 feet, we find from the above table, that the pressure upon a square foot at the depth of 2 feet is

124·6464 lbs.;

and, consequently, the pressure at the depth of 20 feet is

1246·464 lbs.:

to this, let the pressure at the depth of 5 feet, as given in the table, be added,

$$\begin{array}{r} 1246{\cdot}464 \\ 311{\cdot}616 \\ \hline 1558{\cdot}080 \text{ lbs.} \end{array}$$

which is therefore the required pressure.

[redacted]

637. *To find the corresponding pressures produced by other liquids.* —If the liquid contained in the vessel be not water, but any other whose relative weight compared with water is known, the calculation is made first for water, and the result being multiplied by the number expressing the proportion of the weight of the given liquid to that of water, the result will be the required pressure. The manner of determining this relative weight will be given hereafter.

638. *Examples of pressure produced at great depths.*—If an empty bottle tightly corked be sunk in the sea, the pressure of the surrounding water, when the depth is sufficient, will either break the bottle or force the cork into it; if the bottle have flat sides, it will be broken; if it be round, its form being stronger, the cork will be forced in.

If a piece of wood which floats on water be sunk to a great depth in the sea, and held there for a certain time, the great pressure of the surrounding liquid will force the water into the pores, the effect of which will be to increase its weight so that it will no longer be capable of floating or rising to the surface.

Divers plunge with impunity to certain depths, but there is a limit below which they cannot live under the intense pressure. It is probable, also, that there is a limit of depth below which each species of fish cannot live.

639. *Liquids not absolutely incompressible.* — Although the theorems of hydrostatics are established upon the supposition that liquids are incompressible, this is true only in a qualified sense. It was long considered, as has been already explained, that no force whatever was capable of compressing them. Experiments, however, instituted in 1761, by Canton, showed that they were compressible in a slight degree; and these experiments have been corroborated by means of the pressure of liquids at considerable depths, in the following manner. Let A B, *fig.* 186., be a cylindrical vessel, having a mouth C, through which a plunger P M passes water-tight. Let this vessel be completely filled with water, the end of the plunger being inserted in the mouth. Let a ring be placed upon the plunger in contact with the mouth, so that when the plunger is pressed into the vessel, the ring will be forced upwards upon it by the flange of the mouth, and let the friction of the ring with the plunger be sufficient to prevent it from falling back to its first position when the plunger is again drawn out.

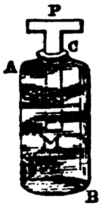

Fig. 186.

Let the vessel thus prepared be immersed to a considerable depth in the sea. Upon drawing it up, it will be found that the pressure of the surrounding water has forced the plunger further into the vessel, and that the water contained in it was therefore compressed into smaller dimensions. Upon drawing up the vessel, the removal of the pressure enabled the water contained in it to resume its former dimensions, and force the plunger back to its first position. When the vessel is raised from the water, the ring which was in contact with the flange of the mouth is found to be raised to a certain height above it.

640. *Method of ascertaining the force necessary to produce a given compression.* — The degree of compression produced by a given force may be found by measuring the total contents of the vessel, the magnitude of a given length, such as one inch of the plunger, the depth of a vessel to which the plunger has been forced, and the depth in the sea to which the vessel has been sunk. At the depth of 6,000 feet, the volume of the water contained in the vessel is diminished by one-twentieth of its original dimensions. Thus, 20 cubic inches of water will be reduced to 19 cubic inches, under the pressure of a column of sea-water, whose base is one square inch, and whose height is 6,000 feet. But this pressure, as shown by the table, would be

2596·8 lbs.

if it were produced by fresh water. But sea-water is heavier than fresh water, in the ratio of 102 to 100. Multiplying, therefore, the preceding number by 1·02 we obtain

2648·0 lbs.

It follows, therefore, that water, when submitted to a compressing force amounting to 2,648 lbs. per square inch, will be reduced in bulk one-twentieth, and that when released from that pressure it will recover its volume.

641. *Examples of effects of pressure.* — *Water-pipes for the supply of towns.* — If a fissure in a rock communicate with an internal cavity of any considerable magnitude, placed at some depth below the mouth of the fissure, rain percolating through, and filling the fissure above it, might produce a bursting force sufficient to split the rock. The pressure in this case, acting against the inner surface of the cavity, will be proportional to the depth of the cavity below the top of the fissure. It appears from the table, page 24., that for every foot in such difference of level, there will be a bursting pressure of 0·4328 lbs. for every square inch of the surface of the cavity.

In the construction of pipes for the supply of water to towns, it is necessary that those parts which are much below the level of the reservoir from which the water is supplied should have a strength proportionate to such difference of level, since they will sustain a bursting pressure of 4·328 lbs. per square inch for every 10 feet by which the level of the river exceeds in height that of the pipe. A pipe, the diameter of whose bore is 4 inches, has an internal circumference of about one foot, and the internal surface of one foot in length of such a pipe would measure a square foot. If such a pipe were 150 feet below the level of the reservoir, the bursting pressure which it would sustain upon one foot of its length may be calculated as follows, from the table, page 24.

[redacted]

	lbs.
Pressure at 100 feet deep - - -	6,232
" 50 " - - -	3,116
Pressure at 150 feet - - - -	9,348

Thus, such a pipe should be constructed of sufficient strength to bear with security nearly five tons bursting pressure on each foot of its length.

CHAP. III.

LIQUIDS MAINTAIN THEIR LEVEL.

642. *The surface of a liquid level when at rest.* — When a liquid contained in any vessel is in a state of rest, its surface will be horizontal; and if the same liquid be contained in different vessels, which have free communication with each other by tubes, pipes, or otherwise, then the surface of these liquids in the different vessels will be at the same level.

This important property of liquids, which is usually expressed by stating that liquids maintain their level, follows immediately from the two properties which have been established in the preceding chapters.

It may be proved that all parts of the surface of a liquid contained in a vessel must be at the same level when at rest, by showing that if they be not at the same level, the fluid must be in motion, and must continue in motion until they attain the same level.

Let A B C D, *fig.* 187, be a vessel containing a liquid whose surface is at different levels, as represented at M N; being higher at M than at N.

Let us suppose a partition introduced into this vessel, dividing its liquid contents into two parts, but having an opening near the bottom at O, the area of which opening we shall suppose to measure a square inch. If we take a column of the liquid whose height is M B, and whose base is a square inch, this column will press at the bottom B with a force equal to its weight, and this pressure will be transmitted equally in every direction throughout the dimensions of the liquid; and consequently it will be transmitted laterally to the orifice O, and through the *orifice O it* will be transmitted to the *liquid on the* other side of the partition.

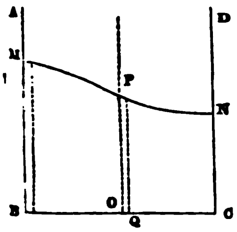

Fig. 187.

[redacted]

It will then likewise press equally in every direction; and if we suppose a column, P Q, having a square inch as its base, this column will be pressed upwards by the same force, but its downward pressure will be equal to the weight of the column P Q. Thus, a horizontal stratum of the liquid, measuring a square inch at O, will be pressed downwards by the weight of the column P Q, and upwards by the weight of the column M B. But the latter being greater than the former, the upward pressure will exceed the downward, and the column P Q will be raised.

The same will be true of every pair of vertical columns into which the liquid on either side of the partition may be resolved; and thus it follows that under such circumstances the liquid will flow from the side M B towards the side N C, the level of the former falling, and that of the latter rising.

But if the column P Q were equal to M B′, *fig.* 188, then the downward pressure would be equal to the upward pressure, and no motion would take place; and if the same were true of every pair of columns into which the liquid on either side of the partition could be resolved, then no motion would ensue; that is to say, if the surface, instead of being at different levels, as in *fig.* 187, were at the same level, as in *fig.* 188, then, and not otherwise, the fluid would remain at rest.

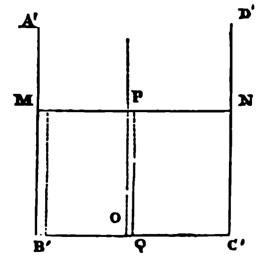

Fig. 188.

We have here imagined a partition to be introduced, dividing the vessel having an orifice near the bottom; but it is evident that the presence or absence of such a partition cannot affect the movement or the equilibrium of the fluid, since such a partition introduces no force to affect the fluid which did not previously exist.

643. *Examples illustrating the principle.* — This property of liquids is so nearly a self-evident consequence of their fundamental property, that it is difficult to demonstrate it. It is nothing more than a manifestation of the tendency of the component parts of a body to fall into the lowest position which the nature of their mutual connection, and the circumstances in which they are placed, will admit. Mountains do not sink and press up the interjacent valleys, because the cohesive principle which binds together the component parts of their masses, and those of the crust of the earth upon which they rest, is opposed to the gravity of their parts, and is much more powerful; but if this cohesion were dissolved in the stupendous masses, — for example, if the Alps or the Andes were liquefied, — these ridges would sink from their lofty eminences, and the circumjacent valleys would rise, a momentary interchange of form taking *place;* and this undulation would continue, until the whole mass

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would settle down into a uniform level surface. All inequalities, therefore, which we observe on the surface of land, are due to the predominance of the cohesive over the gravitating principle; the former depriving the earth of the power of transmitting equally and in every direction the pressure produced by the latter.

On the other hand, if the sea, when agitated by a storm, were suddenly solidified, the cohesive principle being called into action, the mass of water would lose its power of transmitting pressure, and those inequalities which in the liquid form are fluctuating would become fixed; a wave would become a hill, and an intermediate space a valley.

644. *Maintenance of level in communicating vessels.*—The maintenance of level between liquids contained in communicating vessels is established by reasoning similar to that by which the level surface of a liquid contained in any vessel is proved. Let A B C D, *fig.* 189., and A′ B′ C′ D′, be two vessels, between which there is a pipe of communication B′ C.

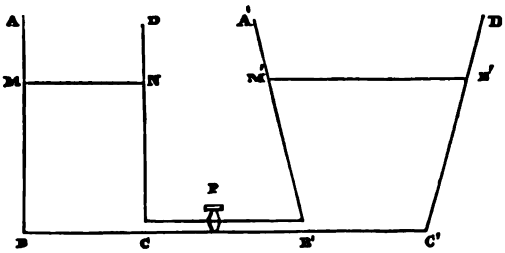

Fig. 189.

If these two vessels be partially filled with the same liquid, the surface M N of the liquid in the one vessel must be at the same level with the surface M′ N′ of the liquid in the other vessel, provided the liquids are at rest.

Let us suppose a stop-cock at P, in the tube of communication B′ C, this tube being horizontal.

By what has been proved it appears, that the pressure exerted by the liquid in A B C D upon the stop-cock will be equal to the weight of a column of the liquid, whose base is equal to the passage of the stop-cock, and whose height is equal to the depth of the stop-cock below the surface M N.

In like manner, the pressure exerted on the other side of the stop-cock by the liquid in the vessel A′ B′ C′ D′, will be equal to the weight of a column of liquid, whose base is equal to the opening of the stop-cock, and whose height is equal to the depth of the stop-cock below the surface M′ N′.

Hence it follows, that the stop-cock will be pressed equally in both directions, if the surfaces M N and M′ N′ are at equal heights above it; and consequently, in this case, if the stop-cock be opened, there will be no tendency of the liquid to flow in either direction through it.

But if, on the other hand, the surface M N be at a greater height above the stop-cock than the surface M′ N′, then there will be a greater pressure upon the stop-cock on the one side C than on the other B′; and if the stop-cock be opened, the liquid will flow from the vessel A B C D to the vessel A′ B′ C′ D′; and in like manner, if the level M N be lower than the level M′ N′, then the pressure on the side C will be less than the pressure on the side B′, and the liquid will flow in the contrary direction.

It therefore follows, in general, that if the levels of the liquid in the communicating vessels be the same, no motion of the liquid will follow; but if they be not the same, then the liquid will flow from the vessel which has the higher level to the vessel which has the lower level.

645. *Experimental illustration.* — The apparatus represented in *fig.* 190. is adapted to explain experimentally these facts. A, B, C, D, E are glass vessels of different shapes, each terminating at the bottom in a short tube which is inserted into a hollow box. In each of these short tubes is a stop-cock K.

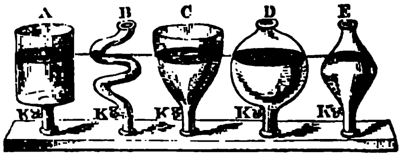

Fig. 190.

When the cocks are all open, a communication between the five vessels is established through the intervention of the box; but each or all of the vessels can be insulated by closing the cocks.

Let the stop-cocks be all closed, and let water be poured into the vessels, so as to stand at different levels, the case below being previously filled with water. If all the cocks be now opened, it will be observed that the higher levels will gradually fall, and the lower ones will rise until all become uniform. If the stop-cocks be again closed, and water be poured into some of the vessels, so as to render the levels again unequal, the same equalization would take place on again opening the stop-cocks.

646. *Position of the spout of a vessel.* — When vessels containing liquids are supplied with spouts, such as those attached to tea-pots, coffee-pots, kettles, watering-pots, and the like, the extremity of the spout must always be above the top of the vessel when the vessel

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stands erect, since otherwise the vessel could not be filled; for as soon as the liquid poured into it would raise it to the level of the top of the spout, any more liquid which might be poured in would flow out at the spout.

Liquids are discharged from a vessel having a spout by inclining the vessel in such a manner that the mouth of the spout shall be below the level of the liquid in the vessel.

647. *This property explains the hydrostatic paradox.* — Thi, property of liquids to maintain their level, when well understood, strips every form of the hydrostatic paradox of that character from which it has taken its denomination.

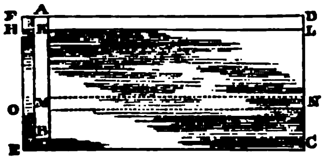

Fig. 191.

Let A B C D, for example, *fig.* 191., be a large vessel with perpendicular sides, and communicating by a short tube B E with a vertical tube E F. If water be poured into this vessel, it will rise to the same level in the tube. Now, let us suppose the water which fills the vessel above the level M N to be removed, and its place supplied by a piston moving water-tight in the vessel; and let this piston be loaded with a weight which shall be equal, including the weight of the piston itself, to the weight of all the water which has been removed. The piston will then press on the water below it with the same force as the water removed previously pressed upon it; and as the water removed was sustained by it, the piston with its load will also be sustained. Thus it appears that this piston is supported by the pressure of the column of water H O in the tube E F; and it will be easily perceived that the effect is identical with those of the hydrostatic bellows and the hydrostatic paradox already explained.

648. *Explanation of the phenomena of streams, rivers, cataracts, springs, &c.* — The play of the property in virtue of which liquids maintain their level, explains an infinite variety of important and interesting phenomena attending the circulation of water on the surface of the globe. By the natural process of evaporation, the clouds become charged with vapor, and are attracted by the lofty ridges of mountains, and all other elevated parts of the land, round which they collect, and upon which they deliver their contents.

The water thus deposited upon the highest parts of the globe, ha a constant tendency, by reason of the quality to which we refer, to return to the general level of the sea, and in finding its way thither gives rise to the phenomena of streams, rivers, cataracts, lakes, springs, fountains, and, in a word, to all the infinite variety of effects attending the movement of water which are witnessed throughout the world.

If the waters which fall from the clouds encounter a soil not easily penetrable, they collect in rills and form streams and rivulets, and descend along the sides of the elevation, seeking constantly a lower level; they encounter in their course, other streams, with which they unite, and at length swell into a river; they follow a winding channel, governed by the course of the valleys and lower parts of the land. Sometimes widening and spreading into a spacious area, they lose the character of a river, and assume that of a lake: then again, being contracted, they recover the character of a river, and after being increased by tributary streams on the one side and on the other, they at length attain their final destination, restoring to the ocean those waters which had been originally drawn from it by evaporation. Throughout the whole of these phenomena, the principle in operation is the tendency of liquids to maintain their level.

But it sometimes happens that the rains on mountainous summits encounter a soil easily penetrable by water. In such cases, the liquid enters the crust of the earth, which it often penetrates to great depths. Sometimes it encounters strata which are impenetrable, and finds itself walled in, so to speak, in a subterranean reservoir. In this case, the liquid is subject to a hydrostatic pressure, arising from the column of water extending from the reservoir to the upper surface, through the veins and channels, through which the reservoir has been filled.

This pressure sometimes forces the water to break its way through the strata which confine it. In such cases, it gushes out in a spring, which ultimately enlarges and becomes the tributary of some river. In other cases, however, the boundaries of the subterranean cistern resist this pressure, and the water is there imprisoned. If the ground above such a cistern be bored to a sufficient depth to penetrate the roof of the cistern, the liquid, having free exit, will rise in the well thus bored until it attain the same level which it has in the channels from which the subterranean cistern has been supplied. If this level be above the surface of the ground, the water will have a tendency to rush upwards, and, if restrained and regulated in its discharge by suitable means, it may be formed into a fountain, from which water will always flow, by simply placing a valve or cock, or from which water may be made permanently to project itself upwards in various forms, so as to produce *jets d'eau*.

If the level of the source, however, be little less than that of the mouth of the pit which has been dug, then the water will rise to such level, and stand there, forming a well. If the original level be considerably below that of the mouth of the pit, then the water will not rise in the pit beyond a certain height corresponding to the level of its source; and in this case a pump is introduced into the pit, and water is raised in a manner which will be explained hereafter.

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The preceding observations will be more clearly understood by reference to the diagram, *fig.* 192.

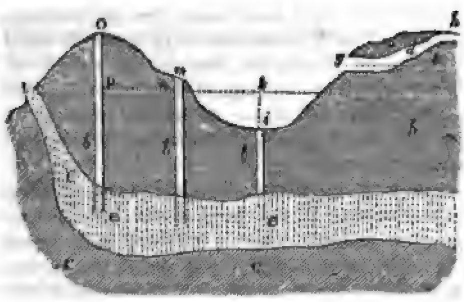

Fig. 192.

This diagram may be considered to represent a vertical section of the strata of the soil, which is penetrated by the pluviose waters, in which *a*, *b*, and *c* represent strata which are impenetrable to water, and *d* and *e* open and porous strata, and crevices which are penetrable.

If we suppose the stratum *d* to reach the surface at *g*, a point below the level of the highest point *h* of the same stratum, then the water will issue from *g* as from a spring, with a force proportional to the pressure due to the height of the level *h* above the level *g*, deducting, nevertheless, more or less force due to the resistance which the fluid encounters in passing through the soil of the stratum.

If a vertical shaft be sunk at *i* through the impervious strata *b*, until it enters the stratum *e*, then water would rise in this shaft until it reaches a height *k* corresponding with the level of its highest point *l*; but since this point *k* is above *i*, the mouth of the shaft, the water would spring upwards towards it, forming a jet. If at the mouth of the shaft *i* a valve or cock be placed which can be opened at pleasure, the water would be supplied as required; or if small orifices of any form be placed at the mouth of the shaft, the water would be forced through these so as to form a *jet d'eau*. It is thus that Artesian wells are formed. Such a spring as that represented at *i* will cause water to rise through pipes, in buildings or elsewhere, to any height not exceeding the level of the line *k l*. If a shaft be sunk at *m*, a point of the surface a little above the level *k l*, and be continued deep enough to enter the stratum *e*, water will rise in this shaft to a point *n* a little below the surface, and will form a well. If the shaft be sunk at a point o considerably above the level of the line *l k*, and be continued, as before, deep enough to enter the stratum *e*, then the water will rise to the point *p* corresponding in its level with *l*, and it will be necessary to raise it to the surface o by means of a pump working in the shaft o *p*.

Water confined in the lower strata of the earth in this manner sometimes bursts its bounds and rushes into the bed of the sea.

649. *Singular effect of hydrostatic pressure in the Rio los Gartos.* — It is stated by Humboldt, that at the mouth of the Rio los Gartos, there are numerous springs of fresh water, at the distance of five hundred yards from the shore. Instances of a similar kind occur in Burlington Bay, on the coast of Yorkshire, in Xagua, in the island of Cuba, and elsewhere.

650. *Examples of the sudden disappearance of rivers.* — In accomplishing their descent to the level of the ocean, rivers sometimes suddenly disappear, finding through subterranean caverns and channels a more precipitate course than any which the surface offers.

After passing for a certain space thus under ground, they reappear and flow in a channel on the surface to the sea. Sometimes their subterraneous passage becomes choked, and they are again forced to find a channel on the surface. The waters of the Oronoko lose themselves beneath immense blocks of granite at the Raudal dé Cariven, which, leaning against one another, form great natural arches, under which the torrent rushes with immense fury. The Rhone disappears between Seyssel and Sluys. In the year 1752, the bed of the Rio del Norte, in New Mexico, became suddenly dry to the extent of sixty leagues; the river had precipitated itself into a newly-formed chasm, and disappeared for a considerable time, leaving the fine plains upon its banks entirely destitute of water. At length, after a lapse of several weeks, the subterraneous channel having apparently become choked, the river returned to its former bed. A similar phenomenon is said to have occurred in the river Amazon, about the beginning of the eighteenth century. At the village of Puyaya, the bed of that vast river was suddenly and completely dried up, and remained so for several hours, in consequence of part of the rocks near the cataract of Rentena having been thrown down by an earthquake.

CHAP. IV.

SOLIDS IMMERSED IN LIQUIDS.

651. *Effects of immersion.* — The immersion, partial or total, of solids in liquids, is attended with effects of great importance in physical science and its application in the arts.

In explaining these effects, we shall limit our observations in the first instance to solids which will not be dissolved in the liquids in

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which they are immersed, and whose surfaces the liquid will not penetrate. The two following principles are demonstrated by theory, and verified by experiment :—

1st. If a solid be immersed in a liquid, it will displace as much of the liquid as is equal in volume to the part immersed.

2d. When so immersed, it will be pressed upwards by a vertical force equal to the weight of the liquid which it displaces.

The former of these propositions may be considered as nearly self-evident. If the liquid do not penetrate the surface of the body immersed in it, it must necessarily be displaced by this body, and the quantity so displaced will evidently be equal to the volume of that part of the solid which is immersed.

If the vessel containing the liquid before immersion were brimful, then the immersion of the solid would cause so much of the liquid to overflow as would be equal in volume to the solid immersed. If the vessel were not brimful, then the surface of the liquid in it would be raised by the immersion of the solid just so much as it would be raised by pouring in so much liquid as is equal in volume to that part of the solid which is immersed.

652. *Volume of a solid measured by immersion.* — In this manner, the magnitude of solids may be easily measured by immersing them in liquids, and measuring the quantity of the liquid which they displace. Thus, if a solid plunged in a vessel brimful of a liquid, cause ten cubic inches of that liquid to overflow, then it may be concluded that the magnitude of the solid immersed is ten cubic inches.

653. *Why liquids are usually expressed by measure, and solids by weight.* — It is difficult directly to measure the volumes of solids, unless they have some regular figure. Liquids, on the other hand, adapting themselves to the form of any vessel in which they are placed, admit of measurement by pouring them into vessels of known capacity. Hence it is, that the quantities of liquids are usually expressed by measure, while those of solids are commonly expressed by weight. But by the method just explained, liquids supply easy means of measuring the volumes of solids, no matter how irregular the shape of the latter may be, provided only that the solids to be measured will not dissolve in, or be penetrated by, the liquid in which it is immersed.

654. *Proof that a solid immersed loses the weight of the liquid it displaces.* — The second of the above propositions may be demon-
ed as follows :—

st A B C D, *fig.* 193., be a vessel containing a quantity of liquid at
, whose level surface is L L′.

If we suppose a part of this liquid having any proposed form M N be rendered solid, but without sustaining any other change in its

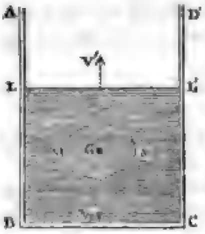

Fig. 193.

internal construction or arrangement, such part will still continue at rest, since no new force will be introduced tending to disturb its equilibrium. Let G be the centre of gravity of the mass M N. It is evident that it will have, in virtue of its weight, a tendency to sink in a vertical line G V directly from G downwards, with a force equal to its weight.

Now, as it does not sink by means of its weight, it must receive from the surrounding fluid pressures, the resultant of which is a single force equal and opposite to G V, and which, therefore, must be directed upwards in the line G V' with an intensity represented by the weight of the mass M N.

Now it is evident, that if this mass be changed in any manner in its internal construction, its form and magnitude being however preserved, it will still continue to be subject to the same pressure as before from the surrounding fluid, and consequently will still be pressed upwards with the same force.

Hence it follows, that if any solid whatever, of any form or magnitude, be submerged in a liquid, it will receive from that liquid upon its surface pressures, the resultant of which will be a single force equal in quantity to the weight of the fluid displaced, and directed vertically upwards from the point which would be the centre of gravity of such fluid, and which is in effect the centre of gravity of the submerged solid, if such solid have uniform density.

655. *Centre of buoyancy or pressure.* — Hence this point, which thus determines the upward action of the fluid surrounding any immersed body, is called the centre of buoyancy, and sometimes the centre of pressure.

656. *Conditions under which a solid will sink or swim.* — From this it will be easily perceived that a solid will either rise to the surface, sink to the bottom, or remain suspended, according as its weight is less than, greater than, or equal to the weight of its own bulk of the liquid; for, since the pressure upwards is equal to the weight of its own bulk of the liquid, if this pressure exceeds its own weight it will necessarily rise by such excess of pressure; if such pressure be less than its own weight, then it will sink with the excess of its own weight above such pressure; and if that pressure be equal to its own weight, then, the upward and downward tendencies being equal, the body will remain suspended, neither sinking nor rising.

It has been customary to express these effects by stating that a solid submerged in a liquid *loses* so much of its own weight as is equal to the weight of the liquid it displaces, or, what is the same, to the weight of its own bulk of the liquid.

657. *Experimental verification.* — This effect can be verified by

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experiment. If a body be weighed in a common balance, and afterwards be suspended from the arm of the balance, submerged in the liquid, and again weighed, it will be found that its weight, when so submerged, will be less than its weight before it was submerged by the weight of as much of the liquid as is equal to its own volume.

It would, however, be an error to infer from this that the weight which the solid in this case seems to lose, is destroyed. It is easy to show that this portion of its weight is supported by the liquid; for if the vessel containing the liquid be weighed with its contents before the solid is immersed, and afterwards, it will be found that after the solid has been submerged, the vessel containing the liquid will be heavier than before by exactly the weight which the solid appears to lose; that is to say, by the weight of so much of the liquid as would fill the space occupied by the solid.

It is therefore more correct to state that when a solid is immersed in a liquid, such a part of the weight of such solid is supported by the liquid as is equal to the weight of so much of the liquid as is equal to the volume of the solid.

658. *Origin of this discovery — Anecdote of Archimedes.* — The hydrostatical principle, in virtue of which a solid submerged in a liquid loses weight equal to that of the liquid it displaces, was discovered by Archimedes. Although this principle is now so generally understood and familiarly known, it is matter of tradition that the discovery made by Archimedes while bathing and reflecting on the effect produced upon his own person by the buoyancy of the water was such, that, frantic with joy, he rushed from the bath through the streets of Syracuse, exclaiming "Eureka! Eureka!" (I have discovered it, I have discovered it!)

If different solids be submerged in the same liquid, the weights which they lose, or appear to lose, will be in the exact proportion of their volumes; for they will be the weights of so much of the liquid as is equal to those volumes.

This supplies a method of estimating comparatively the volumes of different solids, these volumes being in the ratio of the weights they lose when submerged in the same liquid.

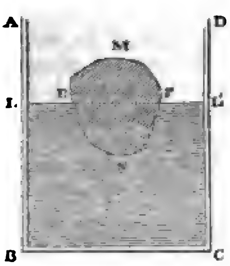

Fig. 191.

659. *A floating body displaces its own weight of the liquid.* — We have here supposed the solids to be totally submerged; let us now consider the case in which they are partially immersed.

Let A B C D, *fig.* 194., be the vessel containing the liquid, M N the body which is partially immersed, L L' the surface of the liquid, E N F the part of the solid which is immersed. According to what has been explained, the body will be subject to an upward pressure equal to the weight of the fluid which it displaces; and since it is also subject to a downward pressure equal to its own weight, it will sink deeper in the fluid, or rise to a less depth, or remain suspended, according as these two opposite forces are related. If the weight of the body be greater than the weight of the liquid which it displaces, it will sink deeper; if the weight of the solid be less than the weight of the fluid it displaces, then, the upward pressure prevailing over the downward pressure, it will rise; and it can only remain suspended, without either rising or sinking, when the weight of the fluid it displaces is equal to its own weight.

Hence it appears, that when a solid floats on a liquid, neither sinking nor rising, it must displace as much of the fluid as is equal to its own weight.

660. ***Solids sink or swim as they are heavier or lighter than their own bulk of the liquid.*** — Solids, therefore, can never float if they be heavier, bulk for bulk, than the liquids in which they are immersed.

If they be equal in weight, bulk for bulk, with the liquid, they will sink, until they are totally immersed; but when once they are totally immersed, then, the upward and downward pressures being equal, the solid will neither sink nor rise, but will remain suspended at any depth at which it may be placed.

To verify this experimentally, let a hollow brass ball be provided with a pipe and stop-cock, so as to admit of fine sand being let into it. Let the quantity of sand be first so adjusted that the weight of the ball shall exactly equal the weight of its own bulk of water. If the ball thus prepared be submerged in water, it will float at any depth at which it is placed, neither rising nor sinking; but if the weight of the ball be increased by the addition of more sand, it will sink more and more rapidly as the excess of weight is augmented; and if, on the other hand, its weight be diminished by withdrawing from it a part of its contents, so as to render it less than the weight of its own bulk of water, it will rise more and more rapidly, according as the excess of the weight of its own bulk of water above its weight is greater.

661. *The buoyancy of a solid depends on the ratio of its weight to the weight of an equal bulk of the liquid.* — The support, whether partial or total, which a solid sustains from a liquid in which it is immersed, is expressed by the familiar term ***buoyancy***. It appears from what has been explained, that a solid is buoyant in a liquid in proportion as it is light and the liquid heavy. Thus the same solid is more buoyant in quicksilver than in water; and in the same liquid, cork is more buoyant than lead.

A solid which will float in one liquid will sink in another: thus glass sinks in water, but floats in quicksilver; ebony sinks in spirits of wine, but floats in water; ash and beech float in water, but sink in ether. All these effects are explained by the fact, that in each case

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the solid sinks or rises according as it is heavier or lighter, bulk for bulk, than the liquid.

662. *Why weights are more easily raised under water.* — A block of stone or other heavy substance is more easily raised at the bottom of the sea than the same block would be on land, because, immersed in the sea, it is lighter by the weight of its own bulk of sea-water than it would be on land.

In building piers and other subaqueous works this is rendered manifest. Those who thus work seem endowed with supernatural strength, raising with ease, and adjusting in their places, rocks which they would vainly attempt to move above water. After a man has worked for a considerable time under a diving-bell, he finds, upon returning to the upper air, that he is apparently weak and feeble; everything which he attempts to lift appears to have unusual weight, and the action of his own limbs is not effected without inconvenience.

663. *The buoyancy of the human body in water — its effects.*— The human body does not differ much from the weight of its own bulk of water; consequently, when bathers walk in water chin-deep, their feet scarcely press on the bottom, and they have not sufficient purchase upon the ground to give them stability. If they are exposed to a current or any other agitation of the fluid, they will be easily taken off their feet.

When air is drawn into the lungs, the body becomes enlarged by its distension; and when it is expired, the dimensions of the body are again diminished. The weight of the body is so nearly equal to that of its own bulk of water, that this change of magnitude, small as it is, is sufficient to make it alternately lighter and heavier than its own volume of water. When a bather, therefore, inspires so as to fill his chest with air, he becomes, in a slight degree, lighter than water, and his head rises above the surface; when, on the other hand, he expires, the body contracting its dimensions without changing its weight, becomes heavier than water, and he sinks. Without some action to counteract this oscillation, the alternate sinking and rising would produce inconvenient effects; but this may be prevented by a slight action of the hands and feet, which resists the intermitting tendency to sink.

The facility with which different individuals are able to float or swim varies according to the proportion which the lighter constituents of the body, such as the softer parts, bear to the heavier, such as the bones.

664. *Any body, however heavy, may float when its form and position fulfil certain conditions.*—A body composed of any material, however heavy, may be so formed as to float on a liquid, however light. The method of accomplishing this is by giving to the solid such a shape that, when immersed in the liquid, some space within

the vessel, below the external surface of the liquid, will be occupied by air or some other substance lighter than the liquid.

Thus, if a tea-cup be placed with its bottom downwards in water it will float, and if water be poured into it, it will still float, but it will be found that the surface of the water in the tea-cup will always be below that of the external water, the air which occupies the difference of the levels producing the buoyancy.

A ship may be composed of materials heavier, taken collectively, than their own bulk of water, and nevertheless it floats, because its hull contains air and other substances much lighter than water; but if such a ship spring a leak it will sink.

Vessels laden with cork, certain sorts of timber, and other substances lighter, bulk for bulk, than water, will often become water-logged, but the vessel and the cargo taken together are lighter than their own bulk of water.

An iron boat will float with perfect security, and if it be formed of double plates of metal, enclosing a sufficient hollow space between them, nothing can sink it, so long as such casing remains uninjured.

665. *Weight of cargo estimated by displacement.*—The weight of a vessel including its cargo being equal to that of the water which it displaces, the weight of the cargo can always be determined by the quantity of displacement. If the displacement of the unladen vessel be subtracted from the displacement of the vessel with its full freight, the difference will be the volume of water which is equal in weight to the cargo.

666. *Heavy bodies supported or raised by light and buoyant bodies.*—The buoyancy of hollow solids is frequently used for the purpose of raising or supporting heavier solids.

Thus bladders are used to support the body in water. Inflated india-rubber bags or belts are used as life-preservers. Hollow boxes or tanks are used for the purpose of raising sunken vessels; these boxes are let down filled with water, and means are provided, when they reach the bottom and are attached by means of diving-bells to the vessels to be raised, of pumping out the water they contain. They thus become empty, and if they have sufficient strength to resist the pressure of the surrounding liquid, and sufficient buoyancy to overcome the weight of the vessel to which they are attached, they will accomplish their purpose.

667. *Method of floating vessels over shoals.*—The same experiment is sometimes used to carry vessels over shoals. An East Indiaman drawing 15 feet of water has been so much elevated by these means as to draw only 11 feet. The largest vessels of war in the Dutch service were enabled by these means to float over the banks of the Zuyder Zee.

668. *Buoyancy of water-fowl and aquatic animals.*—The bodies

of certain species of animals are much lighter than their own bulk of water. Water-fowl, in general, present examples of this, their plumage contributing much to their buoyancy. Fishes have the power of changing their bulk by the voluntary distension of an air-vessel which is included in their organization. By these means they can render themselves at will lighter or heavier than their own bulk of water, and rise to the surface or sink to the bottom. As fishes cannot obtain the air necessary for this voluntary inflation from a surrounding medium, they are provided with an apparatus by which they can generate gas for the purpose. This gas is in general not similar to atmospheric air. In such species of fish as live near the surface, it is found to be generally pure azote or hydrogen; in those species which inhabit strata of the deep having a depth of from 3000 to 4000 feet, the gas generated consists of 90 parts of oxygen and 10 of azote.

669. *The functions of aquatic animals adapted to the depths at which they prevail.* — At a depth of 30,000 feet, the external pressure would render these gases as heavy as their bulk of water, and consequently the apparatus for generating them would lose its efficiency. In fishes which are drawn up from depths of about 3000 feet, the gas included in this apparatus which was subject below to an external pressure of 1500 lbs. per square inch, being a hundred times the atmospheric pressure, swells, when brought above the water, to about a hundred times its original bulk. This produces some curious effects, the internal organs increasing to such an extent that a part of them is driven out of the mouth of the fish, presenting the singular appearance of an inflated bladder.

This circumstance, which is curious and interesting, suggests the probability that the different parts of the sea are each peopled by their inhabitants, varying not only according to climate, but according to depth.

670. *Why drowned bodies rise to the surface.* — *Feats of divers.* — When an animal is first drowned, air being expelled from the lungs, the body is heavier than its bulk of water; but when decomposition takes place, gases are generated in various organs, the vessels become distended, and the body becomes lighter than water, and rises.

A solid having air included, which is exposed to the pressure of the liquid in which it is immersed, may rise to the surface, if it be only sunk to a certain depth; but by sinking it deeper, the pressure of the liquid would condense the air within the solid, so that the weight of the solid, including the air, becomes greater than that of the liquid it displaces, in which case it can no longer rise. A diver who plunges in the sea is lighter when he enters than his own bulk of water; but if he sink to a certain depth, his dimensions will be so contracted by the surrounding pressure, that he will displace a less quantity of water than his own weight, and therefore cannot rise by mere buoyancy, but must ascend by the exertion of his limbs, swimming upwards.

671. *Conditions which determine the position in which a body floats at rest.* — The conditions which determine the equilibrium of a floating body, so far as relates to vertical motion, have been fully explained; but although the body which floats may neither sink nor rise, it does not therefore follow that it will be at rest. If any body of irregular figure float upon water and be at rest, its position may be temporarily changed without either raising or depressing it; and if so changed, upon being disengaged, it will oscillate in the fluid for some time, until at length it settles into a position of rest, and during this oscillation the body will, so far as vertical motion is concerned, be at rest; it will neither sink nor rise, nevertheless it will not be in equilibrium. It remains, therefore, to investigate what are the conditions which determine the absolute repose of a floating body, and what are those forces which give it an oscillatory motion without either rising or sinking.

According to what has been already proved with reference to the centre of buoyancy, this question of the oscillation of floating bodies admits of easy explanation. It has been shown that the upward pressure exerted by a fluid on a solid immersed in it passes through that point which is the centre of gravity of the fluid displaced by it, while the downward pressure due to the weight of the body always passes through the centre of gravity of the body itself.

Now it is clear that if these two points, that is to say, the centre of gravity of the floating body and the centre of buoyancy or the centre of gravity of the fluid which it displaces, be not in the same vertical line, their combined effect must be to turn the body round in the fluid until they are brought into the same vertical line. When this happens, the two forces being in opposite directions and equal will keep each other in equilibrium, and will keep the body at rest. This will be rendered more clearly intelligible by reference to the diagram, *fig.* 195.

Let A B C D be a vessel containing a liquid whose surface is L L'. Let M N be a body floating upon it, and displacing so much of the liquid as is equal to its own weight. Let G be the centre of gravity of the body, and let G' be its centre of buoyancy, or the centre of gravity of that portion of the liquid which the body displaces. Now according to what has been stated, the upward pressure produced by the liquid takes place in a vertical line G' O' from the point G', while the downward pressure due to the weight of the body is presented in a vertical line G O downwards from the point G; and since the body is supposed to be in equilibrium, so far as relates to vertical motion, these two forces must be equal. The body M N therefore, is under the operation of two equal forces, one acting in the line

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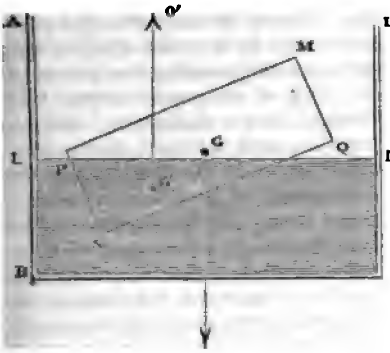

Fig. 195.

G O, and the other G' O'; and it is evident that these forces will have a tendency to turn the body round in the direction M Q N P.

It is evident that such a motion must be imparted to the body so long as the points G and G', through which the two equal and contrary forces pass, are not *in the same vertical line*; but if these points be in the same vertical line, then the two equal forces acting in directions immediately opposed to each other will be in equilibrium, and the body will be at rest.

It may be inferred, therefore, that the condition of equilibrium of a floating body is two-fold.

1°. That it shall displace as much of the liquid as is equal to its own weight.

2°. That the line joining the centre of gravity and the centre of buoyancy shall be vertical.

The former condition determines the equilibrium of the body with reference to vertical motion, and the latter with reference to rotatory motion.

672. *Condition of stable equilibrium of a floating body.* — In a former chapter the characters of stable, instable, and neutral equilibrium have been established. All the conditions incidental to these states are manifested in the case of floating bodies.

It must be remembered, that the centre of gravity of a body is characterized invariably by this property, that it will always endeavour to assume the lowest position which it can have compatibly with the conditions in which the body is placed; and that, consequently, when it is in such lowest position, the body will be in stable equilibrium. But since by any disturbance of that position the centre of gravity must be raised, it will have an immediate tendency to resume it, and the body will oscillate round that position, until finally the centre of gravity settles into it, and the body comes to rest.

A floating body is therefore in stable equilibrium when its centre of gravity has the lowest position which it can have compatibly with the first condition of equilibrium of floating bodies, viz. that the body shall displace its own weight of the liquid. It is evident that the same solid may be immersed in a liquid in an infinite variety of positions, in all of which it shall displace its own weight of the liquid.

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Now in all these positions the centre of gravity of the solid will in general be found at different depths. When the body has that position at which its depth is greatest, it will be in stable equilibrium; but any change from that position necessarily causing the centre of gravity to rise, the body would, in virtue of the general property of the centre of gravity already mentioned, have a tendency to return to that position, and would, in fact, oscillate until it should recover it.

673. *Position of instable equilibrium.*—If the line joining the centre of gravity of the solid and the centre of buoyancy be vertical, but the position of the body be such that any slight disturbance of its position, which shall still cause it to displace its own weight of fluid, will make the centre of gravity descend then the centre of gravity cannot resume its former position; but in virtue of the property already explained it cannot rise, consequently the body will necessarily turn until the centre of gravity descends to the lowest position which it can have compatibly with displacing its own weight of the liquid. This will be the position of stable equilibrium; the former, in which the centre of gravity was at a point from which it could not move without descending, although in the same vertical line with the centre of buoyancy, was a position of instable equilibrium.

674. *Condition of neutral equilibrium.*—It happens, in particular cases, that the centre of gravity of a body is not altered in the height by any change of position of the body compatible with displacing its own weight of fluid. In such a case the body will float in equilibrium, whatever position be given to it, and this corresponds to the condition of neutral equilibrium. A sphere of uniform density presents an example of this. In whatever position it floats, its centre of gravity being at its geometrical centre, and the part immersed being always a segment of the sphere of precisely the same magnitude, the centre of gravity will necessarily be always at the same level; and, consequently, the sphere will float indifferently in any position in which it may be placed.

It is sometimes said that a floating body, subject to these conditions, rests in stable equilibrium whatever position be given to it; but this is incorrect. The essential character of stable equilibrium consists in the fact that the floating body, if disturbed by any external cause, will recover its former position when relieved from such cause. Now a sphere, or any other body which has neutral equilibrium, will not recover its position after a disturbance, but will remain in the new position which has been given to it. In short, it will remain indifferently in any position, and consequently may be overturned by any force which may be applied to it.

675. *Conditions which determine the degree of stability of a floating body.*—The stability of a floating body is susceptible of degrees.

Such a body is more or less stable, according to the force with

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which it recovers its position of equilibrium after any disturbance. In general, the stability will be increased with the increase of the depth of the centre of gravity of the body below its centre of buoyancy. For this reason, vessels which are appropriated to the transport of passengers, or even of cargoes which are light in proportion to their bulk, require to be ballasted by depositing at the lowest part of the hull immediately above the keel a quantity of heavy matter. In packet ships the ballast used for this purpose is usually iron pigs.

The centre of gravity of a vessel may thus be brought so low as to give it such stability that no lateral force of the wind acting on its sails can capsize it. Hence is explained the necessity of stowing the heaviest part of a cargo in the lowest possible position, and so that its centre of gravity shall be immediately over the keel. By such arrangement any inclination of the vessel would cause the centre of gravity to rise, to accomplish which a force would be necessary proportional to the weight of the vessel, and the height through which such centre would be elevated.

The equilibrium of a boat may be rendered instable by the passengers standing up in it. If the centre of gravity of a vessel be not directly over the keel, the vessel will incline to that side at which it is placed, and if this derangement be considerable, danger may ensue. The rolling of a vessel in a storm may so derange its cargo that the centre of gravity would be brought into a position which will throw the vessel on her beam-ends.

676. *Analysis of the effect of a side wind on a ship.* — When the centre of gravity is immediately over the keel, a side wind acting on the sails will incline the vessel the opposite way; this inclination would be much more considerable were it not that the weight of the vessel acting at the centre of gravity counteracts it, and has a tendency to restore the vessel to the upright position. The several forces which maintain the vessel in the inclined position produced by a side wind may be illustrated as follows. Let A B, *fig.* 196., represent the position of the vessel; let S represent the point at which the wind acts upon the sail, and let S W represent the direction of the wind. Let E be the centre of gravity of the vessel and her cargo, and let E F be the direction in which her weight acts. Let E′ be the centre of gravity of the water which the vessel displaces, and E′ F′ the direction of the upward pressure. If the effect of the upward and downward forces at E and E′ be considered for a moment, it will be perceived that they have a tendency to incline the vessel to the side opposite to that towards which it is inclined by the wind. By the principles of the resolu

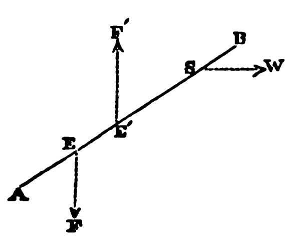

Fig. 196.

tion of forces, the force S W may be replaced by three others, two of which being equal and directly opposed to the upward and downward forces at E and E′ neutralize them, and the third acting parallel to S W merely carries the vessel sideways perpendicular to its keel, producing what is called *lee-way.*

677. *Expedient adopted in steamers.* — In sailing-vessels this sideward inclination is a matter of comparatively slight importance, inasmuch as it does not diminish the impelling power of the wind; but in steam-vessels, in which sails are occasionally used, it is attended with considerable loss of the impelling power, one of the paddle-wheels being lifted out of the water and the other being almost, if not entirely, submerged. The upright position may, however, be generally maintained by the due management of moveable weights placed on the deck of the vessel. In steam-vessels small carriages heavily laden with iron, and furnished with wheels, are usually placed on the deck, and may be rolled from side to side or placed in the middle, so as to regulate the position of the centre of gravity according to the way in which the vessel is affected by the wind. By moving those carriages to the side of the vessel against which the wind is directed, the centre of gravity is moved from over the keel towards that side. Let E, *fig.* 197., represent the place of the centre of gravity when over the keel, and let G represent the point to which the centre of gravity is transferred by moving the carriages to the side of the vessel; let S be the point where the wind acts upon the sail S W: the weight of the vessel acting at G has a tendency to make it incline towards M, and the force of the wind acting at S, in the direction S W, has a tendency to make it incline towards L. These two forces counteract each other and the vessel maintains its upright positon.

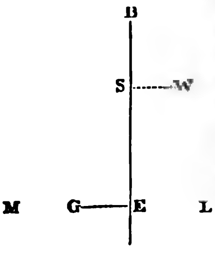

Fig. 197.

CHAP. V.

LIQUIDS IN MOTION.

678 *Subject of hydraulics.* — The branch of the mechanical theory of liquids which comprises the investigation of the principles which govern their motion, is called Hydraulics or Hydrodynamics.

It includes the effects which attend liquids issuing from orifices made in the reservoirs which contain them, or forced by pressure

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through tubes or apertures in pipes or in channels; it includes also the motions of rivers and canals, and the resistances produced by forces, developed by the mutual impact of liquids and solids.

679. *Velocity of efflux from an orifice.* — If a small hole be made in the side of a vessel which contains a liquid, it rushes from it with a certain velocity, depending on the pressure at the point where the orifice is made. Since this pressure is the cause which imparts the momentum or moving force to the fluid, it will necessarily be proportional to such momentum. But this momentum is, according to what has been already established, proportional to the quantity of liquid which is put in motion, and to the velocity imparted to it, and is expressed by multiplying the quantity of liquid which escapes from the orifice in a second by the velocity with which such liquid is moved. Now the column of liquid which passes from the orifice in a second is that whose base is the area of the orifice, and whose length is equivalent to the velocity with which the fluid passes through it; since it is evident that so much of the fluid as passes through the orifice in a second would form a column whose base is the orifice, and whose altitude is the space through which the fluid moves in a second. The effect may not inaptly be illustrated by the process of wire-drawing, in which the metal is forced through a circular orifice. The quantity of metal which passes through in a second would be determined by the area of the orifice and the velocity with which the wire is drawn. If, then, we multiply the area of the orifice by the velocity with which the fluid passes through it, we shall obtain the total quantity of fluid which is discharged in a second. Thus if the area of the orifice be expressed by o, and the velocity with which the fluid passes through the orifice by v, then the total quantity of fluid discharged in a second will be expressed by

$$\mathrm{o} \times \mathrm{v}.$$

But this quantity of fluid being moved with the velocity v has a moving force which will be expressed by multiplying the quantity of fluid discharged by the velocity, and therefore will be expressed by

$$\mathrm{o} \times \mathrm{v}^2.$$

680. *The velocity of efflux not the same as the velocity with which the liquid passes the orifice.* — It is necessary to distinguish the velocity of efflux from the velocity with which the liquid passes through the orifice. By the velocity of efflux must be understood *the quantity of liquid discharged from the orifice per second*, while the velocity of the liquid in issuing from the orifice is measured by the *space through which the liquid would move in a second*, or, what is the same, as has been just explained, the length of the column or vein of liquid which passes through the orifice in a second. If, then, E

express the velocity of efflux, or the total volume of liquid discharged in a second, we shall have

$$E = O \times V.$$

Since the moving force with which the quantity expressed by E is propelled is found by multiplying the quantity E by the velocity with which it is moved, this moving force will be E multiplied by V. But since

$$E = O \times V.$$

we must, therefore, have

$$E \times V = O \times V^2.$$

It appears, therefore, that the moving force impressed per second on the liquid discharged *is proportional to the area of the orifice multiplied by the square of the velocity.*

For orifices of equal magnitude, therefore, the moving force imparted to the liquid will be in the ratio of the squares of the velocities with which the liquid is propelled.

681. *The square of the velocity in escaping from the orifice proportional to the depth.*—But it has been already shown that the moving force imparted to the liquid escaping from the orifice is proportional to the pressure of the liquid at the orifice. This pressure, however, is proportional to the depth of the orifice below the surface of the liquid in the vessel, and consequently it follows that the squares of the velocities of the liquid in passing through the orifice are proportional to the depth.

Thus, if several orifices be made in a vessel containing a liquid at the depth of 1, 4, 9, and 16 feet, the velocities with which the liquid will escape from these will be in the proportion of 1, 2, 3, and 4.

This reasoning shows the manner in which the velocity varies with the depth; and if the velocity corresponding to any particular depth were known, the velocities at other depths could be found. Thus, if the velocity at the depth of 1 foot below the surface were known, then the velocity at 9 feet depth would be three times the former; the velocity at 16 feet would be four times that velocity, and so on.

682. *Velocity of escape equal to that which a body would acquire in falling from a height equal to the depth.*—It can be demonstrated by mathematical principles and verified by experiment, that the velocity of a liquid escaping at any proposed depth is equal to the velocity which a body would acquire in falling freely in a vacuum through a height equal to such depth.

Thus, for example, it is known that a body falling freely through the height of 193 inches, would acquire a velocity, which, if continued uniform, would cause it to move through $193 \times 2 = 386$ inches per second.

Now, if an orifice be made in a vessel containing a liquid at the depth of 193 inches below the surface, it will be found that the liquid

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will flow from such orifice with the velocity of 386 inches per second.

According to the law which regulates the free descent of falling bodies (see Handbook of Mechanics, 248.), the velocity V, acquired in falling through any given height H, is thus expressed:—

$$V = 2\sqrt{H \times g}$$

where $g = 193$ inches.

Hence, since E denotes the velocity of efflux, or the quantity of liquid which escapes in one second by an orifice whose area is O, and whose depth below the surface is H, we shall have

$$E = 2\,O\sqrt{193\,H};$$

and for the quantity Q, which escapes in any number of seconds, t,

$$Q = t \times E$$
$$= 2\,t\,O\sqrt{193\,H}.$$

In these formulæ, O must be expressed in square, and H in linear inches. The quantity Q will be expressed in cubic inches.

If it were required to ascertain in what time a given number of cubic inches, Q, would be discharged by a given orifice at a given depth below the surface, we have from the last formula,

$$t = \frac{Q}{2\,O\sqrt{193\,H}}.$$

In what has been said, it is taken for granted that the surface of the liquid in the vessel is kept constantly at the same level. If the vessel is allowed to empty itself, the pressure at the orifice and the velocity of efflux continually diminish: and, consequently, the time required to discharge a given quantity increases. It can be proved by the higher mathematics, that the time required for a vessel to empty itself is just double that which is required to discharge an equal quantity of liquid, when the vessel is kept full.

Hence, if Q denote the capacity of a vessel in cubic inches, we shall have for the time of exhaustion

$$t = \frac{Q}{O\sqrt{193\,H}}.$$

683. *If the liquid escape by a jet directed upwards, it would rise o the level of the surface, if not resisted by the air.*—It has been proved that if a body be projected upwards with any velocity, it will rise to that height from which it must have fallen to have acquired the velocity with which it is so projected upwards. It follows, therefore, that if the liquid which escapes from an orifice issues vertically upwards, it will rise to a height which is level with the surface of the liquid in the vessel from which it escapes.

In *fig.* 198., o, o′, o″, &c., represent pipes of discharge inserted in a vessel containing a liquid, having their openings turned upwards. The several jets which would escape from these orifices would, if no disturbing force intervened, rise to the level E F G of the liquid in the vessel, as represented in the figure. This result, however, as well as the premises from which it is deduced, require to be submitted to considerable modification before they can be applied in practice.

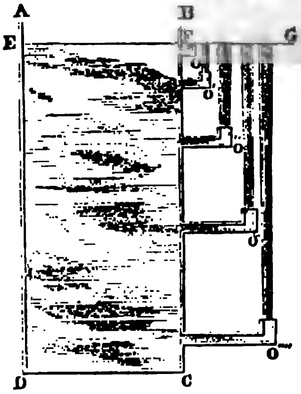


Fig. 198.

684. ***Practical conditions which modify this.*** — In the preceding investigation we have considered the orifice to be indefinitely small, so that every part of it may be considered at the same depth below the surface. If it be not so, the point which determines the velocity would be its centre. We have also considered the fluid in issuing from it as subject to no resistance proceeding from its sides, which will necessarily be the case, if the side of the vessel has any considerable thickness. This cause may be in part obviated by making the side of the vessel at the place where the orifice is made extremely thin.

In fine, we have considered the jet issuing from the orifice to move freely in a vacuum; instead of which, in fact, it encounters the resistance of the air, which not only diminishes the velocity, but scatters the jet.

We have also considered that the jet of liquid issuing from the orifice has the form of a cylindrical rod, the orifice being supposed circular, the thickness of this cylinder corresponding with the magnitude of the orifice.

685. *The contracted vein.* — Now it is shown by Newton, that a jet issuing from a circular orifice made in the thin side of a vessel is not of a cylindrical form; that, in fact, the fluid does not issue from such an orifice, as a small wire would do from the hole through which it is passed in the process of wire-drawing. Newton showed, on the contrary, that the jet, immediately on leaving the orifice, contracts its dimensions, and that at a distance equal to the diameter of the orifice itself this contraction attains its limit, and that the section of the jet at this point will be about two-thirds of the magnitude of the orifice. It is held by Newton, and assumed by others since his time, that beyond this point the section of the jet was enlarged, so as to take the form of a diverging cone. The point of greatest contraction, placed at a distance from the orifice equal to its diameter, is called by New-

ton, and has since been denominated, the *vena contracta* or *contracted vein.*

Now it is evident that the form of the fluid column, on proceeding from the orifice, not being circular, the velocity of the fluid through the orifice must vary, being greater in proportion as the column is more contracted, for exactly the same reason that the velocity of a stream is necessarily augmented in proportion as its bed becomes narrower.

In both cases, the same actual quantity of fluid must pass through the two sections of the stream in the same time, and the narrower the section is, the greater, in the same proportion, must the velocity of the motion consequently be, in order that the same quantity of fluid may pass through.

686. *Velocity of the liquid increases as the area of the section of the jet diminishes.* — It appears, therefore, that since the section of the jet at the orifice is greater than its section at the contracted vein, the velocity of the liquid at the orifice will be less than its velocity at the contracted vein, in the same proportion, and consequently a question arises, which of these varying velocities is that which is determined by the depth of the orifice below the surface?

687. *The velocity at the contracted vein is that due to the depth.* — We have explained that the velocity of the liquid in issuing from the orifice, if it issued in a cylindrical form, would be equal to that which a body would acquire in falling from a height equal to the depth of the orifice; but the jet not being cylindrical, and the liquid, consequently, having a varying velocity, the question arises, at what point of the jet it will have the velocity due to the depth of the orifice. The answer is, that that point will be, not the orifice itself, but the vena contracta, and that, consequently, in the calculation of the velocity of efflux by means of the rules and formulæ given above, the magnitude expressed by O must be taken to signify, not the area of the orifice itself, but the magnitude of the section of the jet of the vena contracta, and the velocity expressed by V will then be that which a body would acquire in falling freely from the surface of the liquid in the vessel to the orifice.

688. *Contracted vein two-thirds of orifice.* — It may be stated in general, therefore, that when the side of the vessel is thin, and the orifice not great, the area of the section of the vena contracta may be taken as nearly equal to two-thirds of the area of the orifice. Various circumstances, however, attending the discharge will modify these conclusions.

According to the conclusions of Newton, the liquid jet, after passing the vena contracta, again enlarged its diameter, thus converging to the vena contracta, and afterwards diverging, as represented in *fig.* 199.

689. *Modification of this theory by Savart and others.* — Re-

cent experimental investigations, however, made by Savart and others, have proved that the phenomena are not in strict conformity with Newton's theory. It is true that the jet contracts its dimensions in issuing from the orifice, and arrives at the limit of its contraction at a distance from the orifice equal nearly to its own diameter.

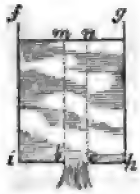
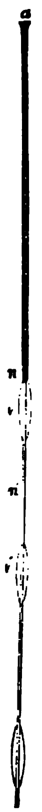


Fig. 199.

Savart has shown that in all cases, except when the jet is discharged upwards, its section goes on diminishing, though much less rapidly than the vena contracta, until it loses its form and is scattered by the resistance of the air. Thus the contraction is at first rapid, and the form of the jet is decidedly conical from the orifice to the contracted vein; but beyond that point the jet has very nearly the form of a uniform rod of glass, having a tendency, however, to become still thinner.



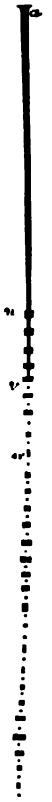


Fig. 200. Fig. 201.

Our limits will not allow us to enter into the details of the curious phenomena developed in the researches of Savart. It may not, however, be uninteresting to reproduce some of his diagrams, showing the form of jets. *Fig.* 200. represents a jet which issues from the bottom of a vessel at *a*, such as it appears to the eye. From *a* to *n* it has the appearance of a uniform and straight glass rod; at *n* it begins to lose its transparency, and also to change its form, swelling and contracting alternately, and at unequal intervals. When this part of the jet, however, was examined with greater precision, it was proved to consist of, not a continuous stream of liquid, but a series of distinct and separate drops, as represented in *fig.* 201.; the dilated parts, represented at *v* and *v'*, *fig.* 200., were formed by large drops, which were dilated horizontally, while the nodes *n*, *n'* were formed by the same drops dilated vertically. Thus it appears that in their descent the drops were subject to a pulsation, by which they alternately enlarged and contracted their dimensions vertically and horizontally; but it was also proved, that besides these drops there were other similar drops, represented by the dots in *fig.* 201., which did not change their form. The pulsations attending these alternate changes of form of the drops produced a distinct sound. When a musical sound of the same pitch was produced near the jet, these alternate pulsations of the drops became more regular.

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It was further proved by these curious researches, that the pressure of the air has no influence on these phenomena.

690. *Resistance of liquids to solids moving through them.*—If a solid presenting a flat surface in the direction of its motion be moved through a liquid which is at rest, it will suffer a certain resistance, depending on the magnitude of such surface and the vessel in which it is moved. This resistance arises, evidently, from the reaction of the liquid which the solid displaces, and to which it imparts motion. Whatever moving force the liquid receives must be lost by the solid, or by whatever agent replaces the solid. It is nearly self-evident, that with the same velocity the resistance will be proportional to the magnitude of the surface, for it is clear that a surface which measures two square feet will drive a column of water twice as great as that which would be driven by a surface measuring one square foot. The resistance therefore, other things being the same, to a flat surface moved against a liquid is proportional to the area of such surface.

691. *A flat surface encounters a resistance which is as the square of the velocity.*—It is evident also, that if the same surface be moved with different velocities, it will encounter different resistances. The greater the velocity with which it is propelled, the greater will be the moving force it will impart to the liquid, and, consequently, the greater will be the resistance it encounters. If the surface be moved with a double velocity, the liquid which it drives before it will also be moved with a double velocity, and will, consequently, have a double momentum or moving force; but when the surface is moved with a double velocity, it advances through a double space in the same time, and, consequently, displaces a double quantity of liquid. Now since this double quantity of liquid is moved with a double velocity, it must have a four-fold momentum, since the momentum is increased in a two-fold proportion in consequence of the double velocity, and again in a two-fold proportion in consequence of the double quantity which is displaced. The moving force, therefore, which is communicated to the liquid by the solid will be four-fold when the velocity is doubled. Now, since this moving force is the measure of the resistance, it follows, that when a flat surface of a given area is moved through a liquid, the resistance which it encounters will be augmented, not in the simple ratio of the velocity, but as its square. Thus a two-fold velocity will give a four-fold resistance, a three-fold velocity a nine-fold resistance, and so on.

692. *How to determine the absolute resistance in moving through water.*—It may be concluded, therefore, in general, that when a solid having a flat surface is moved through a liquid, the resistance encountered by such surface will be proportional to the magnitude of the surface multiplied by the square of its velocity. Such being the law which governs the variation of the resistance, we shall know the

total amount of such resistance in all cases, provided its amount be known for any one surface and any one velocity. Now it is proved that a square foot of surface moved with the velocity which a body would acquire in falling through 193 inches, would suffer a resistance equal to the weight of a column of water of that height, and having a square foot for its base.

If a square foot of surface, then, be moved with any other velocity, greater or less, the resistance can be found by increasing or diminishing this resistance in the ratio of the square of the velocity, and if a surface greater or less than a square foot be so moved, the resistance due to such increase or diminution will be found by increasing or diminishing the resistance encountered by a square foot in the same proportion.

693. *Force with which a liquid in motion strikes a surface at rest.* — If a liquid strike the flat surface of a solid at rest with a certain velocity, it will exert upon such surface a force just double the resistance which the same surface would encounter if it moved in the liquid with the same velocity, the liquid being at rest; that is to say, the force exerted on the surface by the liquid would be equal to the weight of a column of the liquid whose base would be equal to the surface, and whose height would be double the height through which a body would fall freely in order to acquire the velocity with which the liquid strikes the surface.

694. *Force of resistance when the motion is not at right angles to the surface.* — If the surface which is moved against a liquid, or upon which a liquid in motion acts, be not at right angles to the motion, then it will be necessary to resolve the motion into two, by the principle of the composition of forces, one of which shall be perpendicular to the surface, and the other parallel to it. The latter can have no effect, whether the surface strike the liquid or the liquid strike the surface, and that element of the force which is perpendicular to the surface is subject to all the conditions which have been just explained.

695. *Conditions which determine the form of least resistance.* — The effect produced upon the resistance offered to a body moving through a liquid by the obliquity of the different parts of the surface of such body to the direction of the motion forms an important element in the solution of the problem for determining, under different conditions, the shape of the solid moved. A problem which has attained great celebrity in the history of mathematics is one in which it was required to determine the form which should be given to a determinate mass of solid matter, so that it might move through a liquid with the least possible resistance. The form thus determined is known in geometry as the 'solid of least resistance.'

696. *Importance of such principles in naval architecture.* — Nearly similar conditions attend the solution of all the problems which

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are presented in naval architecture. It is this principle which causes the length of the vessel to be presented in the direction of the motion, and which determines the shape of the prow under the various conditions to which different classes of vessels are exposed. The boats which ply on rivers, or other sheets of water not liable to much agitation, nor intended to carry considerable freight, are so constructed that the part of the bottom immersed moves against the liquid at an extremely oblique angle.

697. *Form of fishes.* — It has been often mentioned, as an instance of the felicitous accordance of the works of nature with the principles of science, that the form given by mathematicians as the solid of least resistance accords exactly with the forms of the bodies of fishes. This, however, is not strictly the case, and if it were, so far from being an instance of skill and design in the works of nature, would manifest a certain degree of imperfection.

The solid contemplated in the celebrated problem adverted to has no other function to discharge except to oppose the resistance of the fluid, and the question is one of a purely abstract nature, viz., what shape shall be given to a body, so that, while its volume and surface continue to be of the same magnitude, it may encounter the least possible resistance in moving through a fluid? It must be apparent that many conditions must enter into the construction of an animal, corresponding to its various properties and functions, independently of those in virtue of which it employs itself either to oppose or cleave the air.

698. *Instances of design in creation. — Form of animals intended to move through fluids.* — The discovery of verifications of the principles of physics in the works of nature is in general so seductive, that writers are sometimes tempted to overlook the inevitable causes of discrepancy in their eagerness to seize upon analogies of this kind. Without, however, seeking in natural objects the exact solution of a mathematical problem, which is unencumbered by various conditions which the author of nature has designed to fulfil, innumerable examples may be produced giving striking manifestation of design. Thus, all animals to whose existence or enjoyment a power of easy and rapid motion through fluids is necessary, have been created with a form which, having a due regard to their other functions, is, upon the whole, the best qualified for this end. Birds, and especially those of rapid flight, are examples of this. The neck and breast tapering from before, and increasing by slight degrees towards the thicker part of the body, causes them to encounter the air with a degree of obliquity considerably diminishing the resistance, slight as it is, which this attenuated fluid opposes to their flight. But these conditions are, as might be expected, presented in a much more striking point of view in the form of fishes, and all the species which inhabit the deep.

699. *The force of water in motion a moving power. — Water-wheels, overshot, undershot and breast.* — The force of water in motion

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is rendered available for mechanical purposes as a prime mover principally in three ways:—

1°. By means of the overshot-wheel, represented in *fig.* 105.

2°. By means of the undershot-wheel, represented in *fig.* 106.

3°. By the breast-wheel, represented in *fig.* 107.

(For figures, see Hand-Book of Mechanics.)

In the first, the moving power consists of the weight of water deposited in the buckets on the descending side of the wheel, and which is discharged from them at or near the lowest point. The total mechanical effect is measured by the average quantity of water which descends in the buckets, multiplied by the height through which it falls. In the undershot-wheel, the water acts by its velocity or momentum against the float-boards, and its effect is measured as has been already explained. In the breast-wheel, the water acts chiefly by its weight.

700. *Archimedes' screw.* — The hydraulic instrument called after its inventor the screw of Archimedes, has recently been invested with more than common interest by its successful application to the propulsion of steam-vessels. This machine was invented by Archimedes in Egypt, to aid the inhabitants in clearing the land from the periodical overflowings of the Nile. It was also used as a pump, to clear water from the holds of vessels; and Athenæus states that the name of Archimedes was held in great veneration by seamen on this account.

The instrument varies in form according to the manner and purposes of its application, but its principle may be rendered intelligible as follows:—

Suppose a metal tube bent into the form of a corkscrew, as represented in *fig.* 202. Let it be placed in an inclined position, and so that the mouth A, at the lower end, shall be in the highest position it can have. If a small metal ball be let into the mouth A, it will fall down the curved part till it arrives at B. This point B is evidently so situated, that the ball cannot leave it either on the one side or on the other without ascending; consequently, when the ball arrives there, after a few oscillations, it will remain at rest. If the screw be now turned, without changing its inclination or direction, so that the mouth A, instead of being at the highest position, as represented in *fig.* 202., shall be brought to its lowest position, as represented in *fig.* 203., the point B during such motion of the screw will ascend, and assume the highest position which it can have, as represented in *fig.* 203.

Now suppose the ball for a moment to be attached to the tube so as to be incapable of moving in it. When the screw has been turned to the position represented in *fig.* 203., the ball B would be at the highest point of the body of the tube, and consequently would be raised from the point *b*, which it occupied before the screw was turned, to the point B, which it now occupies. If the ball then be detached

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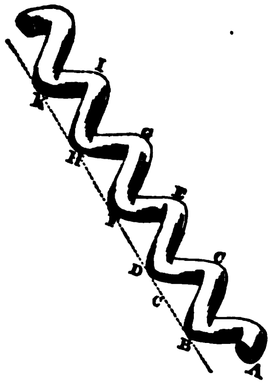

Fig. 202.

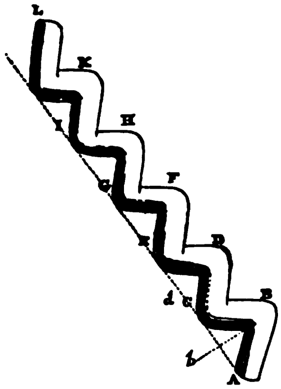

Fig. 203.

the tube, supposing it to be a little short of the summit, it will own that part of the tube from B to C, and arriving at C, it will ain at a point of the tube where it will have an ascent at either f it, and it will consequently come to rest.

the ball be again supposed to be attached to the tube here, and ube be again turned half round, so as to give to it once more the on represented in *fig.* 202., the ball will be at C, having been l from c to C in this half turn. If then the ball be detached at pposing it to be a little beyond the summit, it will fall down the from C to D, when it will again come to rest, because it will have cent at either side of it.

us, in a complete turn of the screw the ball would be carried B to D, *fig.* 202.; in the second turn of the screw it may be n that it would be carried from D to F; in the third from F to H, so on; until at length the ball would be discharged from the r end of the tube, at L. But if we do not suppose the ball to be successively attached to the interior of the tube, this motion from B to L, instead of being effected by intervals, will be made continuously; the process, however, remaining the same.

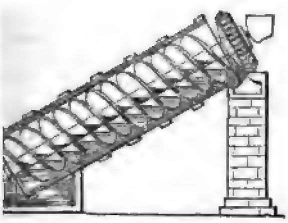

Fig. 204.

All that has been said of the ball in the tube would be equally true if a quantity of liquid were contained in it. Therefore, if the extremity of the screw were im-

BOOK THE FIFTH.

MECHANICAL PROPERTIES OF AIR.

CHAPTER I.

GENERAL PROPERTIES OF AIR.

701. *Atmospheric air, the type of all elastic fluids.*—The class of bodies which exist in, or may be reduced to the form of elastic fluids, or the æriform state, are extremely numerous; indeed, it is probable that all bodies whatever, either by heat, or other physical agents, may be converted into this form. The most universally observable substance of this class is atmospheric air. Many of the qualities found in this substance extend, without modification, to all elastic fluids whatever; but there are some of them, especially when applied to vapours, which require to be restricted and modified by various circumstances, which will be explained in a subsequent chapter. There are also many circumstances to be attended to in explaining the properties of various gases which belong to the department of chemistry, in which the production and constitution of these gases are explained. Our present object is limited to the investigation of the mechanical properties of the atmosphere; it being, however, understood, that the various theorems which we shall establish may be carried into other departments of physics, and applied to all bodies whatever in the gaseous form, subject however to restrictions and modifications peculiar to the vapours and various species of gases to which they may be applied.

Air possesses, in common with all material substances, the qualities of impenetrability, inertia, and weight.

It possesses, in common with liquids, the characteristic properties of fluids, such as the free motion of its particles amongst each other, and the power of transmitting pressure equally in every direction. It possesses, also, its own characteristic properties of compressibility and elasticity, which distinguish it from solids and liquids.

From its extremely attenuated nature, its great levity, the facility with which it is displaced, and the ease with which bodies pass through it, its extreme *transparency*, which renders it imperceptible to sight, immersed in a well or reservoir, so that the water by its weight or pressure would be continually forced into the extremity of the tube, it would be gradually carried along the spiral by turning the screw, until it would attain any height to which the screw might extend.

In practice, the spiral through which the water is carried is not in the form of the tube, but has the character represented in section in *fig*. 204.

As applied to the propulsion of steam-vessels, the screw is horizontal, and exercises its power, not by raising the water, but by driving it backwards: the re-action of the water thus driven gives propulsion to the vessel.

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From its extremely attenuated nature, its great levity, the facility with which it is displaced, and the ease with which bodies pass through it, its extreme transparency, which renders it imperceptible to sight, it might be, and in fact was, doubted whether air were material; and hence the word spirit, from *spiritus* (*air* or *breath*), came to signify an immaterial substance. Nevertheless, it requires but little reflection on the phenomena of nature, as will presently appear, to become convinced that air possesses all the fundamental qualities of matter.

702. *Air is impenetrable.* — Impenetrability, it will be remembered, is that quality in virtue of which a body excludes all others from the space it occupies. If a hollow vessel, such, for example, as a glass tumbler, be inverted and immersed with its mouth downwards in water, it will be found that the water will not fill the tumbler. Let a cork be placed upon the water under the mouth of the tumbler; and when the tumbler sinks, the cork and the surface on which it floats will sink too. The diving-bell exhibits this experiment on a larger scale. A large hollow vessel is sunk by weights with its mouth downwards. Seats and other conveniences are provided within, on which persons may be accommodated, and the whole apparatus is thus let down to the bottom of the sea. Notwithstanding the open mouth and the pressure of the sea, the liquid is nevertheless excluded by the air contained in the bell. The liquid cannot enter the space occupied by the air. The air is impenetrable.

703. *Air has the quality of inertia.* — Inertia, it will be remembered, is manifested by the moving force which matter has when it is in motion, or by the resistance which matter at rest offers to other matter in motion which encounters it. Air exhibits in a most conspicuous manner both of these qualities. Wind is nothing more than air in motion. An example, therefore, of the effects of the power of the wind is a proof of the inertia of air. In a windmill, the moving force of all the heavier parts of the machinery proceeds from the momentum of the wind acting on the sails. A ship is propelled through the deep, and the deep itself is agitated and raised into waves, by the inertia of the atmosphere in motion. As the velocity of the air is augmented, its force becomes almost irresistible, and we find buildings totter, trees torn from their roots, and even the solid earth itself yield before the force of the hurricane.

When the atmosphere is calm and free from wind, a solid body presenting a broad surface moved against it must drive before it and put in motion those parts of the air which lie in its way. If the air had no inertia, it would require no force to impart this motion to it; but universal experience proves that the force encountered by a body moving through the air is great in proportion to the magnitude of the surface which encounters the air, and to the speed with which it is moved. Open an umbrella and endeavour to carry it along swiftly, with the concave side presented forwards, and you immediately encounter a great resistance. This force is nothing more than what is necessary to push the air before it. On the deck of a steamboat propelled with considerable speed, or on the top of a railway carriage, we

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feel on the calmest day a breeze in a direction contrary to that in which we are moved. This arises from the sensation produced by the surface of our body displacing the air as we are carried through it.

704. *Numerous examples of its inertia.* — It is the inertia of the atmosphere which gives effect to the wings of birds. Were it possible for a bird to live without respiration, and in a space void of air, it would no longer have the power of flight. The plumage of the wings, being spread and acting with a broad surface on the atmosphere beneath them, is resisted by the inertia of the atmosphere, so that the air forms a fulcrum, as it were, on which the bird rises by the leverage of its wings. The wings of birds are larger in proportion to their bodies than the fins of fishes, because the fluid on which they act is less dense, and has, proportionally, less inertia than the water upon which the fins of fishes act.

705. *Air is compressible.*—In this quality it is distinguished from the other class of fluids called liquids. It has been shown that liquids are practically incompressible; for although, as a philosophical fact, a mass of liquid may by the action of an extreme force of compression be diminished in a very minute degree in its volume, it does not possess the quality of compressibility in the same manner in which it is manifested in æriform bodies. If air be included in a cylinder in which a piston moves air-tight, the piston, being urged downwards by any force, will compress the air into smaller dimensions, and there is no practical limit to this compression: if the force that urges the piston be doubled or tripled, the air, as will be hereafter proved, will be reduced to one-half or one-third of its dimensions.

When a diving-bell is sunk to a considerable depth in the sea, the water which enters its mouth, though it cannot displace the air, compresses it, and rises to a certain height within the bell, the air giving way to it and being condensed into a smaller space.

706. *Air is elastic.* — This is another quality which distinguishes æriform bodies from liquids. If a liquid be deposited in a cylinder under a piston, it will remain there, its surface maintaining the same position to whatever height the piston may be raised above it; but if air be contained in a cylinder under a piston which moves air-tight, on raising the piston the air will expand, so as still to fill the augmented space below the piston; and this expansion will continue to whatever height the piston may be raised, and to whatever extent the space be augmented, in which the air is free to circulate.

It is evident that this tendency to enlarge its volume, and which is expressed by the term elasticity, will cause the air confined in any vessel to press on the inner surface of such vessel with a force corresponding to its tendency to expand. If no corresponding external pressure act upon the surface of such vessel, the air will have a tendency to burst it, and will, in fact, burst it if it have not strength to resist the elastic force. We are enabled, by means which will be explained hereafter, to remove the atmosphere from around an inflated bladder. On doing this, the elasticity of the air included in the bladder, being unresisted by any external pressure, will burst the bladder, if it have not a strength corresponding to such elasticity.

707. *Air as weight.* — Means will be explained hereafter by which air can be withdrawn from the interior of any vessel which contains it, in the same manner exactly as water can be pumped from a well. Let a copper flask, holding about two quarts, having a narrow neck provided with a stop-cock, be discharged of its air, and let it be correctly weighed. Let the stop-cock be now opened, and the air readmitted, and let it be again weighed. It will be found to be heavier than before, by the weight of the air readmitted to it. Let an instrument, which will be hereafter described, be next applied to the mouth of the flask, and let air be compressed into it so as to make it contain twice as much as before, and the stop-cock being closed, let it be again weighed.

The increase of weight produced in this last case will be found to be exactly equal to the increase of weight produced by readmitting the air into the empty flask. In both cases the augmented weight arises from the increased weight of the air contained in the flask.

Having thus explained, in general, the properties of air, we shall in the following chapters trace these properties through their most important consequences.

CHAP. II.

COMPRESSIBILITY AND ELASTICITY OF AIR.

708. *The diminution of the volume of air is proportional to the force which compresses it.*—It has been explained, in general, that when air is submitted to the action of any compressing force, it will be reduced in its volume. It remains now to investigate in what proportion its volume will be diminished by any given increase of the force which compresses it.

It is found, by experiment, that the diminution of volume will be in the exact proportion of the compressing force. If the compressing force be doubled, the air which is compressed will be reduced to half its volume; if the compressing force be increased in a threefold proportion, the volume of the air compressed will be diminished in a threefold proportion, and so on.

Let A B C D, *fig.* 205, be a glass tube, curved at one end, B C, and having a short leg C D, with a stop-cock at its extremity D. Let the leg A B be more than 6 feet in length. The stop-cock D being opened, so as to allow free communication with the air, and the mouth of the longer leg A being also open, let so much mercury be poured into the tube as will fill the curved part B C, and rise to a small height in each leg. The surfaces E and F will then, according to the principles already explained, stand at the same level. Let the stop-cock D be now closed, so that the air in the leg D F shall be shut off from communication with the external atmosphere.

The surfaces E and F will still remain at the same level.

They are, however, now acted upon by different forces; the surface E is acted upon by the weight of the atmosphere transmitted through the open tube A E. But the weight of the atmosphere does not act upon the surface F, inasmuch as the stop-cock D is closed, and all communication with the external air intercepted. The surface of the mercury at F is therefore acted on only by the elasticity of the air inclosed in the tube D F; and since the surfaces E and F, under these circumstances, continue at the same level, it follows that the weight of the atmosphere acting at E is equal to the elasticity of the atmosphere manifested by the air inclosed in D F.

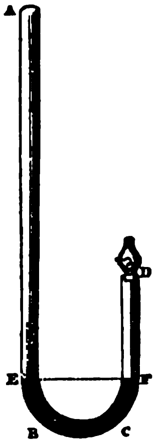

Fig. 205.

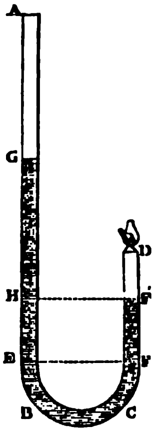

Fig. 206.

The method of ascertaining the actual force with which the atmosphere presses by its weight on the surface of the mercury at E will be explained hereafter. For the present it is necessary for us to assume that this force is equal to the weight of a column of mercury about 30 inches in height. We will assume, therefore, that the *elastic force of the air inclosed* between F and D is such that it presses upon the surface F with the same force as a column of mercury 30 inches in height would press upon it.

Now, if we pour into the tube A E as much mercury as will raise the surface in the leg A B 30 in. above the surface of the mercury in the leg D C, we shall have an additional pressure equal to the weight of a column of 30 inches of mercury transmitted from the leg A E to the surface of the mercury in the leg D F, and therefore acting as a compressing force on the air included in the leg D F. If this be done, it will be found that the surface of the mercury in the leg D F will rise so as to force the air included in D F into half its original volume; that is to say, from F to F′, *fig*. 206, because the level of the mercury will, when the additional column has been introduced into A E, be raised to the point F′, exactly mid-way between D and F, and the air which originally filled the space D F will be now compressed into one-half this space; that is to say, into D F′.

In the same manner, if mercury be again poured into the tube A E until the surface of the column in A E be 60 in. above the level of the mercury in D F, then the air in D F will be compressed into one-third of its original volume.

In the former case, when the column in A E was 30 in. above the column in D F, the air was compressed by a force equal to the weight of 60 in. of mercury, because, as has been already explained, it was compressed by the atmosphere, equal to 30 in. of mercury, and by the additional force of the column of 30 in. of mercury introduced into the tube. In the latter case, the air is compressed by a force equal to 90 in. of mercury, as it is compressed first by the atmosphere, equal to 30 in. of mercury, and, secondly, by the column of 60 in. of mercury introduced into the tube. The compressing forces, therefore, in the three cases, are represented respectively by 30, 60, and 90 in. of mercury; and, consequently, the compressing force in the one case is twofold, and in the other case threefold, the force by which the air is compressed in its natural state. In the same manner, to whatever extent such experiments may be continued, it will be found that the diminution of volume will always be in the exact proportion of the increase of the compressing force, and, in like manner, the augmentation of volume will be in the proportion of the diminution of the compressing force.

The law just enunciated was investigated by Boyle, in 1660, and by Mariotte, in 1668; and is now generally known as ***Mariotte's law*** Dulong and Arago, in 1830, published the results of some experiments, made in Paris, by which they proved that the law holds for atmospheric air up to a pressure of 27 atmospheres.

Oersted and Despretz, having instituted experiments to ascertain if it were applicable to other gases, found that those gases which are easily liquefied, have an increasing compressibility; that is, have their *volume* diminished in a greater proportion than the increase of pressure.

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Pouillet, in his "Élémens de Physique," announces the results of some experiments made by him, as follows: —

1st. Up to 100 atmospheres, oxygen, nitrogen, hydrogen, nitric, oxide, and carbonic oxide, follow the same law of compression as atmospheric air.

2d. Sulphurous acid, ammoniacal gas, carbonic acid, and nitrous oxide, begin to be markedly more compressible than air, as soon as their volume is reduced to a third or a fourth; and it cannot be doubted that they deviate from the law for less changes.

3d. Light-carburetted hydrogen and olefiant gas are not liquefied under the pressure of 100 atmospheres at the temperature of 50°; and yet they have a compressibility sensibly greater than that of air.

Later and more elaborate experiments by Regnault show that the law of Mariotte is not strictly true even for atmospheric air; yet, within practical limits and for practical purposes, it may be assumed to be true for that fluid.

CHAP. III.

WEIGHT OF AIR.

709. *Discovery of the weight of the atmosphere.* — The discovery of the weight of the thin transparent fluid which surrounds the earth, which by respiration supports animal life, and is necessary to the due exercise of the animal and vegetable functions, forms a remarkable epoch in the history of physical science. The ancient philosophers observed that in the instances which fell under their notice space was filled by some material substance. The moment a solid or a liquid was by any means removed, the surrounding air instantly rushed in and filled the space so deserted. Hence they adopted the physical dogma, that nature abhors a vacuum, — a figurative proposition, meant as a statement that it was a law of nature that space could not exist unoccupied by matter.

If a tube be immersed in a liquid, and the suction of the lips be applied at the upper end, the water which surrounds it will rise in the tube as the air is withdrawn by suction. This was explained by declaring that nature abhorred a vacuum, and, therefore, the water necessarily filled the space deserted by the air.

This alleged antipathy of nature to a vacuum served the purposes of Natural Philosophy for 2000 years.

710. *Anecdote of Galileo and Torricelli.* — It happened in the time of Galileo, that is, about the middle of the seventeenth century, that some engineers near Florence, being employed to sink a pump to an unusual depth, found they could raise by no exertion the water

higher than 34 feet in the barrel. Galileo was consulted, and it is said that he answered, half-seriously and half-sportively, that nature's abhorrence of a vacuum extended to the height of 34 feet, but that beyond this her disinclination to an empty space was not carried. The answer, however, whatever it was, does not appear to have been satisfactory, and the question continued to excite attention. After the death of Galileo, Torricelli, his pupil, since become so celebrated, directed his attention to its solution. He argued, that whatever be the cause which sustains a column of water in a pump, the measure of the power thus manifested must be the weight of the column of water sustained; and, consequently, if another liquid were used, heavier bulk for bulk than water, the same force would sustain a column of that liquid, having less height in proportion as its weight would be greater. By using a heavier liquid, therefore, such as mercury, for example, the column sustained would be much shorter, and the experiment would be more manageable. The weight of mercury being bulk for bulk about 13½ times that of water, it followed that, if the force imputed to a vacuum could sustain 34 feet of water, it would necessarily sustain 13½ times less, or about 30 inches of mercury. Torricelli therefore made the following experiment, which has since become so memorable in the history of physical science.

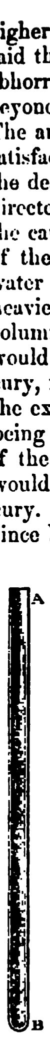

Fig. 207.

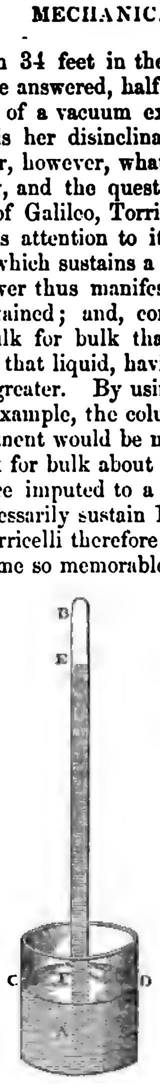

Fig. 208.

711. *Celebrated experiment of Torricelli.* — He procured a glass tube A B, *fig.* 207., more than 30 inches long, open at one end A, and closed at the other B. Filling this tube with mercury, and applying his finger at the open end A, so as to prevent its escape, he inverted it, plunging the end A into mercury contained in a cistern C D, *fig.* 208.

On removing the finger, he observed that the mercury in the tube fell, but did not fall altogether into the cistern; it only subsided until its surface E was at a height of about 30 inches above the surface of the mercury in the cistern.

This result, which was precisely what Torricelli had anticipated, clearly demonstrated the absurdity of the statement imputed to Galileo, that nature's abhorrence of a vacuum extended to the height of 34 feet, since in this case her abhorrence was limited to 30 inches. In fine, Torricelli soon perceived the true cause of this phenomenon.

The weight of the atmosphere acting upon the surface of the mercury in the cistern supports the liquid in the tube. But the surface

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E being excluded from contact with the atmosphere, is free from the pressure of its weight; the column, therefore, of mercury F being pressed upwards by the weight of the atmosphere, and not being pressed downwards by any other force, would stand in equilibrium.

712. *Experimental proof of the weight of the atmosphere.* — This explanation was further confirmed by the fact, that on admitting the air to the upper end of the tube B, by breaking off the glass at that point, or opening a stop-cock placed there, the column of mercury in the tube instantly dropped into the cistern. This was precisely the effect which ought to ensue, inasmuch as the admission of the pressure of air upon the column E balanced the pressure on the surface in the cistern, and there was no longer any force to sustain a column of mercury in the tube, and consequently it fell into the cistern.

713. *Pascal's experimentum crucis.* — This experiment and its explanation excited, at the epoch we refer to, the greatest sensation throughout the scientific world, and, like all new discoveries which have a tendency to explode long-established doctrines, was rejected by the majority of scientific men. The celebrated Pascal, who flourished at that epoch, however, had the sagacity to perceive the force of Torricelli's reasoning, and proposed to submit his experiment to a test which must put an end to all further question about it. "If," said Pascal, "it be really the weight of the atmosphere under which we live that supports the column of mercury in Torricelli's tube, we shall find, by transporting this tube upwards in the atmosphere, that in proportion as it leaves below it more and more of the air, and has consequently less and less above it, there will be a less column sustained in the tube, inasmuch as the weight of the air above the tube, which is declared by Torricelli to be the force which sustains it, will be diminished by the increased elevation of the tube."

Pascal therefore caused Torricelli's tube to be carried to the top of a lofty mountain, called the Puy-de-dome, in Auvergne, and the height of the column to be correctly noted during the ascent. It was found, in conformity with the principle announced by Torricelli, that the column gradually diminished in height as the elevation to which the instrument was carried increased. The experiment being repeated upon a high tower in Paris with like success, there no longer remained any doubt of the fact, that the column of mercury in the tube, as well as the column of water in common pumps, is sustained, not by the force vulgarly called suction, nor by nature's abhorrence of a vacuum, but simply by the weight of the incumbent air acting in one case on the surface of the mercury, and in the other on the surface of the water in the well, in which the pump terminates.

The instrument which we have here described as used in the experiment of Torricelli, is nothing more than the common barometer. By the principle explained in 616., the height of the column sustained by the atmospheric pressure will be the same whatever be the bore of the tube. If we suppose the section of the bore to be equal to one square inch, the column of mercury sustained in the tube will be balanced by the weight of a column of the atmosphere pressing upon a square inch of the surface of the mercury in the cistern. If we suppose, on the other hand, the tube to have a bore equal to half a square inch, then the atmospheric column which balances the mercury will have a base of half a square inch also.

714. *Construction of a barometer.* — In adapting such an apparatus to indicate minute changes in the pressure of the atmosphere there are several provisions to be made.

The height to be measured being that of the surface of the column in the tube above the surface of the mercury in the cistern, it is not enough to ascertain the position of the surface in the tube, unless the surface in the cistern have a fixed level. Now it is evident, that whenever the surface in the tube rises, the surface in the cistern must fall, and *vice versâ*, inasmuch as whatever mercury enters the tube must leave the cistern, and whatever flows from the tube must return to the cistern. If the magnitude of the surface in the cistern be very considerable compared with the bore of the tube, and if extreme accuracy be not necessary, the effects arising from this cause will will be too minute to need any correction; but if that extreme accuracy is desired, which is necessary in barometers used for philosophical experiments, then means must be provided of keeping the mercury in the cistern at a fixed level, or of measuring the change of level.

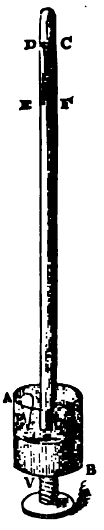

Fig. 209.

In *fig.* 209., the cistern A B is represented having an index at P, showing the point at which the level of the mercury in the cistern should stand. A screw is represented at V, by turning which the bottom can be elevated or depressed, so that when the level in the cistern falls it may be raised, or when it rises it may be lowered, and thus the level may always be adjusted so as to correspond with the point of the index. The scale represented at D E is divided with reference to the level determined by the point of the index P.

715. *Methods of purifying the mercury.* — It is necessary that the mercury should be perfectly pure, since otherwise a column of a given height would vary in its weight, according to the quantity and quality of the impurities which the liquid might contain.

The solid impurities which mercury may contain are usually removed by straining it through chamois leather, the quicksilver passing freely through its pores, while the solid impurities are retained.

Mercury, like water, commonly contains combined with it more or less air or other elastic fluids. If such mercury were used for the barometer tube, this fixed air, when relieved from the pressure of the

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atmosphere, as it would necessarily be in the tube, would become disengaged, and would rise to the upper part of the tube D C, and there exert a pressure which would counteract, to a greater or less extent, the pressure of the atmosphere.

Independently of the impurities, whether of an æriform or a liquid species, which may be combined with the mercury, the tube itself, before it is filled, is liable to be coated with like impurities. Thus particles of air and of moisture will always adhere to the inner surface of it; and even though the mercury were pure, this air and film of moisture would have a tendency, when relieved from the pressure of the atmosphere, to rise and vitiate the vacuum at the top of the barometric column.

These effects are avoided by the following expedients. The mercury before it is poured into the tube, is boiled, in which process all the air it contains is expelled by its increased elasticity, and all the liquid impurities by evaporation. The tube itself is heated over a spirit-lamp, so that all the moisture as well as the particles of air adhering to its surface are expelled; in fine, when the tube has been filled with mercury, the mercury is boiled in it.

716. *Method of indicating the exact height of the column, and of allowing for effects of temperature.* — But supposing the mercury to be perfectly pure, and all the provisions made which are necessary to indicate the exact height of the column sustained, two barometers, equally well constructed, would still differ in their indications, if they are exposed to different temperatures. It will be shown hereafter that mercury, like all other fluids, is subject to a change of density or specific gravity with every change of temperature. If, therefore, two barometers, equally well constructed, be used in different and distant places, where they are exposed to different temperatures, the same pressure of the atmosphere will sustain columns of different heights; that which is exposed to the higher temperature being more elevated than that which is exposed to the lower. In comparing, therefore, the indications of barometers in different places, it is necessary to observe the temperature; and tables and formulæ are supplied in physical science, by which the effects of different temperatures can be ascertained, and the necessary corrections applied.

The changes incidental to the atmospheric pressure, the indication of which is the chief use of the barometer, are so limited and minute, that, owing to the great specific gravity of mercury, they produce extremely minute changes in the barometric column, which are therefore difficult to be observed.

717. *Water barometer.* — More sensible indications would be obtained by adopting a barometer of a lighter fluid than mercury Thus, water is 13½ times lighter than mercury, and, consequently, a water barometer would exhibit a column 13½ times greater than that of mercury.

Such a column would, therefore, measure about 34 feet, and a change which would produce a variation of about the tenth of an inch in the column of mercury, would produce a variation of an inch and a third in the column of water.

But to the use of water, or any other liquid save mercury, for barometric purposes, there are numerous and insuperable practical objections. Independently of the unwieldy height of the column, which would render it impossible to transport the barometer from place to place, all the lighter liquids would produce vapour in the upper part of the tube, which would vitiate the vacuum, would react against the barometric column, and disturb its indications. The consequence of this has been, that mercury has been invariably retained as the only practicable fluid for barometers.

Several expedients, however, have been adopted in barometers used for common domestic purposes to render their indications more sensible. Although these are inapplicable in barometers used for scientific purposes, yet, as they are frequently adopted in domestic barometers, it may be useful here to notice them.

718. *Diagonal and wheel barometers.* — A form of barometer, called the diagonal barometer, is represented in *fig.* 210. In this the upper end of the tube is bent, so that the scale, instead of being limited to the length C D, is extended over the greater length C B.

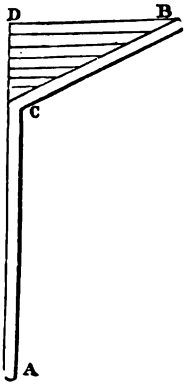

Fig. 210.

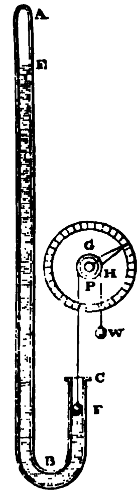

Fig. 211.

A form of barometer, called the wheel barometer, is represented in *fig.* 211. In this, the tube, instead of having a cistern, is continued of the same diameter, having its lower end bent upwards at B C. A float is placed upon the mercury at F, which rises and falls with it. The change of altitude of the level F corresponds with that of E, and the difference between the two levels E and F is the height of the barometric column. The changes of this height are always double the change of level of the surface E F. The float F is connected by a string with a wheel H, which carries an index that plays upon a graduated dial-plate, G. In this manner the magnitude of the graduated scale may be made to bear any proportion, however great, to the change of level of the mercury at E, so that the smallest change of the barometric column will produce a considerable motion of the index.

719. *State of the upper regions of the atmosphere.*—The gravity of air, combined with its elasticity and compressibility, supplies the means of determining, by reasoning alone, the constitution of the superior strata of the atmosphere, which are inaccessible to direct experiment.

If we suppose the atmosphere, which extends from the surface of the earth upwards to a height more or less considerable, to consist of a series of layers or strata, placed one above the other, it is evident that each successive stratum, in ascending, will sustain a weight less than those below it. The first stratum of atmosphere, which is in immediate contact with the surface of the earth is compressed by the entire weight of the atmosphere above it, that is to say, by the weight of the whole atmosphere, except the first stratum; the next stratum is compressed by the weight of the whole atmosphere, except that of the first two strata; the third stratum is compressed by the weight of the whole atmosphere, except the first three strata; and so on Now, it has been already shown that the density of air is always proportional to the force which compresses it; and it follows, therefore, that the density of the first, or lowest stratum, is greater than the density of the second, and the density of the second greater than the density of the third, and so on, the air becoming gradually less dense as it ascends to a greater height.

720. *Height of atmosphere limited.*—It may be asked, under such circumstances, whether the atmosphere must not extend to an unlimited height, because, if its rarefaction augments in proportion as the compression diminishes, there can be no limit to such rarefaction.

But it must be remembered that the constituent particles or ultimate molecules of the air itself have definite weight, and that, so soon as the rarefaction becomes so great that the elastic force proceeding from the mutual repulsion of the particles of air is equal to the weight of these particles, no further rarefaction can take place. We may therefore conceive the particles of air at the upper surface of the atmosphere resting in equilibrium, under the influence of two opposite forces, viz., their own weight tending to carry them downwards, and the mutual repulsion of the particles which constitutes the elasticity of the air tending to drive them upwards.

If a particle of air were raised above this height by the application of any external agency, and then disengaged, it would drop by its gravity to the surface of the atmosphere, in the same manner and by the same law which makes a stone drop to the ground. The limit, therefore, of the altitude of the atmosphere is that point where the rarefaction will diminish the elastic force of the air, so as to render it equal to the proper gravity of its constituent particles.

721. *Average pressure of atmosphere.*—We have seen that the

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height of the column of mercury which balances the atmospheric pressure at the surface of the earth is about 30 inches. Now two cubic inches of mercury weigh in round numbers a pound avoirdupois; consequently it follows, that a column of mercury, whose base is a square inch and whose height is 30 inches, will weigh 15 lbs.

But since such a column measures the pressure of the atmosphere, it follows that the atmosphere presses with a force of 15 lbs. for every square inch of surface upon which it rests.

But the air possesses, in common with all other fluids, the faculty of transmitting pressure equally in every direction; consequently it follows, that every object exposed to the atmosphere is pressed upon every part of its surface with a force amounting to 15 lbs. per square inch.

The surface of a human body of average size measures about 2000 square inches. Such a body, therefore, sustains a pressure from the atmosphere amounting to 30,000 lbs., or very nearly 15 tons.

722. *Method of measuring heights by barometer.* — It has been shown that when a barometer is carried upwards in the atmosphere, the column of mercury in the tube falls, because the force which sustains it is diminished by an amount equal to the weight of the column which it leaves below it. By comparing, therefore, the height of the column in the barometer at any two stations, one of which is above the other, we can ascertain directly the weight of a column of atmosphere extending from the lower to the higher station. Thus, for example, if the column of mercury in the barometer at the lower station be 30 inches, and at the higher station 20 inches, it follows that a column of air whose base is at the lower station, and whose summit is at the higher station, will have a weight equal to that of a column of mercury 10 inches high, and therefore that the quantity of air composing such a column will be one-third of the quantity composing a column extending from the lower station to the summit of the atmosphere.

If the atmosphere were uniformly dense, the barometer would supply a most easy and simple means of determining its actual height.

In the example just given, the column of air between the two stations would weigh one-third of the weight of a column extending from the lower station to the summit of the atmosphere; and, if the air were uniformly dense, it would follow, therefore, that the entire height of the atmosphere would be just three times the height of the upper above the lower station. But, owing to the circumstances already explained, which produce a gradual rarefaction of the air as the height increases, it follows that the heights of columns of air are not proportional to their weights.

If the only cause which produces a gradual rarefaction of the air as we ascend in the atmosphere were that which has been just stated, namely, the weight of the incumbent air, it would not be difficult to

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find a rule by which a change of altitude might be inferred from observing the change of pressure indicated by a barometer. Such a rule has been determined, and is capable of being expressed in the language of mathematics, although it be not of a nature to be rendered intelligible in an elementary and popular treatise.

723. *Density of air affected by its temperature.*—But there are other causes affecting the relation of the change of pressure to the change of altitude. The density of any stratum of air is not alone affected by the incumbent pressure of the superior strata, but also by its own temperature. If any cause increase this temperature, the stratum will become more rarefied, and with a less density will support the same incumbent pressure; and if, on the contrary, any cause produce a fall of temperature, it will require a greater density with the same pressure. In the one case, therefore, a change of elevation which would be necessary to produce a given change in the height of the barometer would be greater than that computed on theoretical principles; and in the other case it would be less. The temperature therefore forms an essential condition in the calculation of heights by the barometer.

Formulæ have been contrived, partly by theoretical principles, and partly from observation, by which the difference of height of two stations may be deduced from observations simultaneously made at them on the barometer and the thermometer. To apply such a rule it is necessary to know, first, the latitude of the place of observation; secondly, the heights of the barometer and thermometer at each of the two stations, besides some other physical data, to comprehend which it would be necessary to have reference to some principles drawn from the physics of heat and from physical astronomy, which cannot be introduced here. Such a formula, therefore, cannot be usefully given here.

724. *Fall of barometer in the balloon ascent of De Luc.*—The barometer in the balloon in which the celebrated De Luc made his scientific voyage, fell at the greatest altitude to 12 inches. Supposing the barometer at the surface to have stood at that time at 30 inches, it follows from this, that he must have left below him in quantity exactly three-fifths of the entire atmosphere, since 12 inches would be only two-fifths of the complete column sustained in the barometric tube. His elevation at this moment was estimated to have been 20,000 feet; but it is certain that he had not attained a point amounting to more than a small fraction of the entire altitude of the atmosphere.

Since the density of air is proportional to its pressure, other things being the same, it would follow that the density of the air in which the balloon floated on this occasion was only four-tenths of the density at the surface.

Now when the barometer is at 30 inches, air is 10,800 times lighter

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than mercury; and, consequently, the air surrounding De Luc's balloon must have been 27,000 times lighter, bulk for bulk, than mercury. The height, therefore, of air above the balloon, supposing its density to be undiminished in rising, would have been 27,000 feet, and in this case the entire height of the atmosphere would be nearly 50,000 feet. But here it is to be considered, as in the former case, that in rising above the level of the balloon, the air would constantly diminish in density; and consequently a column supporting 12 inches of mercury would have a much greater elevation than 27,000 feet.

725. *Extreme variations incidental to the barometer.* — The physical effect of which the barometric column is the measure, is the weight of the atmosphere at the place where this barometric column is situated; and consequently, the variations, whatever they may be, which are incidental to the column, indicate corresponding variations in the weight of the atmosphere. Now it has been found that the barometric column is subject to two species of variation: one of an extremely minute amount, and which takes place at regular periods; the other of much greater amount, and which may be considered as comparatively contingent and accidental. The extreme limit of this latter variation is, however, not great. The greatest height, for example, which the barometer kept at the Paris Observatory has been known to attain is 30·7 inches, and the lowest 28·2 inches, the difference being 2·5 inches, or $\frac{1}{12}$th of the average height of the column.

The mean height of the barometer at Paris, obtained from observations continued for several years, has been found to be 29·77 inches.

726. *Diurnal variation.* — The periodical variations of the barometric column are extremely complicated, though very minute. In winter, it is found that the column attains a maximum height at nine in the morning; it falls from this hour until three in the afternoon; it then begins to rise, and attains another maximum at nine in the evening. In summer, the hour of the first maximum is eight in the morning, and that of the minimum four in the afternoon; that of the second maximum being eleven at night. In spring and autumn, this maximum and minimum take place at intermediate hours.

727. *Supposed connection between barometric changes and the weather.* — The accidental variations of the barometer, or, to speak more properly, those which are not periodic, and which are much greater in magnitude, have been generally supposed to be prognostics of change in the weather, and hence the barometer is sometimes called a weather-glass. Rules have been attempted to be established by which from the absolute height of the mercurial column the coming state of the weather may be predicted; and we accordingly find the words Rain, Fair, Changeable, Frost, &c., engraved upon the scale attached to common domestic barometers, as if, when the mercury

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stands at the heights marked respectively by these words, the weather is always subject to the vicissitudes expressed by them.

It requires but little reflection on what has been stated to show the fallacy of such indications. The absolute height of the mercurial column varies with the position of the instrument. A barometer in Fleet Street, London, will be higher at the same moment than one on the top of St. Paul's, and consequently two such barometers would indicate different coming changes of the weather, though absolutely situate in the same place. Two barometers, one of which is placed at the level of the Thames, and the other at the top of Hampstead Hill, will differ by half an inch, and, consequently, would indicate, according to the usual scales, different coming changes.

728. *Fallacy of the popular rules.* — It is evident, therefore, that the absolute height of the barometer cannot in itself be an indication of anything but the weight of the atmosphere in the place where the instrument stands, and the words engraved on barometric plates, which have been just referred to, are altogether unworthy of serious attention.

It is found that the changes of weather are indicated not by the actual height of the mercury, but by its change of height. One of the most general, though not invariable rules, is, that when the mercury is very low, and therefore the atmosphere very light, high winds and storms are likely to prevail.

729. *Rules by which coming changes may be prognosticated.*— The following rules may to some extent be relied upon, but even these are subject to much uncertainty.

1. Generally the rising of the mercury indicates the approach of fair weather, the falling of it shows the approach of foul weather.

2. In sultry weather, the fall of the mercury indicates coming thunder. In winter the rise of the mercury indicates frost. In frost, its fall indicates thaw, and its rise indicates snow.

3. Whatever change of weather suddenly follows a change in the barometer, may be expected to last but a short time. Thus, if fair weather follow immediately the rise of the mercury, there will be very little of it; and, in the same way, if foul weather follow the fall of the mercury, it will last but a short time.

4. If fair weather continue for several days, during which the mercury continually falls, a long succession of foul weather will probably ensue; and again, if foul weather continue for several days while the mercury continually rises, a long succession of fair weather will probably succeed.

5. A fluctuating and unsettled state in the mercurial column indicates changeable weather.

The domestic barometer would become a much more useful instrument, if, instead of the words usually engraved on the plate, a short list of the best established rules, such as the preceding, accompanied it, which might be either engraved on the plate or printed on a card.

It would be right, however, to express the rules only with that degree of probability which observation of past phenomena has justified. There is no rule respecting these effects which will hold good with perfect certainty in every case.

730. *Calculation of the height of the atmosphere if it were of uniform density, and the limit of its variation of height.*—The weight of mercury is 13½ times greater than that of water, and the weight of water is about 800 times that of air, at the mean density of the latter; consequently, the weight of mercury is, bulk for bulk, 10,800 times greater than the weight of air; therefore a column of air of uniform density, equal in weight to the barometric column, would be 10,800 times higher. Now, taking the average height of the barometric column at 2½ feet, a column of air of equal weight, and having a uniform density equal to that of air at the surface of the earth, would give a height of 27,000 feet; and, since the barometric column is subject to irregular variations, which range within a twelfth of its entire height, the corresponding column of air would be subject to like variations, ranging within a like proportion of its entire height, which according to this calculation would amount to 2250 feet. If, therefore, the atmosphere were, like the ocean, of uniform density, the height of the waves, which would be incidental to its surface agitated by the disturbances to which it is exposed, would be nearly half a mile.

731. *Enormous height of atmospheric waves.*—But as the atmosphere is not of uniform density, but diminishes in density in a rapid proportion, as the height increases, its altitude is much greater than 27,000 feet; and the change incidental to its superficial level indicated by the variations of the barometer must therefore be proportionally greater. The waves of the sea, therefore, even in the most violent storms, are absolutely insignificant compared with the waves which prevail in the upper surface of the ocean of atmosphere under which we live.

732. *Why the atmospheric pressure does not crush bodies exposed to it.*—It might be expected that the great pressure to which all bodies surrounded by the atmosphere are exposed would produce conspicuous effects, in crushing, compressing, or bursting them; whereas it is found that even the most delicate textures are not affected by it. A bag made of the lightest and finest tissue partially filled with air, is practically subject to no external pressure; its sides, though loaded with an enormous pressure, do not collapse. This is explained partly by the equality of the pressure which is directed upon it on all sides, partly by the resistance produced from within, by the elasticity of ir contained in it.

e same circumstances explain the fact that animals are neither

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cted in their movements, nor crushed by the enormous pressure ch their bodies are subjected. The atmosphere pressing them y in every possible direction, laterally and obliquely, upwards wnwards, has no tendency to impel them in any one direction, than another, and consequently offers no other resistance to notion than is produced by the inertia of the atmosphere itself. ternal pores of their bodies being filled with fluids, both liquid seous, producing a pressure outwards exactly equal to the ex- pressure of the air inwards, an equilibrium results, and no part body is crushed.

e effect of the internal fluids in resisting the external pressure atmosphere may be rendered manifest by applying an exhaust- ringe or a cupping-glass to any part of the skin. Such an in- ent has no other effect than that of removing the atmospheric ire from that part of the surface to which it is applied; but it does this, immediately the skin is distended and sucked, as it into the glass, in consequence of the elasticity of the fluids con- l in the organs.

. *Suction the effect of atmospheric pressure.* — The various mena, which are vulgarly called suction, are merely the effects nospheric pressure. If a piece of moist leather be placed in contact with any heavy body having a smooth surface, such as a or a piece of metal, it will adhere to it; and if a cord be at- d to the leather, the stone or metal may be raised by it.

is effect arises from the exclusion of the air between the leather he stone. The weight of the atmosphere presses their surfaces ier with a force amounting to 15 lbs. on a square inch of the e of contact.

4. *Flies walking on ceiling.* — The power of flies, and other s, to walk on ceilings, smooth pieces of wood, and other similar es, in doing which the gravity of their bodies appears to have fect, is explained upon the same principle. Their feet are pro- with an apparatus similar exactly to the leather applied to the .

5. *Respiration.* — The pressure and elasticity of the air are both l into effect in the act of respiration. When we inspire the at- here, we make by a muscular exertion an enlarged space within hest. The atmospheric pressure forces the external air into this . By another and contrary muscular exertion, the chest is then acted, so as to squeeze out the air which has been inhaled, and i, by compression, acquires an elasticity greater than the atmos- c pressure, in virtue of which it is forced out at the mouth and ils.

6. *Action of bellows.* — The action of a common bellows is pre- r similar, except that the aperture through which the air enters ferent from that by which it is expelled. The analogy, how-

ever, would be complete if we inspired by the mouth and expelled by the nose. When the boards of the bellows are separated, the inner chamber is enlarged, and the air is forced in by the external pressure through the aperture governed by the leather valve or clack. The boards being then pressed together, and the escape of the air being stopped by the closed valve, it is compressed until it acquires an elasticity greater than the atmospheric pressure, and is forced out.

Bellows on a large scale are constructed with an intermediate board, so as to consist of two chambers, and to produce a continued instead of an intermittent blast. This is nothing more than a double bellows, one forcing air into the chamber of the other, and the second being urged by an uninterrupted pressure produced usually by a weight suspended from the upper board.

737. *Vent-peg — lid of tea-pot.* — The effect produced by a vent-peg in a cask of liquid is explained by the atmospheric pressure. The cask being air-tight, so long as the vent-peg is maintained in its position, the surface of the liquid in the vessel will be excluded from the atmospheric pressure, and it can only flow from the cock in virtue of its own weight. If the weight of the atmosphere be greater than the weight of a column of the liquid, corresponding with the depth of the liquid in the vessel, the liquid cannot flow from the cask; but the moment the vent-peg is removed, the atmospheric pressure being admitted above the level of the liquid in the cask, the liquid flows from the cock in virtue of its own weight.

If the lid of a teapot or kettle were perfectly close, the liquid would not flow from the pipe, because the atmospheric pressure would be excluded from the inner surface. A small hole is therefore usually made in the lid to admit the air and allow the liquid to flow freely.

738. *Pneumatic ink-bottle.* — Ink-bottles are sometimes so constructed as to prevent the inconvenience of the ink thickening and drying. Such a bottle is represented in *fig.* 212.: A B is a close glass vessel, from the bottom of which a short tube B C proceeds, the depth of which is sufficient for the immersion of a pen. When ink is poured in at C, the bottle being placed in an inclined position, is gradually filled up to the nob A. If the bottle be now placed in the position represented in the figure, the chamber A B being filled with the liquid, the air will be excluded from it, and the pressure tending to force the ink upwards in the short tube C, will be equal to the weight of the column of ink, the height of which is equal to the depth of the ink in the bottle A B, and the bore of which is equal to the section of the tube C. The ink will be prevented from rising in the tube C by the atmospheric pressure, which is much greater than the pressure of the column of liquid in the bottle. As

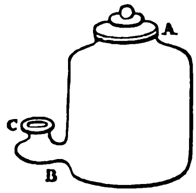

Fig. 212.

[redacted]

the ink in the short tube C is consumed by use, its surface will gradually fall to a level with the horizontal tube B, a small bubble of air will then insinuate itself through B, and will rise to the top of the bottle A B, where it will exert an elastic pressure, which will cause the surface of the ink in C to rise a little higher; and this effect will be continually repeated, until all the ink in the bottle has been used.

Birdcage fountains are constructed on the same principle.

The peculiar gurgling noise produced in decanting wine arise from the pressure of the atmosphere forcing air into the interior of the bottle to replace the liquid which escapes.

CHAP. IV.

RAREFACTION AND CONDENSATION OF AIR.

The effects, infinitely various, produced by the atmosphere on bodies, whether organized or unorganized, cannot be made fully manifest without being enabled to exhibit the same objects under other atmospheric conditions, such as when exposed to an atmosphere much more rare and much more condensed. Instruments for experimental investigation have been accordingly contrived, by which the air surrounding objects of experimental inquiries can be either rarefied or condensed to any desired extent within practical limits. We shall in the present chapter explain the principal instruments by which these processes are exhibited, and give some examples of their use.

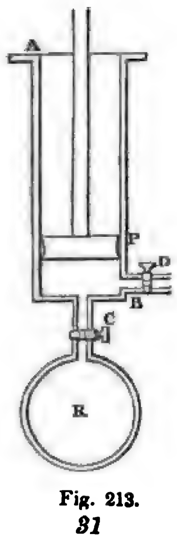

Fig. 213.

739. *The exhausting syringe.* — Let A B, *fig.* 213., represent a cylinder having a solid piston P, moving air-tight in it. Let C be a tube proceeding from its lower end, furnished with a stop-cock C, and let B be another tube furnished with a stop-cock D. Let the tube C be screwed upon any vessel such as R, from which it is desired to extract the air.

If the piston be now raised in the cylinder, the cock D being closed and the cock C being open, the air in R will necessarily expand, in virtue of its elasticity, so as to fill the enlarged space provided by raising the piston. The air which previously filled the vessel R and the connecting tube will, in fact, now fill these, and also the enlarged space in the cylinder. When the piston

[redacted]

is brought to the top of the cylinder, let the cock C be closed and the cock D be opened. Upon driving down the piston, the air which fills the cylinder will be expelled from the tube B through the open stop-cock D. When the piston has reached the bottom of the cylinder, let D be closed and C opened, and let the same process be repeated; the air filling the vessel R will, as before, dilate itself, so as to fill such vessel and the cylinder. The cock C being again closed, and D opened, and the piston driven down, the air which fills the cylinder will be again expelled. This process being continued, any desired quantity of air, short of the whole, can be taken out of the vessel R and expelled into the atmosphere.

It is evident that the escape of the air from R into the cylinder is effected in virtue of its elasticity; while its escape from the stop-cock D into the atmosphere is effected in virtue of its compressibility.

740. ***Rate of the rarefaction.*** — It is easy to explain the *rate* at which the air is drawn from the vessel R by this process. If we suppose the volume of the cylinder through which the piston passes to be $\frac{1}{10}$th, for example, of the entire volume of the cylinder, the tube and the connecting pipe taken together, then it is clear, that on completing the first downward stroke of the piston $\frac{1}{10}$th of all the air included between the piston and the surface of the vessel R will be expelled, and $\frac{9}{10}$ths consequently will remain.

At every succeeding stroke, $\frac{1}{10}$th of what remained after the preceding stroke will be expelled, and in the same way $\frac{9}{10}$ths will remain.

If we suppose the vessel R and the connecting tube to contain ten million grains weight of air, the quantities expelled at each successive stroke, the quantities remaining, and the total quantities expelled from the commencement of the operation, will be thus exhibited in the following table: —

No. of Strokes.	Grains expelled at each Stroke.	Grains remaining under Pressure.	Total Numbers of Grains expelled.
1	1,000,000	9,000,000	1,000,000
2	900,000	8,100,000	1,900,000
3	810,000	7,290,000	2,710,000
4	729,000	6,561,000	3,439,000
5	656,100	5,905,900	4,095,100
6	590,590	5,315,310	4,685,690
7	531,531	4,783,779	5,217,221
8	478,378	4,305,401	5,695,599
9	430,540	3,874,861	6,126,139
10	387,486	3,487,375	6,513,625
11	348,737	3,138,638	6,862,362
12	313,864	2,824,774	7,176,226

[redacted]

Thus, in twelve strokes of the syringe, of the ten million of grains of air originally included, something more than seven million, or seven-tenths of the whole, have been withdrawn, and something less than three-tenths remain.

741. *An absolute vacuum cannot be obtained by this process.* — A rarefaction has been therefore produced in the proportion of something more than three to ten. But it will be apparent, that although by this process the rarefaction may be continued to any required extent, a literal and absolute vacuum can never be produced, because some quantity of air, however small, must always remain in the vessel R. After every stroke of the piston, $\frac{9}{10}$ths of the air which is in the vessel before the stroke remains in it. Now it is evident, that if we successively subtract $\frac{1}{10}$th of any quantity, we must always have some remainder, however long the process be continued; and the same will be true, whatever proportion be thus continually subtracted.

742. *But may be indefinitely approached.* — Nevertheless, although an absolute vacuum cannot be obtained by such means, we can continue the process until the rarefaction shall be carried to any required extent.

In practice, the stop-cocks D and C are replaced by valves. A valve is placed at D which, opening outwards, is forced open by the elasticity of the air compressed under the piston when depressed, but is kept closed by the external pressure of the atmosphere when the piston is raised. The valve at C opens upwards, and is opened by the elasticity of the air in R when the piston is raised, and kept closed by the elasticity of the compressed air in the cylinder when the piston is depressed. Instead of placing a tube and valve at B, it is usual to make the valve in the piston itself, opening upwards; but the action is still the same. An exhausting syringe, therefore, may be shortly described to consist of a cylinder with two valves, one in the bottom, opening upwards, and one in the piston, also opening upwards. When the piston is drawn upwards, the valve in the bottom of the cylinder is opened by the pressure of the air under it, and the air passes through it. When the piston is driven downwards, the valve in the piston is opened by the elasticity of the air compressed under it, which rushes through it.

743. *The air-pump.* — The air-pump is an apparatus consisting usually of two exhausting syringes, B B′, *fig.* 214., mounted so as to be worked by a single winch and handle, as represented at D, and communicating by a common pipe T with a glass vessel R, in which may be placed the objects of experiment. The vessel R, called a receiver, has an edge S ground smooth, resting upon a plate, also ground smooth, and kept in air-tight connexion with it by being smeared with hog's lard. A stop-cock C is provided in the pipe T, by which the communications between the receiver R and the syringes can be made

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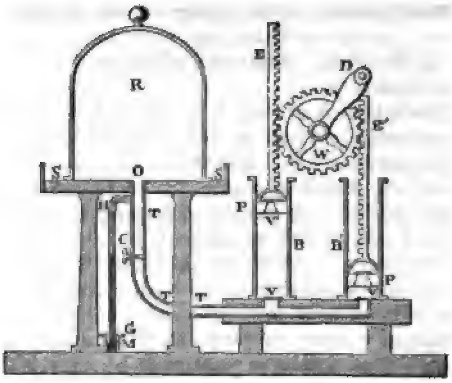

Fig. 214.

and broken at pleasure. Another stop-cock is provided elsewhere, by which a communication can be made at pleasure between the interior of the receiver R and the external air. To indicate the extent to which the rarefaction is carried from time to time by the operation of the syringes, a mercurial gauge H G M is provided, constructed in all respects similar to a barometer. The atmosphere presses on the surface of the mercury in the cistern M, while the column of mercury in the tube H G is pressed upon by the rarefied air in R. The height of the column, therefore, sustained in the tube, indicates the difference between the pressure of the external air and the air in the receiver.

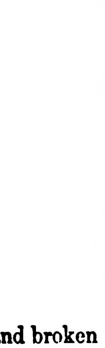

Fig. 215.

When a gauge, of the form represented in *fig.* 214., is used, it is necessary that it should have the height of about 30 inches; since, when a high degree of rarefaction has been effected, a column of mercury will be sustained in the tube H G, very little less than in the common barometer. In small pumps, where this height would be inconvenient, a siphon-gauge, such as that represented in *fig.* 215., is used. This gauge is screwed on to a pipe communicating with the receiver. Mercury fills the leg A B, which is closed at the top A, and partially fills the leg S. When the atmosphere communicates freely with the tube D C, the surface of the mercury in S being pressed by its full force, sustains all the mercury which the tube B A can contain, and this tube, consequently, remains completely filled; but when the pipe D C S is put in communication with the exhausted receiver, the surface of the mercury in S being acted upon only by the pressure of the rarefied air in the receiver, the weight of the higher column in B A will predominate, and the mercury will fall in it, until the difference of the levels in the

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two legs shall be equal to the pressure of the rarefied air in the receiver.

744. *The condensing syringe.*—This instrument differs from the exhausting syringe only in the direction in which the valves are placed. It consists of a cylinder and piston, as represented in *fig.* 213. When the piston is drawn upwards, the cock D is open, and C is closed, and the cylinder is filled with air proceeding from the external atmosphere. When the piston is pressed downwards, the cock D is closed, and C is opened, and the air which filled the cylinder is forced into the vessel R. On raising the piston again, the cock C is closed, and D is opened, and the effects take place as before. It is evident that, by every stroke of the piston, as much air as fills the cylinder is driven into the vessel R.

In practice, the cocks D and C are replaced by two valves, one in the bottom of the cylinder, and the other in the piston, both opening downwards, contrary to the valves in the exhausting syringe.

The operation is explained in the same manner.

745. *The condenser.*—The condenser is an apparatus which bears to the condensing syringe precisely the same relation which the air-pump bears to the exhausting syringe. It consists of one or two condensing syringes, mounted so as to be conveniently worked by a winch, and communicating with a strong reservoir, which is fastened down upon a plate, so as to be maintained in air-tight contact with it, notwithstanding the bursting pressure of air condensed within it. By the operation of syringes, volumes of air corresponding to their magnitude are forced continually into the reservoir, which becomes therefore filled with an atmosphere proportionally more dense than the external air.

746. EXPERIMENTS WITH THE AIR-PUMP.—I. *External bladder-glass.*—A strong glass vessel is provided, open both at top and bottom, and having a diameter of four or five inches. Upon one end is tied a bladder, so as to be completely air-tight. The other end is placed upon the pump-plate, being previously smeared with lard, to make the contact air-tight. The air under the bladder is rarefied by the operation of the pump, and the bladder is subject to a pressure from without proportional to the difference between the pressure of the external air and the pressure of the rarefied air under the bladder. When the rarefaction has been carried to such an extent that the strength of the bladder is less than this pressure, the bladder bursts with a loud report.

II. *Internal bladder-glass.*—A glass vessel, open at one end only, similar to the last, is provided, on the mouth of which a bladder is tied, so as to be air-tight. This vessel is placed under a receiver on the air-pump plate, and the air around it within the receiver is rarefied by the pump. The elasticity of the air included within the bladder being greater than that of the rarefied air outside it, there is a bursting pressure within the bladder proportional to the difference between the elasticity of the air within and without. If the bladder be not too strong, the rarefaction may be carried to such an extent that the bladder will burst.

III. *Bottle burst by elasticity of common air.*—Let two bottles composed of very thin glass be provided, one having flat sides, and the other round, and their mouths being closed, so as to be air-tight, air being included within. If these bottles be placed under a receiver, and the air around them be exhausted, that with the flat sides will be burst by the elasticity of the air it contains; that which has round sides will resist the elastic force in consequence of the strength depending on its shape.

IV. *Effect of gases contained in dried fruit.*—If shrivelled fruit, such as apples or grapes, be placed under a receiver, and the air around them exhausted, they will lose their shrivelled appearance, and seem to be fresh and ripe. This effect is caused by the gases which they include distending them so soon as their external surface is relieved from the pressure of the surrounding air, so as to give effect to the elasticity of these gases.

V. *The Magdeburgh hemispheres.*—The apparatus known by this title is represented in *fig.* 216. It consists of two hollow brass hemispheres with evenly ground edges, which admit of being brought into air-tight contact when smeared with lard. The apparatus when screwed upon the plate of an air-pump may be exhausted, so that the space within the hemispheres may be rendered a partial vacuum. The external air will thus press the two hemispheres together with a force proportional to the difference between the pressure of the external air and the pressure of the rarefied air within. When a sufficient exhaustion has been produced, the stop-cock attached to the lower hemisphere is closed, the apparatus is unscrewed from the pump-plate, and a handle screwed upon the lower hemisphere. It will be found that two of the strongest men will be unable to tear the hemispheres asunder, provided they are of a moderate magnitude, owing to the amount of the pressure with which they are held together. If, for example, the pressure of the rarefied air within is equivalent to a column of two inches of mercury, while the external air has a pressure represented by 30 inches of mercury, there will be a force amounting to 14 lbs. per square inch in the section of the hemispheres.

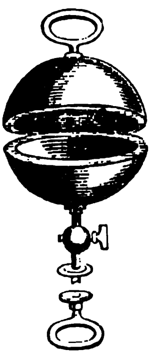

Fig. 216.

If the hemispheres have 4 inches diameter, the area of their sec-

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tion will be nearly 12¼ square inches, and consequently the force with which they will be pressed together will be about

$$12\frac{1}{4} \times 14 = 175 \text{ lbs.}$$

This apparatus derives its name from the place where the inventor of the air-pump, Otto Guericke, first exhibited the experiment, in the year 1654. The section of the hemispheres employed by him measured 113 square inches, and they were held together by a force equal to about three-fourths of a ton.

CHAP. V.

MACHINES FOR RAISING WATER.

747. ***The bucket in a well.***—The most simple apparatus by which water can be raised from a depth is a bucket suspended from a rope, which is wound upon a windlass. The bucket being let down and dipped into the water, is drawn up by working the windlass. The bucket may be filled without being inverted, by providing a valve in its bottom opening upwards; the weight of the empty bucket pressing on the water is sufficient to open the valve.

In this process, however, the moving power which works the windlass raises not only the weight of the water and the bucket which contains it, but also the weight of the rope by which the bucket is suspended.

748. ***The lifting-pump.*** — If, instead of a bucket and rope, a pipe or tube be let down into the well, and in this pipe a piston be provided having a valve in it opening upwards, this piston being worked in the usual manner upwards and downwards, the water would be lifted in the pipe. Such an apparatus is called a lifting-pump, and is represented in *fig.* 217.: W is the water, *c d* the piston, *u* the valve in it which opens upwards. When the piston is moved downwards, this valve opens, and the water passes through it. When the piston is moved upwards, the column of water which is above it is pushed up, and the valve is kept closed by the pressure of the water upon it. A valve *x* is placed at C D in a fixed position, through which the column of water passes when the piston rises, and which prevents the return of such water downwards, the valve being kept closed by the weight of the water above it. The column of water driven upwards by the piston is pushed to any required height, through the pipe E F. In such an apparatus, the moving power must be equal to the weight of the water raised, together with the weight of the pump-rod and frame by which the piston is worked, as well as the friction of the moving parts.

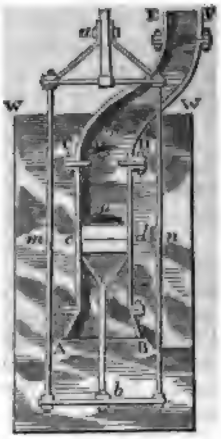

Fig. 217.

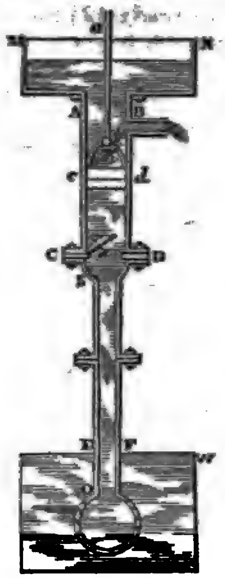

Fig. 218.

749. *The suction-pump.* — By far the most common form of the water-pump is the common suction-pump usually provided for domestic purposes. This instrument consists of a pipe or barrel, S O, *fig.* 218., which descends into the well, and the length of which must be less than 34 feet. Attached to the top of this pipe, which is called the suction-pipe, is a large syringe, acting precisely on the principle of the exhausting syringe already explained.

At the commencement of the operation, the pipe S E is filled with air to the level of the water in the well. The operation of the syringe, according to what has been already explained, draws the chief part of the air out of this pipe S E. When the water within the pipe is partially relieved from the atmospheric pressure, the weight of the atmosphere, acting upon the external surface of the water in the well, forces it up in the pipe S E; and according as the air is withdrawn by the syringe, the water continues to rise, until it passes through the valve *x*. This valve opening upwards, prevents its return, since the weight of the column above it will keep it closed. When the barrel A C becomes filled with water, the syringe no longer acts as such, but works on the principal of the lifting pump, already explained. When the piston descends, the valve *x* is closed, and the valve *v* opened, the *water* passing through the piston. When the piston is raised, the

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valve *v* is closed, and the column of water above the piston is projected upwards.

Meanwhile the pressure of the atmosphere on the water in the well causes more water to rise in the pump-barrel following the piston.

It has been already explained that the atmospheric pressure is capable of supporting a column of about 34 feet of water. It is evident, therefore, that such a pump as is here described can only be efficient when the piston is at a height of less than 34 feet above the surface of the water in the well, since otherwise the atmospheric pressure would not keep the water in contact with the piston.

The suction-pump, therefore, as compared with the lifting-pump, saves 34 feet length of pump-rod; but otherwise there is no comparative mechanical advantage.

It might appear at first view that the pressure of the atmosphere sustaining a column of water in the suction-pipe, supplies aid to the power that works the pump, and spares an equivalent amount of that power.

This, however, is not the case, as will appear from a due consideration of all the forces which are in operation.

The atmosphere presses with a force of about 15 lbs. per square inch on the surface of the water in the well, and produces therefore an upward force upon the column raised, the amount of which is estimated in lbs. by multiplying the number of square inches in the section of the column by 15. But, on the other hand, the atmosphere also presses upon the upper surface of the column raised with exactly the same force, and consequently the upward and downward pressure of the atmosphere upon the column of water raised being equal and opposite, are mutually destructive; the weight, therefore, of the entire column of water extending from the level of the surface in the well to the pipe of discharge above, must be raised by the power, whatever it be, that works the pump. This power, therefore, must overcome not only the weight of the column of water above the piston, but also of that part of the column which is between the piston and the well.

750. *The forcing-pump.* — This instrument is represented in *fig.* 219. The suction-pipe C E is similar to the suction-pump. The piston *c d* is a solid plug without a valve.

The forcing pipe G H has at its base *e f* a valve V' which opens upwards. When the piston *c d* is raised, the valve V is opened, and the water rises from the suction-pipe into the pump-barrel. When the piston *c d* is pressed downwards, the valve V is closed, and the water is forced by the pressure of the piston through the valve V' into the force-pipe, and thus while the operation is continued, at each upward motion of the piston, water is drawn from the suction-pipe into the pump-barrel, and at each downward motion it is forced from the pump-barrel into the force-pipe.

751. ***Forcing-pump with air-vessels.*** — In order to produce a continued flow of water in the force-pipe, an air-vessel is often attached to force-pumps. Such an appendage is represented in *fig.* 220.

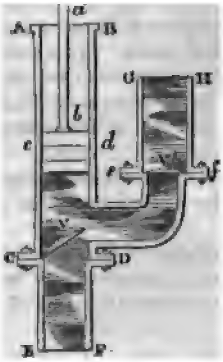

Fig. 219.

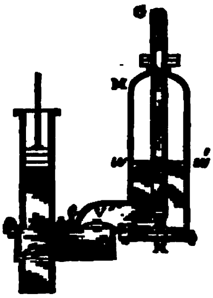

Fig. 220.

When the piston descends, the water is driven through the valve-pipe V′ into the vessel which is closed and contains air. The forces G H descends into this vessel, and terminates near the bottom. The water which is forced in rises in it to a certain level, w w′, the air above it being compressed. The return of the water through the valve V′ being stopped, it is subject to the elastic pressure of the air confined in the air-vessel M N. This pressure forces the water through the tube H G, from the top of which it issues in a constant stream.

In the force-pump, as represented in this figure, the upper surface of the piston is exposed to the atmosphere and does not act; the lower surface acts alternately by suction and pressure. When the piston is drawn upwards it acts by suction, and when the piston is driven downwards it acts by pressure. Now there is no reason why the upper surface of the piston might not be similarly employed.

To accomplish this it would be only necessary to put the upper part of the pump-barrel in communication with the suction-pipe and the force-pipe with proper valves.

This is accordingly accomplished in some forcing-pumps in which the piston acts upon the column in the force-pipe both in ascending and descending; and, consequently, acts continuously without the intervention of an air-vessel.

752. *The fire-engine.* — The fire-engine is nothing more than two force-pumps, worked by a common winch or handle, in a manner exactly similar to the two syringes which work an air-pump. The pistons ascend and descend alternately in the pump-barrels, and

drive a continual stream of water into an air-vessel, so that a continual jet can be projected from the discharge pipe.

753. ***The siphon.*** — A siphon is an apparatus by which a liquid can be decanted from one vessel to another without inverting or otherwise disturbing the position of the vessel from which the liquid is removed. Let D, *fig.* 221., be a vessel containing a liquid, and let B be the height over which it is necessary to conduct the liquid, so as to transfer it to the vessel F G. Let A B C be a bent tube open at both ends, and let the leg B A be immersed in the liquid which it is required to transfer, and let the leg B C be directed into the vessel into which the liquid is to be removed. Let the air which fills the tube A B C be drawn from it by the mouth placed at C, or by an exhausting syringe. The liquid in the vessel D will then be forced up in the pipe A B by the pressure of the atmosphere, and will fill the entire tube to the mouth C. It will then flow through the siphon, and continue to be discharged at C so long as the level of the liquid in the vessel D is above its level in the vessel F G.

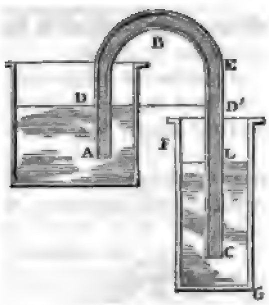

Fig. 221.

It is evident that the bend of the siphon B cannot be at a greater height above the level of the liquid in D than corresponds with the height of a column of the liquid which the atmospheric pressure can support. Thus, if the liquid to be decanted were mercury, the height of B above D should be less than 30 inches; and if it were water, it must be less than 34 feet.

The principle of the siphon is easily explained. Supposing the entire tube to be filled with liquid, we have the column extending in the siphon from the leg D to the highest point B, pressed upwards by the atmosphere acting upon the surface D, and transmitted to it by the liquid in the vessel and in the tube. Against this there is the weight of the column of liquid in the siphon, extending from the leg D to the point B. The pressure therefore acting at the point B towards E is that of the atmosphere, diminished by the weight of a column of the liquid, whose height is that of the point B above the surface D.

Now we have at the point B another pressure opposed to this. The atmosphere pressing on the surface of the liquid at L is in like manner transmitted to the point B by means of the liquid in the vessel F G, and in the tube; but it is diminished by the weight of a column of the liquid, whose height is that of the point B above the surface L. If the height of B, therefore, above the surface of the liquid in the two vessels were the same, the liquid at B would be pressed equally in opposite directions by the common force of the atmosphere, diminished by the weight of two equal columns of the liquid. But so long as the column B L is greater than the column B D, the effect of the atmospheric pressure acting from B towards D will be more diminished than its effect acting from B towards E, and consequently the liquid will flow in the latter direction. In a word, the liquid will flow through the siphon towards that vessel in which its level is lowest, and will continue so to flow until the levels be equalized.

The process of exhausting the siphon by suction or otherwise is often difficult, and always inconvenient. It may be avoided by inverting the siphon, and filling up the end of its longer leg, that of the shorter leg being temporarily stopped.

The siphon being once filled, and its mouths plugged, it may be inverted and placed in the vessel in the required position, when, the plugs being removed, it will commence to act.

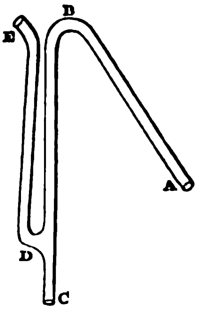

Fig. 222.

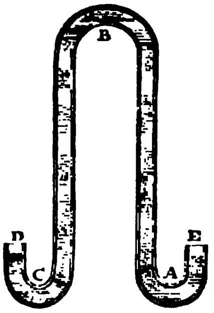

Fig. 223.

A siphon is sometimes constructed, as represented in *fig.* 222., having a suction or exhausting pipe D E attached to it, by which the process is facilitated.

The Wurtemberg siphon is so formed that when once filled it will always remain so, provided the waste by evaporation be supplied. This instrument is represented in *fig.* 223.

* * *

CHAP. VI.

THE AIR-GUN, THE BALLOON, AND THE DIVING-BELL.

751. *The air-gun.* — The air-gun is an instrument by which balls or other missiles are projected by the elastic force of compressed air instead of the expansive force of gunpowder.

A strong hollow chamber, usually having the form of a metallic ball, is provided, into which air is condensed by means of a condensing syringe. This is screwed upon the gun near the breach, so as to communicate with the interior of the barrel behind the ball, the pipe of communication being governed by a valve or cock which is connected with the trigger. On drawing the trigger, the valve is opened, and the barrel put in free communication with the condensed air, which, pressing behind the ball, propels it towards the mouth, from which it is projected with a corresponding force. The stock of the gun may contain a supply of balls, and be furnished with a simple mechanism, by which they may be successively transferred to the barrel, so that the gun may be immediately loaded after each discharge.

755. *The air balloon.* — It has been explained in the last book that any body lighter bulk for bulk than a liquid will ascend through the liquid to its surface. There is nothing in the conditions which determine this question which renders them inapplicable to aeriform fluids; and it accordingly follows by the same reasoning, that if a body, lighter bulk for bulk than the atmosphere, be placed in it, it will rise upwards, in the same manner and upon the same principles as a cork would rise in water.

If a hollow vessel of sufficient magnitude could be exhausted by an air-pump, and if it could be constructed with sufficient strength to resist the external pressure of the atmosphere, and at the same time so light that its entire weight would be less than the weight of the air extracted from it by the pump, such a body would necessarily rise in the atmosphere, its weight being less than that of the air it displaces. But these conditions are impracticable; there is no material of which such a body could be constructed, so as to be at the same time sufficiently light and sufficiently strong.

If a fluid could be found lighter bulk for bulk than air, having the same pressure, then a hollow vessel filled with such a fluid would be subject to no external pressure tending to crush it, and might be lighter bulk for bulk than air, and under such circumstances it would ascend in the atmosphere.

756. *Fire balloons.* — The first attempts to realize these conditions were by means of heated air. When air is heated it expands, and bulk for bulk becomes lighter than it is at a lower temperature.

If, then, a large bag, composed of paper or silk, or other light material, be filled with heated air, the weight of such a bag, including its contents, might be less than its own bulk of air in the natural state, and it would consequently have a buoyancy proportional to such difference of weight.

757. *Montgolfier's balloon.* — The application of this principle formed the first successful attempt in aerostation. In the year 1722, the celebrated Montgolfier, residing at Annonai, made a series of experiments which ultimately terminated in the formation of a balloon

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of the spherical form, containing 23,000 cubic feet of heated air, and having such a buoyancy as to be capable of raising a gross weight of 500 lbs. This machine rose in the atmosphere to the height of 6000 feet. In this, and subsequent similar experiments, the air within the balloon was kept heated by a fire which was lighted below it, the balloon having an open mouth at its lowest point, through which the flame of the fire was transmitted.

758. *Hydrogen balloons.* — The step from the fire balloon to balloons filled with gas, lighter bulk for bulk than the atmosphere, was easy and obvious. The gas denominated hydrogen was no sooner discovered than it was applied to this purpose.

This gas, being more than fourteen times lighter than atmospheric air, has considerable buoyancy; balloons, accordingly, filled with it, would rise to a great height in the atmosphere.

It has been already explained, that as we ascend in the atmosphere, the strata of air have less and less density: a balloon, therefore, containing gas whose pressure balances the lower strata, will, if it be completely filled, have a tendency to burst when it ascends into the rarer strata; for the gas, not having room to expand, will maintain its original elastic force, while the atmospheric pressure, being diminished in the ascent, will cease to balance it. There will therefore be a bursting pressure equivalent to the excess of the atmospheric pressure at the lower strata, over the pressure in the superior strata to which the balloon ascends.

These effects are prevented practically by inflating, only imperfectly, the balloon at the moment of its ascent. When it rises into the superior strata, the gas accordingly expands, and the balloon becomes comparatively filled, gaining at the same time increased buoyancy by the increased expansion of the gas within it. If it ascend to a still greater height than that at which the inflation becomes complete, it is relieved from the bursting force by means of a safety valve.

When the æronaut desires to descend, he is provided with a valve, by which he can discharge a part of the gas, so as to diminish the buoyancy of the balloon; and when he requires to ascend, he is provided with ballast composed of sand-bags, by casting out which he diminishes the weight of the balloon.

759. *Aerial voyage of Gay Lussac and Biot.* — The impracticability of governing balloons in their course through the air has prevented, and probably will continue to prevent, them from being applied to any purpose of permanent or extensive utility. Scientific voyages have, however, been made into the atmosphere with some success, for the purpose of observing, at great elevations, meteorological phenomena. In 1804, MM. Gay Lussac and Biot made an ascent from Paris, taking with them meteorological apparatus, and attained a height of 13,000 feet. Soon afterwards, M. Gay Lussac ascended alone to a

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height of 23,000 feet; and a like elevation has recently been attained by MM. Barral and Bixio.

760. *Voyage of Garnerin.*—In 1807, M. Garnerin made a nocturnal ascent, and, rising with unusual rapidity, attained a prodigious elevation. By some neglect, the apparatus, on discharging the gas, became unmanageable, and the æronaut was obliged to make an incision in the balloon, which then descended with such rapidity that he was obliged to counteract its motion, by casting out his ballast. The balloon, in this way, alternately rose and sunk for eight hours, during which he experienced the phenomena of a thunder-storm, by which, in fine, he was driven against the mountains, and landed at Mont Tonnerre, a distance of 300 miles from the place of his ascent.

761. *Attempts to use balloons in military operations.*—Attempts have been made to render balloons useful in military operations. A captive balloon is held attached to a cord of sufficient length, so that a person can ascend to a corresponding height and obtain a bird's-eye view of the enemy's movements. An academy for the practice of this manœuvre was formerly established at Meudon, near Paris, where a corps of æronauts was trained. The project, however, was soon abandoned.

762. *The diving-bell.*—This machine depends for its effect on the impenetrability which air enjoys in common with all material substances. The diving-bell is a large vessel closed at the sides and top, but open at the bottom, impenetrable to air and water. When pressed with its mouth downwards into the water, the water partially enters the mouth, compressing the air within it. As it descends, this compression increases, and at a depth of thirty-four feet, being equal to that of the atmosphere, the air included in the bell will be compressed into half its volume, and consequently the bell will be half filled with water. Apparatus is provided by which this effect is counteracted, by forcing in air by means of a condenser worked at the surface, communicating by a pipe which descends and enters the bell by being brought under its mouth. It is forced in until the water is brought nearly to the mouth of the bell, which is thus filled with air in a compressed state.

According as the air included in the bell is rendered impure by respiration, it is discharged, and fresh air is received by means of the condenser and pipe just mentioned. Strong glass lenses, similar to those fixed in the deck of a ship, are set in the top of the bell, by which light is admitted to the interior. The shape of the machine is generally oblong, with seats for the divers, shelves for writing materials and other articles being placed at the sides. Messages are communicated from the bell to the surface above, either by writing or by signals. A board is carried in the bell, on which a written message may be chalked, and which is connected by a cord with the superintendent above.

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When the bell is of cast-iron, signals are given by striking it with a hammer, which produces a sound distinctly audible at the surface of the water. The bell is usually suspended by a crane placed at the surface of the water, so that its position can be changed within certain limits. Means, however, are also provided, by which the divers can emerge from the bell, and move about in the water, having dresses by which they are enabled to respire the air included in the bell.

BOOK THE SIXTH.

SPECIFIC GRAVITY.

CHAPTER I.

STANDARDS OF SPECIFIC GRAVITY.

763. *Weight with reference to bulk.* — When substances are compared one with another in reference to their weight, one is denominated heavier or lighter than another, without any special reference either to the bulk or weight of any particular quantity of the substance in question.

Thus, when we say cork is lighter than oak, and mercury is heavier than water, we speak intelligibly, although it be true that a particular mass of cork may be found which is heavier than a particular quantity of oak, and a certain mass of water may be heavier than another mass of quicksilver. It is, however, implied in such estimates, that they refer to equal volumes of the two substances which are thus compared as to weight. When we say that cork is lighter than oak, we mean that a piece of cork is lighter than a piece of oak of the same size; and when we say that water is lighter than mercury, we mean the observation to apply to equal measures of the two liquids.

A substance is sometimes said to be heavy or light without express reference to any other substance: thus air is said to be light, and lead heavy. A comparison is, however, here tacitly implied. It is meant that air is lighter, and lead heavier, bulk for bulk, than the average of the substances that fall under common observation. This use of positive terms to express comparative qualities prevails in all applications of language: thus we say of a certain individual that he is very tall, and of a certain house that it is very high; meaning that the man is taller than the average of men, and the house higher than the average of houses.

764. *Absolute and relative weight.*—It appears, therefore, that the term weight is used in two distinct, and sometimes opposite, senses. A mass of cork may be at once very light and very heavy, according to the sense in which the terms are used. A mass of cork which weighs twenty tons is heavy because the absolute weight is consider-

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able. It is, however, in another sense light, because, bulk for bulk, compared with most other solid substances with which we are familiar, its weight is inconsiderable.

The absolute weight of a body is that of its entire mass, without any reference to its bulk; the relative weight is the weight of a given magnitude of the substance compared with the weight of the same magnitude of other substances. The term weight is commonly used to express the absolute weight, while the relative weight is called specific gravity. This denomination of relative weight implies that bodies of different species have different weights under equal volumes. Thus, a cubic inch of cork has a weight different from a cubic inch of oak or of gold; a cubic inch of water contains a less weight than a cubic inch of mercury.

Each different species of body has a different weight corresponding to the same bulk; and hence the name specific gravity, which expresses the weight of a given bulk.

765. *Standard of specific gravity for solids and liquids.*—But as specific gravity is a comparative term, it is convenient that some standard should be selected to which all substances may be referred. Water has accordingly been taken, by common consent, as this standard for all bodies in the solid or liquid form. If we say, then, that the specific gravity of quicksilver is 13½, it is meant that the weight of any particular measure of quicksilver is 13½ times greater than the weight of the same measure of water.

But in adopting water as a standard, it is important to consider whether that liquid itself be not subject to variation in its weight. Now it will be shown, hereafter, that every change of temperature which a substance undergoes causes a change in its volume; and water shares in this universal quality. When the temperature is raised, it becomes lighter: a pint of boiling water is lighter than a pint of cold water. If a pint vessel be exactly filled with boiling water, it will be something less than full when it becomes cold, the water contracting its dimensions as its temperature is lowered. In adopting water, therefore, as a standard, it is important that it should be taken at some known temperature.

In some tables of specific gravities, water is taken as the standard at the temperature of 60°, assumed as the average temperature in our climate. There is a further convenience in the adoption of this temperature as that of the standard, since it happens that a cubic foot of water at this temperature weighs, with great precision, 1000 ounces. The numbers, therefore, which express the specific gravities of other bodies with reference to this, will also express the number of ounces which are contained respectively in a cubic foot of their dimensions. Thus, if 13,580 express the specific gravity of mercury, that of water being 1,000, then it will follow that a cubic foot of mercury, at the temperature of 60°, weighs 13,580 ounces.

In some tables, however, the standard temperature of water has been taken at 40°, for a reason which will be hereafter explained.

766. *Standard for gases.* — Bodies which exist in the æriform state are so much lighter than water, that a practical inconvenience would result from taking water as their standard of specific gravity, since the numbers which would then express their specific gravities would be inconveniently small. The standard of specific gravity, therefore, for bodies in the gaseous or æriform state, is atmospheric air. This form of matter being still more susceptible of change in volume from change of temperature, it is the more necessary in fixing the standard that a certain temperature should be agreed upon. The temperature selected for this purpose has been universally that of melting ice, a point which is independent of the arbitrary scales of thermometers used in different countries.

But as the weight of a given bulk of air depends not only on its temperature, but also upon the pressure to which it is subject, and as this pressure varies within certain limits, it becomes as necessary in fixing the standard of specific gravity to assign a certain pressure as a certain temperature. The pressure generally selected has been that which the atmosphere has when the barometer has the height of thirty inches.

It is therefore to be understood that the standard to which the specific gravities of all bodies in the gaseous form are referred is atmospheric air in a pure state, at the temperature of melting ice, and having a pressure of thirty inches of mercury.

If it be desired to determine the relative weight of any body in the gaseous form in relation to water, it is only necessary to determine the weight of atmospheric air in the standard state in reference to water. Now it has been ascertained that a quantity of atmospheric air, equal in volume to 1000 grains of water, will weigh 1·22 grains; and consequently, since a cubic foot of water weighs 1000 ounces, a cubic foot of atmospheric air will weigh 1·22 oz.

CHAP. II.

SPECIFIC GRAVITY OF SOLIDS.

The methods of determining the specific gravity of solid bodies are different, according as they are heavier or lighter than equal volumes of water.

767. *Methods of determining the specific gravity of solids which are heavier than water.* — Let the solid be accurately weighed, and let it then be suspended in pure water and again accurately weighed

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The difference between the two weights in water and out of water, will, according to what has been explained, be the weight of a volume of water equal in bulk to the solid. Let the weight of the solid be divided by this weight, and the quotient will be the specific gravity of the solid, that of water being 1·000. Thus, for example, if a solid which weighs 8 ounces, is found to weigh only 6 ounces being weighed in pure water, it will follow that the weight of the water which is displaced, and which is equal to its own volume, will be 2 ounces. Such a solid, therefore, is four times heavier than its own bulk of water, and consequently its specific gravity is 4, that of water being 1.

In like manner, if the weight out of water has been 9 ounces, the weight in water being 7, then the specific gravity would be found by dividing 9 by 2, and would be 4·5.

This method is not practicable for solids which are dissolved when immersed in water. The specific gravities of such solids may be determined by immersing them in some other liquid in which they are not soluble, and determining their specific gravity with reference to such liquid.

The specific gravity of this liquid being then determined in relation to water by the methods which will be explained hereafter, the specific gravity of the solid in relation to water will be known. Thus, for example, if the solid in question be five times heavier than the liquid in which it is immersed, and in which it is not soluble, and this liquid be itself twice as heavy as water, then it is clear that the solid will be ten times the weight of its own bulk of water, and its specific gravity would accordingly be 10.

Such solids may, however, be sometimes measured in water by coating them with a varnish not affected by water. The specific gravities of salts and like substances may thus be found.

This method is subject to a slight error, owing to the increased volume produced by the coating, and therefore is not admissible where extreme accuracy is necessary.

If the solid consists of many minute pieces, or be in form of a powder, a cup to receive it ought to be previously suspended in water and accurately counterpoised.

768. *Methods of determining the specific gravities of solids which are lighter than water.*—Let the solid be first correctly weighed, and then attached to another solid also accurately weighed, and which is so much heavier than water that the two solids connected may sink. Let the weight which they lose by immersion be noticed. This will be the weight of as much water as is equal in bulk to the two solids taken together. Let the heavier solid be then immersed, and let the weight it loses be ascertained. If this loss of weight be subtracted from the loss sustained by the combined solids, the remainder will be the weight of as much water as is equal in bulk to the lighter solid.

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he ratio of the weight of the latter solid to this will determine its pecific gravity.

Thus, for example, let the weight of the lighter solid be 3 ounces, nd that of the heavier solid 15 ounces. Let the weight which the wo together lose when submerged in water be 5 ounces, and let the reight which the heavier alone loses when immersed be 1 ounce. ince, then, both together lose by immersion 5 ounces, and the eavier alone loses by immersion 1 ounce, the weight of water equal n bulk to the combined volumes will be 5 ounces, while the weight f water equal in bulk to the heavier alone is 1 ounce.

The weight of water, therefore, which is equal in bulk to the ighter, must be 4 ounces. But the weight of the lighter solid itself s 3 ounces; therefore it will weigh three quarters of its own volume f water; and, consequently, its specific gravity must be $\frac{3}{4}$, or 0·75; nd, by what has been previously explained, a cubic foot of such a olid would weigh 750 ounces.

The specific gravity of a solid lighter than water may also be determined by observing the magnitude of the part immersed when it floats; for when it floats, according to what has been proved, it displaces as much water as is equal to its own weight; consequently, the solid will be just so much lighter, bulk for bulk, than water, as the part of its volume immersed when it floats is less than its entire volume. If, therefore, we divide the part immersed by its entire volume, we shall obtain a fraction which will express its specific gravity.

Thus, for example, if a piece of wood floating on water has half its volume immersed, then it follows that the specific gravity of the wood will be 0·5.

CHAP. III.

SPECIFIC GRAVITY OF LIQUIDS.

769. *Methods of determining the specific gravity of liquids.*—To determine the specific gravity of a liquid, let any solid be selected which is heavier, bulk for bulk, than the liquid and than water, and let the loss of weight it suffers by being immersed in the one and in the other be ascertained. This loss will be the weight of so much of the liquid and of so much water as is equal to the bulk of the solid. If, then, the loss of weight sustained by the solid in the liquid be divided by the loss of weight it sustains in water, the quotient will be the specific gravity of the liquid.

For example, let a piece of glass immersed in sulphuric acid lose

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3,700 grains of its weight, and let the same piece of glass immersed in water lose 2,000 grains. It follows, therefore, that the weight of sulphuric acid will be the weight of an equal bulk of water in the ratio of 37 to 20, or $\frac{37}{20}=1{\cdot}85$ to 1. If 1,000, therefore, express the specific gravity of water, 1,850 will express the specific gravity of sulphuric acid.

The specific gravity of a liquid may also be found by means of a solid which is lighter bulk for bulk than the liquid and water. Let such a solid be successively floated on the liquid and on water, and let the magnitudes of the parts immersed be observed. These magnitudes will be the volumes of the liquid and of water which are equal in weight to the solid, and, consequently, the specific gravity of the liquid will be found by dividing the part of the solid which is immersed when it floats in water by the part immersed when it floats in the liquid.

For example, if, the same solid, floated successively on water and muriatic acid, have the parts immersed in the proportion of 12 to 10, then the weight of muriatic acid will be to the weight of an equal bulk of water as 12 to 10, and consequently the specific gravity of muriatic acid will be 1·200, that of water being 1·000.

The specific gravity of liquids may also be ascertained by providing a floating body, which can be loaded until a certain known magnitude of it shall be immersed. A float provided with a small disk or cup at its upper part, in which known weights may be placed, serves this purpose. If such a float be placed successively on the liquid whose specific gravity is to be ascertained, and on water, and be so loaded as to have equal parts immersed in both, then the two weights of the float will be the weights of the quantities of the liquid and of water which it displaces.

These weights will therefore be in the proportion of the specific gravities; and the specific gravity of the liquid may be ascertained by dividing the weight of the float in the liquid by the weight of the float in the water.

Thus, for example, if the weight of float, when immersed in water, be 2,000 grains, while its weight, when immersed in sulphuric acid, is 3,700 grains, the immersion being the same, then the specific gravity will be ascertained by dividing 3,700 by 2,000, which will give as the result 1·850.

The most direct method, however, of determining the specific gravity of a liquid is to provide a vessel of known capacity, such as a cubic inch, for example, and of known weight. Let such a vessel be filled with pure water, and weighed; and then filled with the liquid whose specific gravity is to be ascertained, and weighed. If the weight of the empty vessel be subtracted, in each case the remainder will be the weight of the contents, and the comparison of these weights will give, as already explained, the specific gravity.

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CHAP. IV.

SPECIFIC GRAVITY OF GASES.

770. *Methods of determining the specific gravity of gases.* — The specific gravity of gases is determined, as has been already explained, by comparing their weights with the weight of an equal volume of pure atmospheric air at a given temperature and pressure. A flask of known capacity is first exhausted by means of the air-pump, and then weighed. This flask is then filled with the gas whose specific gravity is to be ascertained, and again weighed. The difference between the weights in the two cases will be the weight of so much of the gas as fills the flask. The weight of atmospheric air itself can be similarly ascertained, and if the weight of the gas be divided by the weight of the same volume of atmospheric air, the quotient will express the specific gravity of the gas in relation to atmospheric air.

It must be observed that in such an experiment the gas must be taken at known pressures and temperatures, since the specific gravities of gases, from their susceptibility of change of density by temperature and pressure, are much more liable to vary than those of solids. The specific gravity, therefore, of gases in general, is determined at the standard temperature of melting ice, and under the standard pressure of 30 inches of mercury.

CHAP. V.

INSTRUMENTS FOR DETERMINING SPECIFIC GRAVITY.

THE instruments for determining specific gravity are the hydrostatic balance, a flask or measure of known capacity, and various forms of floating instruments called hydrometers.

771. *The hydrostatic balance.* — This instrument consists of an ordinary balance, as represented in *fig.* 224., so mounted as to supply convenient means of weighing bodies in and out of liquids. The beam and scales *a b c d* are supported above a stage G H, below which vessels X Y to receive the liquid are placed. The solids are first placed in the scales, and weighed. They are then attached to hooks connected with the bottoms of the scales, and weighed in the liquids, so that the losses of the weight in the liquid can be determined.

Glass flask. — A glass flask, with an accurately ground glass stopper, serves the purpose of a measure for the determination of specific

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gravities. The quantity of liquid which it contains, when exactly filled, is easily ascertained; and where extreme accuracy is required, the flask must be reduced to a standard temperature.

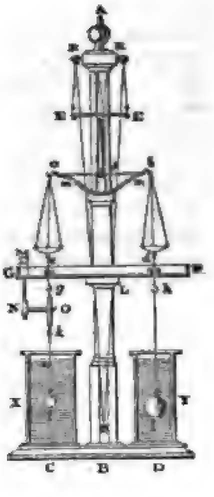

Fig. 224.

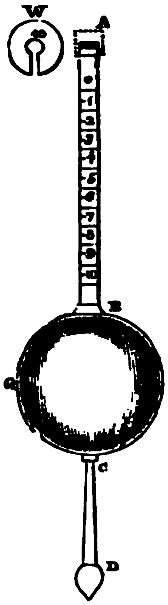

Fig. 225.

772. *Hydrometers.* — Various forms of instruments thus designated have been invented for ascertaining the specific gravity of liquids for the common purposes of commerce. Their indications depend upon the fact that a body, when it floats in a liquid, displaces a quantity of liquid equal to its own weight. The accuracy of these indications depends upon giving them such a shape that the part of them which meets the surface of the liquid in which they float is a narrow stem, of which even a considerable length displaces but a very small weight of the liquid. Thus, any error in observing the immersion produces but a slight effect upon the result.

773. *Sykes's hydrometer.* — This instrument is represented in *fig.* 225. It consists of a brass ball G, the diameter of which is 1·6 inch, into which a conical stem C D, terminating in a pear-shaped bulb, is inserted, and which is so loaded that, being heavier than the rest of the instrument, the graduated stem B A will always remain uppermost and vertical. The instrument is provided with sliding weights W, which will cause it to sink more or less in the liquid. In using

the instrument to ascertain the specific gravity of spirits, it is first plunged in the liquid, so as to be wetted to the highest degree on the scale, and is then allowed to rise and settle in equilibrium. The degree upon the scale at the surface of the liquid indicates the magnitude immersed, and by the aid of tables and a thermometer, by which the temperature of the spirits is observed, the specific gravity is computed.

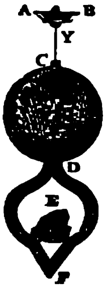

Fig. 226.

774. *Nicholson's hydrometer.* — This instrument is represented in *fig.* 226., and is similar in principle to the last, but is provided with a dish A B, which is loaded until the instrument is made to sink to a standard point, marked about the middle of the stem at Y. The instrument is so constructed that the weight of a quantity of distilled water at 60°, equal in volume to the part of the instrument below the standard point, will be equal to the weight of the instrument, together with 1000 grains.

To find the specific gravity of any other liquid, let the instrument float upon it, and let weights be put in the dish A B until the standard mark on the stem is brought to the surface of the liquid. The weight of the instrument, together with the weight in the dish, will then express the weight of the liquid which the instrument displaces. Thus the weight of equal bulks of the liquid and distilled water at the temperature of 60° will be ascertained, and the specific gravity of the liquid may be thence inferred.

775. *Specific gravity indicates other important qualities.* — The power of determining the specific gravity of bodies often supplies the means of detecting other qualities, and sometimes of indicating their component parts, when they are formed of different constituents. Thus spirits, used in commerce and domestic economy, are a mixture of pure alcohol with other bodies, of which water is the principal. The value, therefore, of the liquid depends upon the proportion of pure alcohol which it contains, and this portion is indicated by its specific gravity. In like manner, the precious metals, whether applied to useful or ornamental purposes, are generally alloyed with others of a baser species, the presence and quantity of which would be determined by their specific gravities.

The first attempt to apply the buoyancy of solids to the detection of their component parts, is attributed to Archimedes. It is related that Hiero, king of Syracuse, having bought a crown of gold, desired to know whether the article were of pure metal; and as the workmanship was costly, he desired to accomplish this without defacing it. The problem was referred to Archimedes. The philosopher, while meditating on the solution of it, was bathing. He reflected on the buoyancy of his own body in the water, and then reasoned upon the general effects produced on the weights of solids by immersion. The whole train of reasoning, which has been developed in the preceding chapters, passed through his mind. He perceived that by ascertaining the degree of buoyancy which the crown would exhibit when immersed in water, he could ascertain if it were pure gold. It was on this occasion that he rushed forth in a transport of joy, exclaiming "Eureka! Eureka!"

CHAP. VI.

TABLES OF SPECIFIC GRAVITY.

776. *Standards adopted in different tables.* — Various investigations have been made in different countries, and by different experimental enquirers, with a view to determine and record with precision the specific gravities of bodies. The standard substances invariably adopted, have been water for solids and liquids, and atmospheric air for gases. But these standards have differed one from another in some particulars. Thus, some tables of specific gravities have been calculated with reference to the specific gravity of water at the temperature of melting ice; some at the temperature which determines the maximum density; a condition which will be explained hereafter. This latter temperature is taken at 4° of the centigrade thermometer, which is equal to 39·2° of Fahrenheit's thermometer. The standard temperature has been in some cases taken at 62° Fahrenheit.

Similar varieties have prevailed with respect to the standard temperature of atmospheric air, in which however, also, a standard pressure must be adopted. In some experiments, the standard adopted has been pure dry air, at the temperature of 60°, the barometer standing at 30 inches.

Whatever be the standard adopted in the one or the other class of specific gravities, such tables can be rendered the means of determining, not only the absolute weights of given volumes, but the absolute volumes of given weights of the bodies registered in them, provided that the absolute weight of a given volume of the standard, whatever it be, be known.

It is therefore important to indicate the weights of these standards.

777. *Weight of a cubic inch of water under standard conditions.* — It has been ascertained that one cubic inch of water at the temperature of 62°, weighs 252·458 grains. It follows from this, that a cubic foot of water under these conditions will weigh 997·125 ounces.

It appears from the experiments of Despretz, that between the temperature of greatest density and 62°, the specific gravity of water differs only by one part in a thousand; and, consequently, the weight of a cubic foot of water at 39·2° would be 998·125 ounces.

For purposes, therefore, where the most extreme accuracy is not necessary, it may be assumed, as a convenient standard of calculation, that a cubic foot of water weighs 1000 ounces.

It has been ascertained by recent experiments, that dry atmospheric air at 32° temperature, the barometer standing at 30 inches, is 773·28 times lighter than water.

778. *Weight of a cubic foot of air under standard conditions.*—We shall find, therefore, the weight of a cubic foot of air at this temperature and pressure, by dividing the number of ounces in a cubic foot of water by 773·28. It consequently follows, that a cubic foot of air at this temperature and pressure will weigh 1·291 ounces, or 554·8 grains.

Since the weight of a cubic foot of water is 1000 ounces, it follows that if the specific gravity of water be expressed by 1000, the numbers which express the specific gravities of all other liquids and solids, will also express the number of ounces contained in a cubic foot of their dimensions. Thus, the specific gravity of gold being 19,360, it follows that a cubic foot of gold will weigh 19,360 ounces.

By the tables of specific gravities, the volume of any proposed weight of a body can be readily calculated, for it is only necessary to divide the number expressing the weight in ounces by the number expressing the specific gravity, omitting the decimal point; the quotient will express the number of cubic feet in the volume. Thus, for example, if it be desired to ascertain the bulk of a ton weight of gold, it is only necessary to reduce the ton weight to ounces, and to divide the number of ounces by 19,360, and the quotient will be the number of cubic feet in the ton weight.

These methods of calculation will be applicable to all tables of specific gravities, composed with reference to water as a standard.

If it be desired to find the absolute weight of a cubic foot of any of the gases or vapours, whose specific gravities are referred to atmospheric air as a standard, it is only necessary to multiply the number expressing their specific gravity by 554·8; the result will express in grains the weight of a cubic foot of the gas or vapour.

If it be desired to find the volume of any gas or vapour which shall have a given weight, let the weight of a cubic foot be first ascertained by the preceding rule, and let this weight be then divided into the proposed weight, the quotient will be the volume in cubic feet.

The following tables of specific gravities, taken from the Annuaire of the French Board of Longitude for 1850, contain the most extensive, recent, and correct results of experiments on the specific gravities of bodies.

TABLE I.

779. Specific Gravities of Gases at 32° Fahr.; Barom. 30 inches

Names.	Specific Gravity by Observation.	Specific Gravity by Calculation.	Observers.
Air	1·000		
Oxygen	1·106	—	Dumas, Boussing.
Hydrogen	0·0691	—	Id. Id.
—— carburetted, of the marshes	0·555	0·559	Thomson.
Methyle	—	0·490	
Hydrogen, bicarburetted (olefiant gas)	0·978	0·980	Th. de Saussure.
—— bicarburetted, of Faraday	1·920	1·960	Faraday.
—— phosphoretted	1·214	1·193	Dumas.
—— arseniuretted	2·695	2·695	Id.
Chlorine	2·470	—	Gay Lussac, Then.
Oxide of chlorine, or hypochloric acid	—	2·340	
Hypochlorous acid of Balard	—	2·980	
Nitrogen	0·972	—	Dumas, Boussing.
Protoxide of nitrogen	1·520	1·525	Colin.
Deutoxide of nitrogen	1·0388	1.0360	Berard.
Cyanogen	1·806	1·818	Gay Lussac.
Chloride of cyanogen	—	2·116	Id.
Ammonia	0·596	0·591	Biot, Arago.
Carbonic oxide	0·957	—	Cruikshank.
Carbonic acid	1·529	—	Dumas, Boussing.
Chloro-carbonic acid	—	3·399	
Sulphurous acid	2·234	—	Thenard.
Acid, chlorohydric	1·247	1·260	Biot, Arago.
—— bromohydric	—	2.731	
—— iodohydric	4·443	4·350	Gay Lussac.
—— sulphohydric	1·191	—	Gay Lussac, Then.
—— selenohydric	—	2·795	Bineau.
—— tellurohydric	—	4·490	Id.
—— fluoboracic	2·371	—	John Davy.
—— fluosilicic	3·573	—	Id.
—— chloroboracic	3·420	—	Dumas.
Monohydrate of methyle	1·617	1·601	Dumas, Peligot.
rate of do.	1·731	1·737	Id. Id.
e of do.	1·186	1·170	Id. Id.

TABLE II.

PECIFIC GRAVITIES OF VAPOURS REDUCED BY CALCULATION TO 32° FAHR.; AND BAROMETER 30 INCHES.

Names.	Specific Gravity by Observation.	Specific Gravity by Calculation.	Observers.
	1·000		
ine	5·540	5·390	Mitscherlich.
	8·716	8·700	Dumas.
ur	6·617	6·650	Id.
horus	4·420	4·320	Id.
ic	10·600	10·360	Mitscherlich.
ry	6·976	6·970	Dumas.
arsenious	13·850	13.300	Mitscherlich.
anhydrous sulphuric	3·000	2·760	Id.
selenious	4·030	—	Id.
hypo-nitrous	1·720	—	Id.
Nitric quadrihydrated	1·270	—	Bineau.
chloride of sulphur	4·700	4·650	Dumas.
o. of do.	3·700	—	Id.
chloride of phosphorus	4·870	4·790	Id.
ide of arsenic	6·300	6·250	Id.
of arsenic	16·100	15·640	Mitscherlich.
chloride of mercury	8·350	8·200	Id.
ride of mercury	9·800	9·420	Id.
bromide of mercury	10·140	9·670	Id.
bromide of mercury	12·160	12·370	Id.
odide of mercury	15·600	15·680	Id.
uret of mercury (cinna-	5·500	5·400	Id.
chloride of antimony	7·800	—	Id.
chloride of bismuth	11·100	10·990	Jacquelain.
ichloride of chromium	5·520 5·900	5.500	Bineau, Walter.
ride of tin	9·199	8·990	Dumas.
chloride of cyanogen	6·390	—	Bineau.
ide of cyanogen	3·610	—	Id.
ide of silicon	5·939	5·959	Dumas.
hor	5·468	5.314	Id.
ce of turpentine	4·763	4·765	Id.
ne or benzol	2·770	2·730	Mitscherlich.
haline	4·528	4·492	Dumas.
ur des Hollandais	3·443	3·450	Gay Lussac.
uret of carbon	2·644	—	Id.
ol	1·6133	1·6010	Id
	2·5860	2·5830	Id.
acetic	3·067	3.066	Dumas, Boullay.
oxalic	5·087	5·081	Id. Id.
benzoic	5·409	5·240	Id. Id.

Names.	Specific Gravity by Observation.	Specific Gravity by Calculation.	Observers.
Wood-spirit	1·120	1·110	Dumas, Peligot.
Sulphate of methyle	4·565	4·370	Id. Id.
Acetate of do.	2·563	2·570	Id. Id.
Potato oil	3·147	3·070	Dumas.
Acetone	2·019	2·020	Id.
Mercaptan	2·326	2·160	Bunsen.
Aldéhyde	1·532	1·530	Liebig.
Essence of bitter almonds	—	3·708	Wohler, Liebig.
Hydruret of salicyle	4·270	4·260	Piria.
Essence of cinnamon	—	4·620	Dumas, Peligot
—— of cumin	5·200	5·100	Gerb. Cahours.
Acid, acetic	2·770	2·780	Dumas.
—— benzoic	4·270	4·260	Id.
—— valerianic	3·680	3·550	Dumas, Stas.
—— cyanhydric (Prussic)	0·947	0·936	Gay Lussac.
Kakodyle	7·100	7·280	Bunsen.
Oxide of kakodyle	7·550	7·830	Id.
Cyanuret of kakodyle	4·630	4·540	Id.
Chloride of kakodyle	4·560	4·800	Id.
Water	0·6235	0·6240	Gay Lussac.

TABLE III.

781. Specific Gravity of Liquids at 39·2° Fahr.

Name.	Sp. Grav.
Water, distilled	1·000
Bromine	2·966
Mercury at 32°	13·596
Acid, sulphuric, most concentrated	1·841
—— hyposulphuric	1·347
—— fuming nitric	1·451
—— quadrihyd. nitric	1·420
—— nitric of commerce	1·220
—— hyponitric	1·451
—— concentrated liquid hydrochloric	1·208
—— acetic monohydrate	1·068
—— acetic, greatest density	1·079
—— oleic	·898
—— cyanhydric (Prussic)	·696
Sulphuret of carbon	1·263
Protochloride of sulphur	1·680
Alcohol, absolute	·792
Do., greatest density (hyd. de Rudberg)	·927
Ether	·715
—— hydrochloric	·874
—— acetic	·868
Wood-spirit	·798
Potato oil	·818
Acetone	·792
Mercaptan	·840
Essence of turpentine	·869
—— citron	·847
Aldéhyde	·790
Essence of bitter almonds	1·043
Oil of spiræa	1·173
Essence of cumin	·969
—— of cinnamon	1·010
Sea-water	1·026
Milk	1·030
Wine, Bordeaux	·994
—— Burgundy	·991
Olive oil	·915
Naphtha	·847

TABLE IV.

2. Specific Gravity of Solids at 39·2° Fahr.

783. *Simple Bodies.*

Names.	Specific Gravity.	Observers.
	4·948	Gay Lussac.
r	2·086	Leroyer, Dumas.
m	4·300	
orus	1·770	
	5·670	Herapath.
Diamonds	3·530	
	3·500	
Graphite	2·500	
um	·865	Gay Lussac, Then
	·972	Id. Id.
ese	8·010	
	7·788	
st	7·200	
ot hammered	7·810	
	7·190	
m, hammered	8·690	
	7·291	
cast	7·812	
cast	8·279	
rged	8·666	
lenum	8·600	
en	17·600	Frères d'Echuyart.
um	5·900	
ny	6·720	
m	5·300	
nfm	6·240	
m	9·000	Bucholz.
h	9·822	
ast	11·350	
, cast	8·850	
lled or forged	8·950	
y at 32°	13·598	
n	10·000?	
(cast by electric battery)	18·680	Children.
um	11·300	
lled	11·800	
m	11·000?	
cast	10·470	
orged	19·360	
st	19·260	
m	21·530	
lled	22·060	
(fused)	21·800	Hare.
ative)	23 to 26	Breithaupt.

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784. *Binary Compounds.*

Names.		Specific Gravity.	Observers.
Acid, silicic	Quartz hyalin.	2·653	M.*
	Agate	2·615	M.
	Opal (sil. hyd.)	2·250	M.
—— hydrated boracic (sassoline)		1·480	M.
Lime		3·150	Boullay.
Chloride of calcium		2·230	Id.
Fluoride of calcium (fluor spar)		3·200	M.
Chloride of barium		3·900	Boullay.
Chloride of potassium		1·836	Wenzel.
Iodide of potassium		3·000	Boullay.
Chloride of sodium		2·100	Kirwan.
Hydrochlorate of ammonia (sal. ammon.)		1·520	M.
Alumina	Corundum, sapphire, and oriental ruby	4·160	M.
	Emery	3·900	M.
Acid, arsenious		3·700	Leroyer, Dumas.
Protoxide of antimony		5·778	Boullay.
Sulphuret of antimony		4·334	M.
Oxide of silver		7·250	Boullay.
Sulphuret of silver		7·200	M.
Chloride of silver (cast)		5·548	Boullay.
Iodide of silver (cast)		5·614	Id.
Deutoxide of mercury		11·000	Id.
Protochloride of mercury		7·140	Id.
Bichloride of mercury		5·420	Id.
Deutiodide of mercury		6·320	Id.
Protiodide of mercury		7·750	Id.
Bisulphuret of mercury		8·124	Id.
Oxide of bismuth		8·968	Id.
Sulphuret of bismuth		6·540	M.
Sulphuret of molybdenum		4·600	M.
Acid, tungstic		6·000	M.
Protoxide of copper		5·300	Boullay.
Deutoxide of copper		6·130	Id.
Protosulphuret of copper		5·690	M.
Deutoxide of tin		6·700	M.
Protosulphuret of tin		5·267	Boullay.
Bisulphuret of tin		4·415	Id.
Protoxide of lead (cast)		9·500	Id.
Peroxide of lead		9·200	Id.
Iodide of lead		6·100	Id.
Seleniuret of lead		7·690	M.
Sulphuret of lead (Galena)		7·580	M.
Oxide of zinc		5·600	Boullay.
Sulphuret of zinc (blende)		4·160	M.

* M. indicates the numbers taken from the "Traité de Minéralogie" of Beudant. The mean has generally been taken.

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Names.		Specific Gravity.	Observers.
de of iron		5·225	Boullay.
tic oxide of iron		5·400	Id.
urets of iron	Bisulphuret of iron (pyrites)	5·000	M.
	Id. (white pyrites)	4·840	M.
	Magnetic pyrites	4·620	M.
ide of manganese		4·480	Boullay.
ioxide of manganese		4·810	M.
xide of manganèse		4·722	M.
sulphuret of manganese		3·950	M.
ide of titanium (rutiline)		4·250	M.

785. *Simple Salts.*

Names.		Specific Gravity.	Observers.
nate of lime	Iceland spar	2·728	Malus.
	Arragonite	2·946	Thenard.
nate of magnesia (giobertite)		2·880	M.
nate of iron (iron spar)		3·850	M.
nate of manganese		3·550	M.
nate of zinc		4·500	M.
nate of baryta		4·300	M.
nate of strontia		3·650	M.
nate of lead (white lead)		6·730	M.
ate of baryta (heavy spar)		4·700	M.
ate of strontia (celestine)		3·950	M.
ate of lead		6·300	M.
ate of silver		5·340	Karsten.
hates of lime	Anhydrite	2·900	M.
	Gypsum	2·330	M.
hate of potassa		2·400	M.
drous sulphate of soda		2·630	Karsten.
mate of potassa		2·700	Kopp.
mate of lead (native)		6·600	M.
te of potassa		1·930	M.
te of baryta		3·185	Karsten.
te of strontia		2·890	Id.
te of lead		4·400	Id.
bdate of lead		6·700	Gmelin.
gstate of lead		8·000	Id.
gstate of lime		6·000	Karsten.
ninate of magnesia (Spinelli)		3·700	M.
ninate of zinc (spinel. zinc.)		4·700	M.
ate of zirconia (zircon)		4·400	M.
te of magnesia (Boracite)		2·500	M.

786. *Compound Minerals.*

Names.	Specific Gravity.	Observers.
Emerald	2·700	M.
Garnet	3·350 to 4·240	M. M.
Mesotype	2·250	M.
Idocrase	3·000 to 3·400	M. M.
Epidote	3·300 to 3·400	M. M.
Triphane	3·190	M.
Chabasite	2·700	M.
Amphigene	2·450	M.
Feld-spar Orthose Albite	2·400 to 2·600	M. M.
Stilbite	2·160	M.
Tourmaline	3·400	M.
Axinite	3·210	M.
Lazulite	2·900	M.
Ilvait (Lievrite)	4·000	M.
Calamine	3·400	M.
Crysocale	2·150	M.
Peridot	3·400	M.
Serpentine	2·470	M.
Steatite	2·800	M.
Magnesite (écume de mer)	2·500	M.
Pyroxene Diopside	3·300	M.
Hedenbergite	3·150	M.
Hypersteno	3·380	M.
Amphibole Tremolite	3·000	M.
Actinote	2·300	M.
Dolomite	2·800	M.
Malachite	3·500	M.
Streaked copper	5·000	M.
Copper pyrites	4·160	M.
Red silver	5·800	M.
Bournonite	5·700	M.
Grey copper	4·300 to 5·000	M. M.
Grey nickel	6·100	M.
Grey cobalt	6·290	M.
Arsenical iron (mispikel)	6·120	M.
Alunite	2·690	M.

Names.	Specific Gravity.	Observers.
Alum	1·700	M.
Muriated lead (kerasine)	6·000	M.
Atakamite (muriated copper)	4·430	M.
Cryolite	2·900	M.
Topaz	3·500	M.
Tellure sélenié bismuthefère	7·800	M.
Tellure auro-plombifère	9·220	M.
Appatite (chloro-phosphated lime)	3·250	M.
Pyro-morphite (chloro-phosphated lead)	7·010	M.
Blue phosphated iron	2·660	M.
Uranite	3·100	M.
Mercure argental	14·100	M.
Sphene	3·600	M.
Wolfram	7·300	M.

787. *Various Substances.*

Names.		Specific Gravity.	Observers.
Graphite (the most dense)		2·500	M.
Bituminous coal		1·250	M.
Anthracite		1·800	M.
Compact coal		1·380	M.
Charcoal in powder		1·500	Rumfort.
Charcoal in pieces, made from	**Walnut**	·625	Marcus Bull.
	White oak, chestnut	·421	Id.
	American ash	·547	Id.
	Beech	·518	Id.
	Horn-beam	·455	Id.
	Wild apple	·455	Id.
	Sassafras	·427	Id.
	Virginian cherry-tree	·411	Id.
	American elm	·357	Id.
	Virginian cedar	·238	Id.
	Yellow pine	·333	Id.
	Birch	·364	Id.
	American chestnut	·279	Id.
	Italian poplar	·245	Id.
Ligneous fibre		1·460 to 1·530	Rumford.

Names.		Specific Gravity.	Observers.
Woods...	Pomegranate-tree	1·350	Brisson.
	Lignum vitæ, ebony	1·330	Id.
	Dutch box	1·320	Id.
	Heart of oak of 60 years old	1·170	Id.
	Medlar	·940	Id.
	Olive	·920	Id.
	French box	·910	Id.
	Spanish mulberry	·890	Id.
	Beech	·852	Id.
	Ash	·845	Id.
	Yew	·807	Id.
	Elm	·800	Id.
	Apple-tree	·733	Id.
	Orange-tree	·705	Id.
	Yellow fir	·657	Id.
	Lime	·604	Id.
	Cypress	·598	Id.
	Cedar	·561	Id.
	White Spanish poplar	·529	Id.
	Sassafras	·482	Id.
	Common poplar	·383	Id.
	Cork-tree	·240	Id.
Yellow amber		1·080	
Oriental ruby		4·280	
Oriental sapphire		3·990	
Brazilian sapphire		3·130	
Oriental topaz		4·000	
Saxon topaz		3·560	
Oriental beryl		3·540	
English flint-glass		3·330	
Glass of St. Gobain		2·380	
Jasper, onyx		2·800	
Pearls		2·750	
Coral		2·680	
Chinese porcelain		2·380	
Porcelain clay		2·210	
Porcelain of Sèvres		2·310	
Silex meulière		2·480	
Flint		2·600	
Porphyry		2·670 to 2·750	
Granite		2·650 to 2·750	
Slate		2·810 to 2·850	
Plaister stone		2·200	

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Names.	Specific Gravity.	Observers.
n marbles	2·650 to 2·750	
, Parian	2·830	
Carrara	2·720	
g stone, large	1·700 to 1·900	
one	2·250 to 2·450	
	2·450 to 2·850	
n	2·300	
stone	2·320	
er	2·700	
	8·300	
r	7.180	
for statues and tam tam	8·950	
tal	8·460	
rs' solder	9.550	
Tuterrague	8.480	
	0·865	

BOOK THE SEVENTH.

THEORY OF UNDULATION.

CHAPTER I.

PRELIMINARY PRINCIPLES AND DEFINITIONS.

A VAST mass of discoveries produced by the labour of modern enquirers in several branches of physics, and more especially in those where the phenomena of sound, heat, light, and the imponderable agents generally are investigated, have conferred upon the subject consigned to the present book much interest and importance.

788. *Tendency to oscillate round a position or state of stable equilibrium.* — When a mass of matter, whatever be its form or conditions, being in a state of stable equilibrium, is disturbed, either collectively or in the internal arrangement of its constituent parts, by any external force which operates for a moment, such body will have a tendency to return to the state from which it was disturbed, and will so return, provided the disturbing force have not permanently deranged its structure. After it has returned to the position of equilibrium, it will have a tendency, by reason of its inertia, to depart from such position again, and to make an excursion in a contrary direction, and so continually to pass on the one side and the other of this position, with an alternate motion more or less rapid, until, at length, by the resistance of the medium in which it is placed, and other causes, it is gradually brought to rest, and settles finally in its previous position of stable equilibrium.

Alternate motions, thus produced and continued, are variously expressed by the terms vibrations, oscillations, waves, or undulations, according to the state and form of the body in which they take place, and to the character of the motions which are produced.

One of the most familiar and generally known examples of this class of motion has already been noticed in the case of the pendulum. There the oscillation is produced by the alternate displacement of the entire mass of the body, which partakes in the common motion of vibration.

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ı. ***Formation of waves or undulations.***—It does not always , however, that the particles of the vibrating body thus share ommon motion. If an elastic string be extended between two points, and be drawn laterally from its position of rest by a ıpplied at its middle point, it will return to that position of rest ıss beyond it, and will thus alternately oscillate on the one side n the other of such a position. In this case the oscillatory ı bears a close analogy to that of the pendulum, as will be fully noticed hereafter.

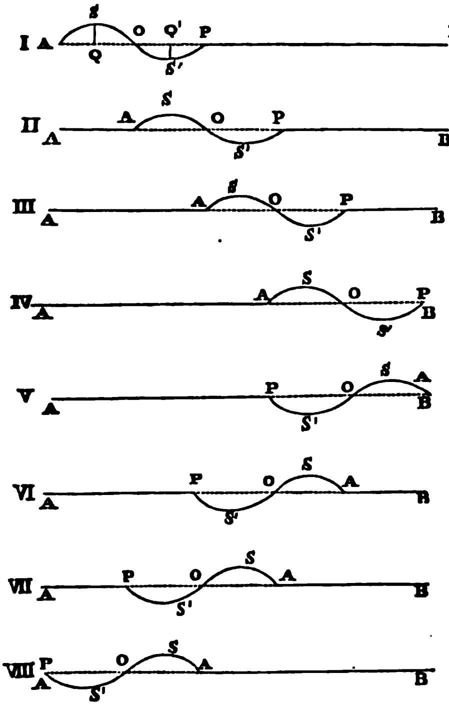

Fig. 227.

ı A B, *fig.* 227. I, be a flexible cord attached to a fixed point at l held by the hand at A. If this cord be jerked smartly once ıce up and down by the hand at A, it will immediately change rm, and an apparent movement will be produced, passing from nd A towards the end B, similar to that of waves upon water. irst effect of the motion will be to cause the cord to assume the curved form A S O, rising above the position of equilibrium. This will be succeeded by a corresponding curved form O S′ P, depressed to the same extent below the position of equilibrium. If the cord be jerked but once, then the point O will appear to advance towards B, the elevation A S O following it, and the depression of O S′ P preceding it, so that the appearances produced successively by the cord will be those represented in *fig.* 227. II. III. IV.

The curve A S O S′ P is called *a wave*.

The curve A S O, which rises above the position of equilibrium, is called the *elevation of the wave*, S being the summit or point of greatest elevation.

The curve O S′ P is called the *depression of the wave*, the point S′ being that of greatest depression.

The distance S Q of the highest point above the position of equilibrium is called the *height of the wave*; and in like manner the distance S′ Q′ of the lowest point of the depression below the position of equilibrium is called the *depth of the wave*.

The distance A P between the beginning of the elevation and the end of the depression is called the *length of the wave*; the distance A O the *length of the elevation*, and O P that of the *depression*.

It is found that such a wave, on arriving at the extremity B, as represented in IV, will return from B to A, as represented in V, VI, VII, VIII, in the same manner exactly as it had advanced from A to B.

Having thus returned to A, it will begin another movement towards B, and so proceed and return as before.

790. *Undulations progressive and stationary.* — A wave which thus moves in some certain direction, is called a *progressive undulation*.

Let a cord be extended between two fixed points, A and B, *fig.* 228., and let it be divided into any number of equal parts, three for example, at C and D. Let the points C and D be temporarily fixed, and let the three parts of the cord be drawn from their position of rest in contrary directions, so that the cord will assume the undulating form represented in the figure. If the parts of the cord thus drawn from their position of equilibrium be simultaneously discharged, each part will vibrate between the fixed points, the adjacent vibrations being always in contrary directions.

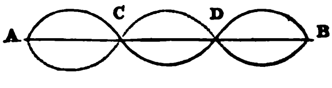

Fig. 228.

Now let the fixed points C and D be removed, so as to leave the cord free. No change will then take place in the vibratory motion of the cord, and it will therefore alternately throw itself into the positions represented in the fig. by the continuous line and the dotted line. But as it continues to vibrate, the parts C and D, although free, will be stationary, and waves will be formed, whose elevation

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and depression will be alternately above and below the lines joining the points A, C, D, and B.

Such an undulation not having any progressive motion, is accordingly called a *stationary undulation.*

The points C and D of the wave, which never change their position, are called ***nodal points*** or ***nodes.***

This species of undulation may be considered to be produced by the alternate elevation and depression of the several parts of the cord above and below its position of equilibrium.

As the circumstances attending, and the laws which govern, the vibrations or undulations of bodies vary with the state in which they are found, according as they are solid, liquid, or gaseous, it will be convenient to consider such effects as exhibited in these states severally.

CHAP. II.

UNDULATION OF SOLIDS.

791. *Vibrations of cords and membranes.* — Solid bodies exhibit the phenomena of vibration in various forms and degrees, according to their figure and to the degree of their elasticity. Cords and wires have their elasticity developed by tension. The same may be said of bodies which have considerable superficial extent with little thickness, such as thin membranes like paper or parchment.

When these are stretched tight and struck, they will vibrate on the one side and on the other of their position of equilibrium, in the same manner as a stretched cord.

Elastic substances, whatever be their form, are susceptible of vibration, the manner and degree of this varying in an infinite variety of ways, according to the form of the body and to the manner in which the force disturbing this form and producing the vibration is applied.

792. *Vibrations of cords or wires transverse, longitudinal, or torsional. — Apparatus of Prof. August.* — Those solids whose breadth or thickness is very small in proportion to their length, such as thin rods, cords, or wires, are susceptible of three kinds of vibration, which have been denominated the transverse, the longitudinal, and the torsional.

An apparatus to exhibit these effects experimentally, contrived by Professor August, is represented in *fig.* 229. This apparatus consists of a piece of brass wire formed into a spiral, one end of which is attached to a frame from which it is suspended, and the other end supports a weight by which it is strained. The transverse vibrations are produced by fixing the lower end of the wire

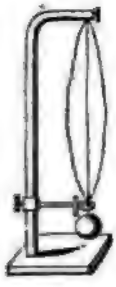

Fig. 229.

by means of the movable clamp represented in *fig.* 229. The wire is then drawn aside from its position of equilibrium and suddenly let go, after which it vibrates on the one side and on the other of this position.

To show the longitudinal vibrations, the weight suspended from the wire is drawn downwards by the hand, the wire yielding in consequence of its spiral form. When the weight is disengaged, the wire draws it up, the spiral elasticity being greater than the weight. The weight, however, rises in this case above the position of equilibrium, then falling returns to it; but in consequence of its inertia descends below it, and thus alternately rises above and falls below this position, until at length it comes to rest.

The torsional vibrations are shown by turning the weight round its vertical diameter. When so turned and let go, it will turn back again until it attains its position of equilibrium; but by reason of its inertia it will continue to turn beyond that position until stopped by the resistance of the wire, when it will return, and thus alternately twist round in the one direction and in the other, until it comes to rest

793. *Vibrations of an elastic string.* — Of the various forms of solid bodies susceptible of vibration, that which is attended with the greatest interest and importance is an extended cord; inasmuch as it not only produces the phenomena in such a manner and form as to render the laws which govern them more easily ascertained, but also constitutes the principle of an extensive class of musical instruments, and is therefore of high importance in the theory of musical sounds.

Let A B, *fig.* 230., be such an extended string. If it be drawn aside at its middle point C from its position of equilibrium, so as to

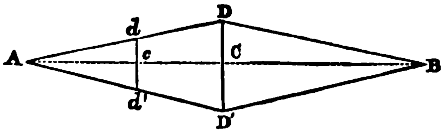

Fig. 230.

be bent into the form A D B, and then disengaged, it will in virtue of its elasticity return to the position A C B; the point D approaching C with an accelerated motion, exactly in the same manner as the ball of a pendulum approaches the centre point of its vibration. Having arrived at the position A C B, the string in consequence of its inertia will be carried beyond that position, and will arrive at a position A D' B on the other side of A C B, nearly at the same distance as A D B was. The motion of the middle point C from C to D' is gradually retarded, until it entirely ceases at D', precisely similar to the motion of the ball of a pendulum in ascending from the middle point to the extreme limit of its vibration. All these observations will be equally applicable to any other point of the string, such as c, which oscillates in like manner between the points d and d'. All the circumstances which were explained in the case of the pendulum, and which showed that the oscillations, whether made through longer or shorter arcs, were made in the same time, are equally applicable to this case of a vibrating string. Thus, the force which impels any point, such as D, towards the line A B, increases as the distance of D from the line A B increases. Therefore, the greater the extent of the excursion which the string has to make, the greater in proportion will be the force which will impel it; and consequently, the time of vibration will be the same although the amplitude of the vibrations be greater. It is, therefore, the general property of all extended strings, when put in vibration, that they will oscillate on either side of their position of rest in equal times, whether the amplitude of the vibrations is great or small. It follows from this, that the time of oscillation will be the same during the continuance of the vibration of the same string, although the amplitude of the oscillations it performs be continually diminished.

These observations, with the necessary qualifications, are applicable to all vibrating bodies. In all cases, the force tending to bring them back to the position of equilibrium is great in proportion to the extent of their departure from it; and consequently, the time of oscillating on either side of their position of equilibrium will be the same, although the amplitude of each oscillation is variable.

794. *General laws affecting them.* — The following laws which govern the vibration of strings have been demonstrated by theory and verified by experiment.

Let N express the number of vibrations per second which the string makes.

Let L express the length of the string.

Let S express the force with which the string is stretched.

Let D express the diameter of the string.

I. *The number* N *will be inversely proportional to* L, *other things being the same.* — That is to say, the number of vibrations made by a string per second will be increased in the same proportion as the length of the string is diminished, and *vice versâ*, the tension of the string and its thickness remaining the same.

II. *The number* N *varies in the proportion of the square root of* S, *other things being the same.* — That is to say, the number of vibrations performed by a string per second will be increased in proportion to the square root of the force which stretches the string. If the string be extended by a four-fold force, the number of vibrations which it performs per second will be doubled; if it be extended by a nine-fold force, the number of vibrations it performs per second will be increased in a three-fold proportion, and so on.

III. *The number of vibrations performed per second is in the in-*

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verse proportion of the diameter of the string, other things being the same. — That is to say, if two strings of the same length and composed of the same material be stretched with the same force, one having double the diameter of the other, the latter will perform twice as many vibrations per second as the former.

The three preceding rules may be expressed in combination by the following formula:

$$N = a \times \frac{\sqrt{S}}{L D};$$

in which *a* is a number depending on the quality of the material of the string, and which will vary in the formula if two different strings be compared together.

It follows, from this formula, that

$$A = \frac{N L D}{\sqrt{S}}.$$

The constant number *a*, therefore, is found by dividing the product of the number of vibrations per second, the length of the string, and its thickness, by the square root of the force which stretches the string.

IV. *The numbers of vibrations of cords of different materials are in the inverse proportion of the square roots of their densities.* — That is to say, if we take, for example, a cord of copper whose density is about 9, and a catgut cord whose density is nearly 1, their diameters, tensions and lengths being equal, the number of vibrations of the copper cord will be to that of the catgut as 1 to 3.

The manner in which the preceding laws may be verified by experiment will be explained in the next Book, when we shall treat of the doctrine of *sound*.

795. *Vibrations of an elastic rod with one end fixed.*—If an elastic rod, fixed at one end and free at the other, be drawn aside from its position of equilibrium, and let go, it will vibrate, and its vibrations will be isochronous, for the reasons which have been already explained in a general manner. It is demonstrated by theory, and verified by experiment, that the number of vibrations per second made by such a rod will, other things being the same, be inversely as the square of its length. If, for example, two pieces of the same elastic steel were to be fixed in a vice at one end, the other ends being free, the number of vibrations performed by the one per second will be four, nine, or sixteen-fold the number of vibrations performed by the other, if the first be two, three, or four times shorter than the second.

The vibrations produced by elastic wires, fixed at one end, are not like the vibrations of a common pendulum, generally made in the same plane; in other words, the free extremity of the wire does not

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scribe a circular arc between its extreme positions. It appears to be npressed with, at the same time, two vibratory motions in planes at right ngles to each other, and moves in a curve produced by the composition f these motions. These effects are rendered experimentally appaent in a beautiful manner, by the following expedient. Let several lastic steel wires, knitting-needles, for example, be fixed at one end n a vice or in a board, and let small balls of polished steel, capable f reflecting light intensely, be attached to the vibrating ends. Each f these small polished balls will reflect to the eye a brilliant point, nd when they are set in motion this brilliant point will produce a ontinued line of light, in the same manner and upon the same priniple (which will be explained hereafter) on which the end of a lighted tick made rapidly to revolve appears one continued circle of light. Now, when the needles are put into a state of vibration, the brilliant oints will appear to describe a complicated curve, exhibited to the ye by an unbroken line of light reflected from the polished ball.

796. *Nodal points experimentally shown.*—Elastic rods are susceptible of the stationary undulations already described as well as trings. The nodal points in the one and the other can be ascertained xperimentally by placing the vibrating string or wire in a horizontal position, and suspending upon it light rings of paper. They will be thrown off so long as they rest upon any part of the string or wire xcept the node, but when they come to a node, they will remain there unmoved, although the vibration of the string or wire may continue.

This experiment may be easily performed upon a string stretched in a horizontal position. If such a string be taken between the fingers at two points, each distant by one-fourth of its length from the two extremities, and being drawn aside in opposite directions, be disengaged, it will vibrate with a stationary undulation, the nodal point being in the centre, and each half of the string vibrating independently of the other. If a light paper ring be suspended on such a string at the middle point, it will remain unmoved; but if drawn aside from the middle point, it will be thrown off and agitated until it returns to that point, where it will again remain at rest.

A solid, in the form of a thin elastic plate, made to vibrate, will always be susceptible of stationary undulations, and will have a regular series of nodal points. Such a plate may be considered as consisting of a series of rods or wires, placed in contact and connected together, and the series of their nodal points will form upon the plate a series of nodal lines.

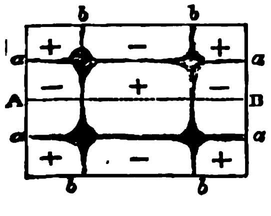

Fig. 231.

To render these nodal lines experimentally apparent, it is only necessary to spread upon the plate a thin coating of

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fine sand; when the plate is put into vibration, the sand will be thrown from the vibrating points, and will collect upon the nodal lines, and affect an arrangement, of which an example is given in *fig.* 231. This will be more fully explained hereafter when we treat of SOUND

CHAP. III.

UNDULATION OF LIQUIDS.

797. *Waves diverging round a centre.* — If a vessel containing a liquid remain at rest, the liquid being subject to no external disturbance, the surface will form a uniform level plane. Now, if a depression be made at any point of this surface by dropping in a pebble, or by immersing the end of a rod and suddenly withdrawing it, a series of circular waves will immediately be formed round the point where such depression is made as a centre, and each such wave will expand in a progressively increasing circle, wave following wave until they encounter the bounding sides of the vessel.

798. *Apparent progressive motion of the liquid deceptive.* — In this phenomenon a curious deception is produced. When we perceive the waves thus apparently advancing, one following another, we are irresistibly impressed with the notion that the fluid itself is advancing in the same direction; we consider that the same wave is composed of the same water, and that the entire surface of the liquid is in progressive motion. A little reflection, however, on the consequences of such a supposition will prove that it is unfounded. The ship which floats on the waves of the sea is not carried forward with them; they pass beneath her in lifting her on their summits, and in letting her sink into the abyss between them. Observe a sea-fowl floating on the water, and the same effect will be seen. If, however, the water itself partook of the motion of the waves, the ship and the fowl would each be carried forward with a motion in common with the liquid. Once on the summit of a wave, there they would constantly remain; or if once in the depression between two waves, they would likewise continue there, one wave always preceding and the other following them.

It is evident, therefore, that the impression produced, that the water is in progressive motion, is an illusion But, it may be asked, to what then does the progressive motion belong? That such a progressive motion does take place in something, we have proof from the evidence of sight; and that no progressive motion takes place in the *liquid* we have still more unquestionable evidence. To what, then, does the motion belong? We answer, to the form of the surface, and not the liquid composing it.

799. ***Progression of waves explained.*** — To render intelligible the manner in which the waves upon a liquid are produced, let A B C D, *fig.* 232., be a vessel containing a liquid whose surface when at rest is L L. Let us imagine a siphon M N O inserted in this vessel, filled with water to the same level as the vessel. It is evident that the water included within the siphon will hold the same position precisely as the water of the vessel which the siphon displaces. If we suppose a piston inserted in the leg M N to press down the water from the level L L to the depth D′, the water in the leg N O will rise to the height E. If the piston be suddenly withdrawn, the water in the leg M N will again rise, and the water in the leg N O will fall, the surfaces D′ and E will return to the common level L L, but they will not remain there, for, in consequence of the inertia, the ascending motion of the column D and the descending motion of the column E will be continued, so that the surface D′ will rise above L L, and the surface E will fall below it, and having attained a certain limit, they will again return respectively to the level L L, and oscillate above and below it until, by friction and atmospheric resistance, they are brought to rest at the common level L L.

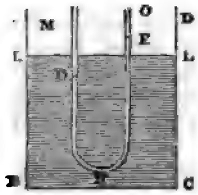

Fig. 232.

Now if we imagine the siphon to be withdrawn, so that the water which occupies its place may be affected by the same pressure at D′, the same oscillation will take place; but at the same time, the lateral pressure which is obstructed by the sides of the siphon will cause other oscillations, by the combination of which the phenomenon of a wave will be produced.

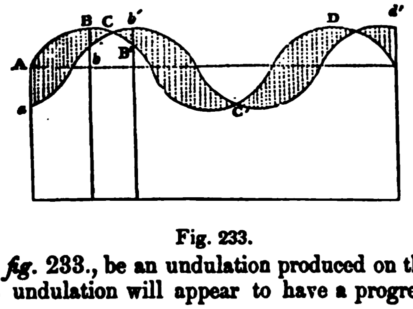

Fig. 233.

Let A B C D, *fig.* 233., be an undulation produced on the surface of a liquid. This undulation will appear to have a progressive motion from A towards X.

Let us suppose that in the interval of one second the summit of the wave B is transferred to *b*′. Now let us consider with what

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motion the particles forming the surface of the water are affected during this interval.

The particle at B descends vertically to *b*, while the particle B′ ascends vertically to *b*′. The several particles of the wave in the first position between B and C descend in the vertical lines represented by dotted lines in the figure to the several points of the surface between *b* and C. At the same time, the several points of the surface of the wave in its first position between C and B′ rise in vertical lines, and form the surface of the wave in its second position between C and *b*′.

In like manner, the particles of the wave in the first position between B′ and C′ rise in vertical lines, and form the surface of the wave in its new positions between *b*′ and C′.

In the same manner, during the same interval, the particles of liquid forming the surface B A descend in vertical lines and form the surface *b a*.

Thus it appears that in the interval of one second the particles of water forming the surface A B C *fall* in vertical lines, and those forming the surface C B′ C′ *rise* in vertical lines, and at the end of a second the series of particles form the surface *a b* C *b*′ C′.

In this manner, in the interval of one second, not only the crest of the wave is transferred from B to *b*′, but all the parts which form its profile are transferred to corresponding points holding the same relative position to the new summit *b*′. Thus we see that the *form* of the wave has a progressive motion, while the particles of water composing its surface have a vertical motion either upwards or downwards, as the case may be.

800. *Stationary waves explained.* — Hence it appears that each of the particles composing the surface of a liquid is affected by an alternate vertical motion. This motion, however, not being simultaneous but successive, an effect will be produced on the surface which will be attended with the form of a wave, and such wave will be progressive. The alternate vertical motion by which the particles of the liquid are affected will, however, sometimes take place under such conditions as to produce, not a progressive, but a stationary undulation. This would be the case if all the particles composing the surface were simultaneously moved upwards and downwards in the same direction, their spaces varying in magnitude according to their distance from a fixed point.

To explain this, let us suppose the particles of the surface of a liquid between the point *a e*, *fig.* 234., to be simultaneously moved in vertical lines upwards, the centre particle *c* being raised through a space than the particles contiguous to it on either side. The o which the other succeeding particles are raised will be ly diminishing, so that at the end of a second the particles which, when at rest, formed the surface *a e*, will form the rface *a b c d e*.

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In like manner, suppose the particles of the surface *e i* to be depressed in vertical lines, corresponding exactly with those through which the particles *a e* were elevated. Then the particles which originally formed the surface *e i* would form the curved surface *e f g h i*, and they would become the depression of a wave. Thus the elevation of the wave would be *a b c d e*, and its depression *e f g h i*.

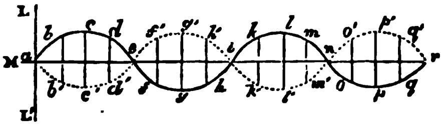

Fig. 234.

Having attained this form, the particles of the surface *a b c d e* would fall in vertical lines to their primitive level, and having attained that point, would descend below it; while the particles *e, f, g, h, i*, would rise to their primitive level, and having attained that position, would continue to rise above it. In fine, the particles which originally formed the surface of the undulation *a b c d e f g h i* would ultimately form the surface *a b' c' d' e f' g' h' i*, represented by the dotted line.

Having attained this form, the particles would again return to their primitive level, and would pass beyond it, and so on alternately.

In this case, therefore, there would be an undulation, but not a progressive one. The nodal points would be *a, e, i, n, r*, and these points during the undulation would not be moved; they would neither sink nor rise, the undulatory motion affecting only those between them.

This phenomenon of a stationary undulation produced on the surface of a liquid may easily be explained, by two systems of progressive undulation meeting each other under certain conditions, and producing at the points we have here called nodal points the phenomenon of interference, which we shall presently explain.

801. *Conditions under which a stationary undulation may be produced.* — Stationary undulations may be produced on a surface of liquid confined in a straight channel by exciting a succession of waves, separated by equal intervals, moving against the end or side of the channel, and reflected from it. The reflected waves, combined with the direct waves, will produce the effect here described.

It may also be produced by exciting waves in a circle from its central point. These waves being reflected from the circular surface, will produce another series, which, combined with the former, would be attended with the effect of a stationary undulation.

802. *Depth to which the effect of waves extend.* — When a system

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of waves is produced upon the surface of a liquid by any disturbing force, a question arises to what depth in the liquid this disturbance of equilibrium extends. It is possible to suppose a stratum of the liquid at any supposed depth below which the vertical derangement would not be continued. Such a stratum would operate as the bottom of the agitated part of the fluid.

The Messrs. Webber, to whose experimental inquiries in this department of physics, science is much indebted, have ascertained that the equilibrium of the liquid is not disturbed to a greater depth than about three hundred and fifty times the altitude of the wave.

803. *Reflection of waves.* — If a series of progressive waves impinge against any solid surface, they will be reflected, and will return along the surface of the fluid as if they emanated from a centre equally distant on the other side of the obstructing surface.

To explain this, it is necessary to consider that when any part of a wave encounters the obstructing surface, its progress is retarded, and the particles composing it will oscillate vertically in contact with the surface exactly as they would oscillate if they had at this point been first disturbed. They will therefore, at this point, become the centre of a new system of waves, which will be propagated around it, but which will form only semicircles, since the centre of undulation will be against the obstructing surface, which will, as it were, cut off half

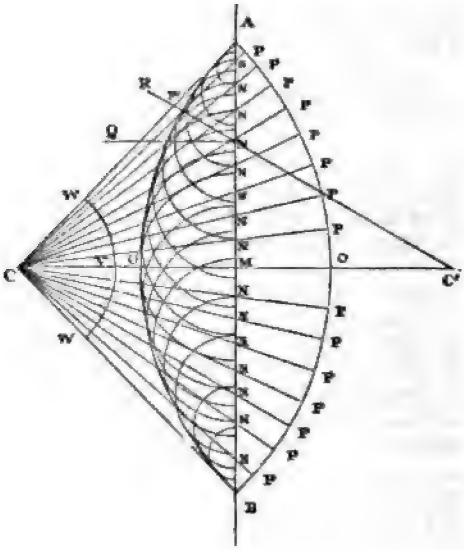

Fig. 235.

of each circular undulation. As the several points of the wave meet the obstructing surface in succession, other series of semicircular waves will be formed, and we shall see that by the combination of these various systems of semicircular waves, a single wave will be formed, the centre of which will be a point just so far on the other side of the obstructing surface as the original centre was on the side of the fluid.

Let C, *fig.* 235., be the original centre of undulation, and let a wave W W issuing from this centre move towards the obstructing surface A B. The first part of this wave which will meet the obstructing surface will be the point V, which moves along the line C M perpendicular to it. After this, the other points of the wave on the one side and on the other will successively strike it.

Let us take the moment at which the surface is struck at the points B and A equally distant from the middle point M by two parts of the wave. All the intermediate points between B and A will have been previously struck; amd if the wave had not been intercepted by the obstructing surface, it would at the moment at which it strikes the points B and A have had the form of the circular arc A O B, having the original point C as its centre.

But as the successive points of the wave strike the surface A B, they will, according to what has been explained, each become the centre of a new wave which will have a semicircular form; and to ascertain the magnitude of such wave at the moment the original wave strikes the points A and B, it is only necessary to ascertain the distance through which each semicircular wave will expand, and the interval between the moment at which the vertex of the original wave strikes the point M, and the moment at which the two extremities of the wave strike the points A and B. It is evident that if the wave had not been interrupted at M, its vertex would have moved on to O; and as the new wave reflected from M will have the same velocity, it follows that at the moment the original wave would have arrived at O, the reflected wave will have expanded through a semicircle whose radius is M O. Therefore, if we take the point M as a centre, and a line equal to M O as a radius, and describe a semicircle, this semicircle will be the position of the new wave formed with M as a centre at the moment that the extremities of the original wave struck the points A and B.

In like manner, it may be shown that if P be the position which the point of the original wave which struck N would have attained had it not been interrupted, the distance from which the semicircular wave having N as a centre would have expanded in the same time will be determined by describing a semicircle with N as a centre and N P as a radius. In the same manner it may be shown that the forms of all the semicircular waves produced with the points N of the obstructing surface between A and B as centres, will be determined by taking the several parts of the radii C P, which lie beyond the obstructing surface as radii, and the points N where they cross the obstructing surface as centres. This has been accordingly done in the diagram, by which it will be perceived that the space to the left of the obstructing surface is intersected by the numerous semicircular waves which have been formed. But it appears also that the series of points where they intersect each other most closely is that of a circular arc A O′ B, having for its centre the point O′ whose distance behind the surface M is equal to the distance of the centre C before it, so that C M shall be equal to C′ M. The effect will be, that a circular wave A O′ B will be be formed, the intersection of the semicircles within this being so inconsiderable as to be imperceptible. This wave A O′ B will accordingly expand from the surface A B towards C on the left in the same manner as the wave A O B would have expanded on the right towards C′, if it had not been interrupted by the obstructing surface.

If any radius of the original wave, such as C P, and the corresponding radius C′ P′ of the reflected wave be also drawn, these two radii will evidently make equal angles with the line C M C′ which is perpendicular to the obstructing surface; and consequently, if from the point N a line N Q be drawn parallel to C M, and therefore perpendicular to A B, the lines C N and N R will form equal angles with it.

804. *Law of reflection—angles of incidence and reflection equal.* —The angle C N Q is called *the angle of incidence* of the wave, and the angle Q N R is called *the angle of reflection*; and hence it is established as a general law, that in the reflection of waves from any obstructing surface, the angle of incidence is equal to the angle of reflection, — a law which has already been shown to prevail when a perfectly elastic body is reflected by a perfectly hard surface.

When a wave strikes a curved surface, it will be reflected from it in a different direction, according to the point of the surface at which it is incident. It will be reflected from such point in the same direction as it would be if it struck a plane which coincides with the curved surface at this point.

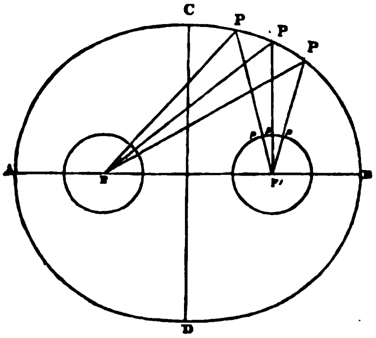

Fig. 236.

805. *Elliptic and parabolic curves.* — There are two species of curves, which in those branches of physics which involve the principles of undulation are attended with considerable importance. These figures are the ellipse and the parabola. *Fig.* 236. represents an ellipse: A B is its major axis, and C D its minor axis; F F′ are two points upon its major axis

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foci, which have the following property. If lines be drawn foci to any point P in the ellipse, these lines will form equal ith the ellipse at P, and their lengths taken together will be the major axis A B.

Waves propagated from the foci of an ellipse. — A remark- sequence of this property follows, relative to undulations hav- their centres one or other of the foci. If a series of pro- circular waves, propagated from the focus F as a centre, strike ace, they will be reflected from the surface at angles equal to which they strike it, because, by the law which has been stablished, the angles of reflection will be equal to the angles nce. If, then, we suppose several waves of the same system g from the focus F, to strike successively the elliptical surface at ts P, they will be reflected in the direction P F' towards the other But as all the points of the same wave move with the same they will describe equal spaces in the same time. Let the *p p* upon the lines P F' be those at which the points of the wave ve simultaneously. It then follows, that the lines F P and P *p* en together, be equal, being in each case the spaces described me time by different points of the same wave. If, then, these ngths F P *p* be taken from the lengths F P F', which are also each other, as has been already explained, the remainders F' *p* ssarily be equal; therefore the points *p* will lie at equal dis- om F', and will therefore form a circle round F' as a centre. e it follows, that each circular wave which expands round F will, after it has been reflected from the surface of the ellipse, form another circular wave round F' as a centre.

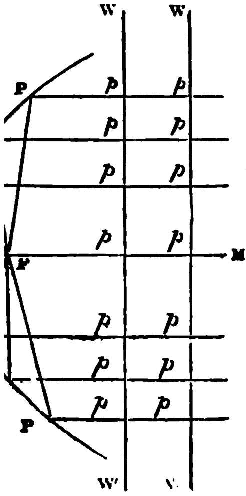

Fig. 237.

807. *Waves propagated from the focus of a parabola.* — The curve called a parabola is represented in *fig.* 237. The point V is its vertex, and the line V M is its axis.

A certain point F upon the axis near the vertex, called the focus, has the following property. Let lines be drawn from this point F to any points such as P in the curve; and let other lines be drawn from the points P severally parallel to the axis V M, meeting lines W W' drawn perpendicular to the axis, and terminated in the curve. The lines F P and P *p* will be inclined at equal angles to the curve at the points P, and the sum of their lengths will be everywhere the same; that is, if the length of the line F P be added to the length of the line P *p*, the same sum will be obtained whichever of the points P may be taken; and this will be the case whatever line W W′ be drawn perpendicular to V M.

It follows from this property, that if the focus of a parabola be the centre of a system of progressive waves, these waves, after striking the surface, will be reflected so as to form a series of parallel straight waves in the direction of the lines W W′, and moving from F towards M.

This may be demonstrated in precisely the same manner, as it has been proved in the case of the ellipse that the reflected waves form a circle round the focus F′; for the lines F P and *p* P, *fig.* 237., forming equal angles with the curve, will necessarily correspond with the direction of the incident and reflected waves, and the sum of these lines being the same wherever the point P may be situated, the several points of the same wave striking different points of the parabola will arrive together at the line W W′, inasmuch as they move with the same velocity, and have equal spaces to move over.

On the other hand, it follows, by precisely similar reasoning, that if a series of parallel straight waves at right angles to V M, moving from M towards V, should strike the parabolic surface, their reflections would form a series of circular waves of which the focus F would be the centre.

If two parabolas, A V B and A′ V′ B′, *fig.* 238., face each other so as to have their axes coincident and their concavities in opposite directions, a system of progressive circular waves issuing from one focus F, will be followed by a corresponding system, having for the

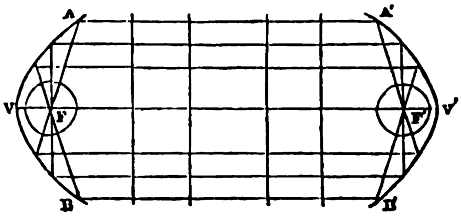

Fig. 238.

centre the other focus F′. The waves which diverge from F, after striking on the surface A V B, will be converted into a series of straight parallel waves moving at right angles to V V′, and towards V′. These will strike the surface A′ V′ B′, and after being reflected from it will form another series of circular waves, having the other focus F′ as their common centre.

A circular surface, if its extent be not great, compared with the length of its radius, may be considered as practically coinciding with

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a parabolic surface whose focus is at the middle point of the radius of the circular surface.

For example, let A B, *fig.* 239., be a circular arc, whose centre is C, and whose middle point is V. Let F be the middle point of the radius C V. Then A B may be considered as so nearly coinciding with a parabola whose focus is F, and whose vertex is V, that it will possess all the properties ascribed to the parabola; and consequently spherical surfaces, provided their extent be small compared with their diameters, will have all the properties here ascribed to parabolic surfaces.

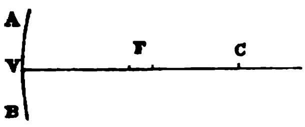

Fig. 239.

808. *Experimental illustrations of these principles.* — All these effects have been beautifully verified by experiment by means of expedients contrived by the Messrs. Webber, whose arrangements, nevertheless, for this object admit of still further simplification.

Experiment 1. — Let a trough of convenient magnitude be partially filled with mercury, so as to present a surface of that fluid of sufficient extent. Let a piece of writing-paper be formed into a funnel, with an extremely small opening at the point, so as to allow a minute stream of mercury to flow from it. Let a piece of sheet-iron, having a perfectly plane surface, be now immersed vertically in the mercury, and let a small stream descend from the funnel at any point upon the surface of the mercury in the vessel. A series of progressive circular waves will be produced around the point where the mercury falls, which will spread around it. This will strike the plane surface of the sheet iron, and will be reflected from it, forming another series of circular waves, whose centre will be a point equally distant on the other side of the sheet iron, as already described.

Experiment 2. — Let a piece of sheet-iron be bent into the form of an ellipse, such as that represented in *fig.* 236.; and let the position of the foci be indicated by a small wire index attached to it. Let this be immersed in the mercury in the trough; and let the funnel be brought directly over the point of the index which marks the position of one of the foci. When the mercury is allowed to fall, a series of circular waves will be produced round that focus, and, striking on the surface of the iron, will be reflected from it, forming another series of circular waves, of which the other focus is the centre as already expressed.

Experiment 3. — Let a piece of sheet-iron be bent into the form of a parabola, as represented in *fig.* 237., the position of the focus being, as before, marked by an index. If this be immersed in the mercury, and the stream be let fall from the funnel placed at the point of the index, a series of circular waves will be produced around the focus, which, after being reflected from the parabolic surface. will

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be converted into a series of parallel straight waves at right angles to its axis, as already explained.

Experiment 4. — Let two pieces of sheet-iron formed into parabolic surfaces, with indices showing the foci, be immersed in the mercury in such a position that their axes shall be in the same direction, and their concavities facing each other. From the funnel let fall a stream upon one focus F, *fig.* 238. Circular waves will be formed which, after reflection from the adjacent parabola, will become parallel waves, and after a second reflection from the opposite parabola will again become circular waves with the other focus as a centre.

Experiment 5. — If pieces of sheet-iron be bent into the form of small circular arcs whose length is small compared with their radius, the same effects will be produced as those which were produced by parabolic surfaces.

809. ***Phenomena produced when two systems of waves encounter each other.*** — When two waves which proceed from different centres encounter each other, effects ensue which are of considerable importance in those branches of physics whose theory is founded upon the principles of undulation.

I. If the elevation of one wave coincides with the elevation of another, and the depressions also coincide, a wave would be produced, the height of whose elevation, and the depth of whose depression, will be equal to the sum of the heights and depths of the elevation and depression of the two waves which are thus, as it were, superposed.

II. If, however, the elevation of one wave coincide with the depression of the other, and *vice versâ*, then the effect will be a wave whose elevation will be equal to the difference of the elevation, and whose depression will be the difference of the depression of the two waves which thus meet.

III. If, in the former case, the heights and depressions of the waves superposed be equal, the resulting wave will have double the height of the elevation, and double the depth of the depression.

IV. If the heights and depressions be equal in the second case, the two waves will mutually destroy each other, and no undulation will take place at the point in question; for the difference of elevations and the difference of depressions being nothing, there will be neither elevation nor depression.

In fact, in this latter case, the depression of each wave is filled up by the elevation of the other.

810. *Interference of waves.* — This phenomenon, involving the effacement of an undulation by the circumstance of two waves meeting in the manner described, is called in the theory of undulation an *interference*, and is attended with remarkable consequences in several branches of physics.

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811. *Experimental illustration of it.* — The two systems of waves formed by an elliptical surface, and propagated, one directly around one of the foci, and the other formed by reflection around the other, exhibit, in a very beautiful manner, the phenomena not only of reflection, as has been already explained, but also of interference, as has been shown with remarkable elegance by the Messrs. Webber already referred to. These phenomena are represented in *fig.* 240., where *a* and *b* are the two foci. The strongly marked circles indicate the elevation of the waves formed around each focus, and the more lightly traced circles indicate their depression. The points where the strongly marked circles intersect the more faintly marked circles, being points where an elevation coincides with a depression, are consequently points of interference, according to what has been just explained. The series of these points form lines of interference, which are marked in the diagram by dotted lines, and which, as will be seen, have the forms of ellipses and parabolas round the same foci.

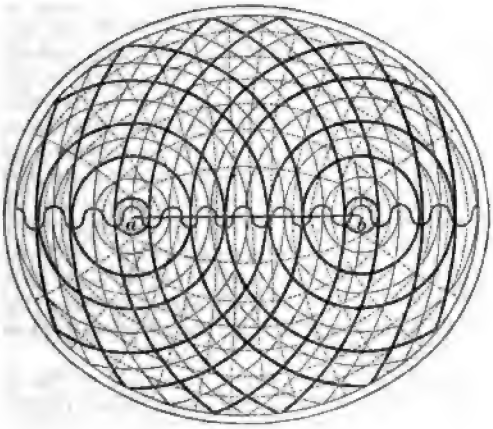

Fig. 240.

812. *Inflexion of waves.* — If a series of waves encounter a solid surface in which there is an opening through which the waves may be admitted, the series will be continued inside the opening, and without interruption; but other series of progressive waves having a circular form will be generated, having the edge of the opening as their centres.

Let M N, *fig.* 241., represent such a surface, having an opening whose edges are A and B, and let C be a centre from which a series of progressive circular waves is propagated. These waves, entering at the

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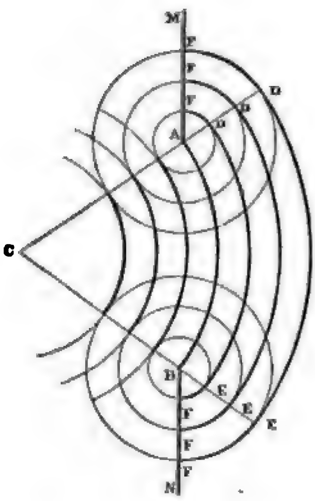

Fig. 241.

opening A B, will continue their course uninterrupted, forming the circular arcs D E. But around A and B as centres, systems of progressive circular waves will be formed which will unite with the waves D E, completing them by circular arcs D F and E F, meeting the obstructing surface on the outside; but these circular waves will also be formed throughout the remainder of their extent, as indicated in the figure, on both sides of the obstructing surface, and intersecting the original system of waves propagated from the centre C. They will also form, with these, series of points of interference according to the principles already explained.

The effects here described as produced by the edges of an opening through which a series of waves is transmitted are called *inflection*, and it will appear hereafter that they form an important feature in several branches of physics whose theory is based upon the principles of undulation.

813. *Undulation of the waters of the globe.*—The undulations produced upon a large scale in the oceans, lakes, rivers, and other large collections of water upon the surface of the globe, are attended with important effects on the economy of nature. Without these the ocean would be soon rendered putrid by the mass of organized matter which would be mingled with it, and which would chiefly float at its surface.

The principal physical cause which produces these undulations, where they take place on a moderate scale, is the motion of the atmosphere, but on a large scale they are produced by the combined effects of the attraction of the sun and moon exerted upon the surface of the ocean. The immense undulations excited by these attractions produce the phenomena of the tides, which will be explained more fully in a subsequent part of this work.

CHAP. IV.

UNDULATION OF ELASTIC FLUIDS.

If any portion of the atmosphere, or any other elastic fluid diffused through space, be suddenly compressed and immediately relieved from the compressing force, it will expand in virtue of its elasticity, and, like all other similar examples already given, will, after its expansion, exceed its former volume to a certain limited extent, after which it will again contract, and thus oscillate alternately on the one side and on the other of its position of repose.

814. *Undulations of a sphere of air.* — We may consider this effect to be produced upon a small sphere of air having any proposed radius, as, for example, an inch.

Let us suppose that it is suddenly compressed, so as to form a sphere of half an inch in radius, and being relieved from the compressing force it expands again, and surpassing its former dimensions swells into a sphere of an inch and a half. It will again contract and return to the magnitude of a sphere, with a radius somewhat greater than half an inch, and will again expand, and so oscillate, forming alternately spheres with radii less and greater than an inch, until at length the oscillation ceases, and it resumes permanently its original dimensions.

These oscillations will not be confined to the single sphere of air in which they commenced; the circumambient air will necessarily follow the contracting sphere when first compressed, so that a spherical shell of air which lies outside the sphere will expand and become less dense than in its state of equilibrium.

When the central sphere again expands, this external spherical shell will contract, and will become more dense than in its state of equilibrium. This shell will act in a similar manner upon another spherical shell outside it, and this upon another outside it, and so forth.

If then we suppose a number of successive spheres surrounding the point of original compression, we shall have a series of spherical shells of air, which will be alternately condensed and expanded in a greater degree than when in a state of repose.

This condensation and expansion thus spreading spherically round the original centre of disturbance, is in all respects analogous to a series of circular waves forming round the central point upon the surface of a liquid, the elevation of the wave in the case of the liquid corresponding to the condensation in the case of the gas, and the depression of the wave corresponding to the expansion of the gas.

815. *Analysis of the propagation of an undulation through an elastic fluid.* — We will limit our observations in the first instance to

a single series of particles of air, expanding in a straight line from the centre of disturbance A, *fig.* 242., towards T. Let S A represent the space through which the disturbing force acts, and let us imagine this air suddenly pressed from S to A by some solid surface moving against it, and let us suppose that this motion from S to A is made in a second. Now, if air were a body devoid of elasticity, and like a perfectly rigid rod, the effect of this motion of the solid surface from S to A would be to push the remote extremity T through a space to the right corresponding with and equal to S A.

But such an effect does not take place, first, because air is highly elastic, and has a tendency to yield to the force exerted by the solid surface upon it, while it moves from S to A; and secondly, because to transmit any effect from A to a remote point, such as T, would require a much greater interval of time than that which elapses during the movement of the surface from S to A. The effect, therefore, of the compression in the interval of time which elapses during the motion from S to A is to displace the particles of air which lie at a certain definite distance to the right of A. Let this distance, for example, be A B. All the particles, therefore, of air which lie in succession from A to B will be affected more or less by the compression, and will consequently be brought into closer contiguity with each other; but they will not be equally compressed, because to enable the series of particles of air lying between A and B to assume a uniform density requires a longer time than elapses during the motion of the solid surface from S to A. At the instant, therefore, of the arrival of the compressing surface at A, the line of particles between A and B will be at different distances from each other; and it is proved, by mathematical principles, that the point where they are most closely compressed is the middle point *m*, between A and B, and therefore, departing from this middle point *m*, in either direction, they are less and less compressed.

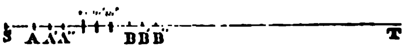

Fig. 242.

The condition, therefore, of the air between A and B is as follows. Its density gradually increases from A to *m*, and gradually decreases from *m* to B. Now, it is also proved that the effect of the elastic force of the air is such that, at the next moment of time after the arrival of the compressing surface at A, the state of varying compression which has been just described as prevailing between A and B will prevail between another point in advance of A, such as A', and a point B' equally in advance of B, and the point of the greatest compression will, in like manner, have advanced to *m'*, at the same distance to the right of *m*. In short, the conditions of the air between A' and B'

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ll be in all respects similar to its condition the previous moment tween A and B; and in like manner, in the next moment, the same ndition will prevail between the particles A'' and B'' to the right of and B'. Now, it must be observed that as this state of varying nsity prevails from left to right, the air behind it, in which it rmerly prevailed, resumes its primitive condition. In a word, the ate of varying density which has been described as prevailing tween A and B at the moment the compressing surface arrived at A ill, in the succeeding moments, advance from left to right towards T, id will so advance at a uniform rate; the distance between the points B, A' B', and A'' B'', &c. always remaining the same.

816. *Aërial undulations.*—This interval between the points A and is called a *wave* or *undulation*, from its analogy, not only in form at in its progressive motion, to the waves formed on the surface of quids, already described; the difference being, that in the one case e centre of the wave is the point of greatest elevation of the surice of the liquid, and in the other case it is the point of greatest ndensation or compression of the particles of the air. The distance etween A and B, or between A' and B', or between A'' and B'', which lways remains the same as the wave progresses, is called the *length f the wave.*

In what precedes we have supposed the compressing surface to adance from S to A, and to produce a compression of the air in advance f it. Let us now suppose this surface to be at A, the air contiguous it having its natural density.

If the wave *proceed contrariwise* from A to S, the air which was ontiguous to it at A will rush after it in virtue of its elasticity, so hat the air to the right of A will be disturbed and rendered less ense than previously. An effect will be produced, in fine, precisely ontrary to that which was produced when the wave advanced from S o A; the consequence of which will be that a change will be made pon the air between A and B exactly the reverse of that which was reviously made, that is to say, the middle point *m* will be that at rhich the rarefaction will be greatest, and the density will increase radually, proceeding from the point *m* in either direction towards the oints A and B.

The same observations as to the progressive motion will be appliable as before, only that the centre of the progression *m*, instead of eing the point of greatest condensation, will be the point of least ensity.

817. *Waves condensed and rarefied.* — The space A B is also in his case denominated a wave or undulation. But these two species f waves are distinguished one from the other by being denominated, he former a *condensed wave*, and the latter a *rarefied wave.* Now, et it be supposed that the compressing surface moves alternately backrards and forwards between S and A, making its excursions in equal

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times. The two series of waves, as already defined, will be produced in succession. While the condensed wave moves from S towards T, the rarefied wave immediately follows it, and in the same manner this rarefied wave will be followed by another condensed wave, produced by the next oscillation, and so on.

The analogy of these phenomena to the progressive undulation on the surface of a liquid, as already described, is obvious and striking.

What has been here described with reference to a single line of particles extending from the centre of the distance A in a particular direction, is equally applicable to every line diverging in every conceivable direction around such centre, and hence it follows that the succession of condensed and rarefied waves will be propagated round the centre, each wave forming a spherical surface, which is continually progressive and uniformly enlarges, the wave moving from the common centre with a uniform motion.

818. *Velocity and force of aërial waves.* — The velocity with which such undulations are propagated through the atmosphere depends on, and varies with, the elasticity of the fluid.

The degree of compression of the wave, which corresponds to the height of a wave in the case of liquids, depends on the energy of the disturbing force.

All the effects which have been described in the case of waves formed upon the surface of a liquid are reproduced, under analogous conditions, in the case of undulations propagated through the atmosphere.

819. *Their interference.* — Thus, if two series of waves coincide as to their points of greatest and least condensation, a series will be formed whose greatest condensation and rarefaction is determined by the sum of points, as prevailing in the separate undulations; and if the two series are so arranged that the points of greatest condensation of the one coincide with the greatest rarefaction of the other, and *vice versâ*, the series will have condensations and rarefactions determined by the difference of each of the separate series; and, in fine, if in this latter case the condensation and rarefactions be equal, the undulations will mutually efface each other, and the phenomena of interference, already described as to liquids, will be reproduced.

As the undulations produced in the air are spread over spherical surfaces having the centre of disturbance as a common centre, the magnitude of these surfaces will be in the ratio of the squares of their radii, or what is the same, of the squares of their distances from the point of central disturbance; and as the intensity of the wave is diminished in proportion to the space over which it is diffused, it follows that the effects or energy of these waves will diminish as the squares of their distances from the centre of propagation increase.

BOOK THE EIGHTH.

SOUND.

CHAPTER I.

PRODUCTION AND PROPAGATION OF SOUND.

Sound is the sensation produced in the organs of hearing when they are affected by undulations transmitted through the atmosphere around them. These undulations are subject to an infinite variety of physical conditions, and each variety is followed by a different sensation.

820. *Cause of sensation of sound.* — To investigate the manner in which the sensation of sound is produced by the vibrations imparted to the tympanum of the ear by the undulations of the atmosphere is the province of physiology; but to trace the connexion between the various sensations of which we are conscious, and the corresponding variety of physical conditions affecting the undulations of the air which produce them, is the proper business of physics.

The atmospheric undulations which thus produce the sensation of sound are themselves excited usually by the vibration of some elastic bodies, whose condition of equilibrium is momentarily disturbed, and which impart to the air in contact with them undulations which correspond with and are determined by such vibration.

The vibrating bodies which thus impart undulation to the air are called sounding or sonorous bodies, and the air is said to be a propagator or conductor of sound, and is sometimes called a soniferous medium.

The sounding body does not, however, invariably act in a direct manner upon the air which conveys the undulation to the organ of hearing. It often happens that the vibrations of the sounding body are first imparted to other bodies susceptible of vibration, and after passing through a succession of these, the undulation is finally imparted to the air, which is invariably the last medium in the series, and that from which the organ of hearing receives it.

821. *Presence of air necessary to the production of sound.* — That the presence of air or other conducting medium is indispensable for the production of sound, is proved by the following experiment.

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Let a small apparatus called an alarum, consisting of a bell, the tongue of which is governed by a string, be placed under the receiver of an air-pump, through the top of which a rod slides, air-tight, the end of the rod being connected with a detent which governs the motion of the tongue through the intervention of the string. This rod can, by a handle placed outside the receiver, be made to disengage the string, so as to make the bell within it ring whenever it is desired.

This arrangement being made, and the alarum being placed within the receiver, upon a soft cushion of wool, so as to prevent the vibration from being communicated to the pump-plate, let the receiver be exhausted in the usual way. When the air has been withdrawn, let the bell be made to ring by means of the sliding rod. No sound will be heard, although the percussion of the tongue upon the bell, and the vibration of the bell itself, are visible. Now if a little air be admitted into the receiver, a faint sound will begin to be heard, and this sound will become gradually louder in proportion as the air is gradually readmitted.

In this case the vibrations which directly act upon the ear are not those of the air contained in the receiver. These latter act upon the receiver itself and the pump-plate, producing in them sympathetic vibration; and those vibrations impart vibrations to the external air which are transmitted to the ear.

If in the preceding experiment a cushion had not been interposed between the alarum and the pump-plate, the sound of the bell would have been audible, notwithstanding the absence of air from the receiver. The vibration in this case would have been propagated, first from the bell to the pump-plate and to the bodies in contact with it, and thence to the external air.

822. *A continuous body of air not necessary.* — Persons shut up in a close room are sensible of sounds produced at a distance outside such room; and they may be equally sensible of these, even though the windows and doors should be absolutely air-tight. In such case the undulations of the external air produce sympathetic vibration on the windows, doors, or walls by which the hearers are enclosed, and then produce corresponding vibrations in the air within the room, by which the organs of hearing are immediately affected.

823. *Propagation of sound progressive.* — It has been shown in the last Book that the propagation of undulations through the atmosphere is progressive; and if it be admitted that such undulations are the agencies by which the sense of hearing is affected, it will follow that an interval of time, more or less, must elapse between the vibration of the sounding body and the perception of the sound by a hearer, and that such interval will be proportionate to the distance of the hearer from the sounding body, and to the velocity with which sound is propagated through the intervening medium. But this pro-

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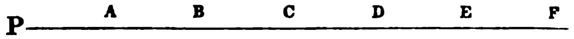
gressive propagation of sound can also be directly proved by experiment.

Let a series of observers, A, B, C, D, &c., be placed in a line, at distances of about 1000 feet asunder, and let a pistol be discharged at P, about 1000 feet from the first observer.



This observer will see the flash of the pistol about one second before he hears the report. The observer B will hear the report one second after it has been heard by A, and about two seconds after he sees the flash. In the same manner, the third observer at C will hear the report one second after it has been heard by the observer at B, and two seconds after it has been heard by the observer at A, and three seconds after he perceives the flash. In the same way, the fourth observer at D will hear the report one second later than it was heard by the third observer at C, and three seconds later than it was heard by the observer at A, and four seconds after he perceives the flash.

Now it must be observed, that at the moment the report is heard by the second observer at B, it has ceased to be audible to the first observer at A; and when it is heard by the third observer at C, it has ceased to be heard by the second observer at B, and so forth. It follows, therefore, from this, that sound passes through the air, not instantaneously, but progressively, and at a uniform rate.

824. *Breadth of sonorous waves.*—As the sensation of sound is produced by the wave of air impinging on the membrane of the ear-drum exactly as the momentum of a wave of the sea would strike the shore, it follows that the interval between the production of sound and its sensation is the time which such a wave would take to pass through the air from the sounding body to the ear; and since these waves are propagated through the air in regular succession, one following another without overlaying each other, as in the case of waves upon a liquid, the breadth of a wave may always be determined if we take the number of vibrations which the sounding body makes in a second, and the velocity with which the sound passes through the air. If, for example, it be known that in a second a musical string make 500 vibrations, and that the sound of this string take a second to reach the ear of a person at a distance of 1000 feet, there are 500 waves in the distance of 1000 feet, and consequently each wave measures two feet.

The velocity of the sound, therefore, and the rate of vibration, are always sufficient data by which the length of a sonorous wave can be computed.

825. *Distinction between musical sounds and ordinary sounds.* — It has not been ascertained, with any clearness or certainty, by what

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physical distinctions vibrations which produce common sounds or noises are distinguished from such as produce musical sounds. It is nevertheless certain, that all vibrations, in proportion as they are regular, uniform, and equal, produce sounds proportionably more agreeable and musical.

Sounds are distinguished from each other by their pitch or tone, in virtue of which they are high or low; by their intensity, in virtue of which they are loud or soft; and by a property expressed in French by the word *timbre*, which we shall here adopt in the absence of any English equivalent.

826. *Pitch of a sound.* — The pitch or tone of a sound is grave or acute. In the former case it is low, and in the latter high, in the musical scale. It will be shown hereafter that the physical condition which determines this property of sound is the rate of vibration of the sounding body.

The more rapid the vibrations are, the more acute will be the sound. A bass note is produced by vibrations much less rapid than a note in the treble. But it will also be shown that the length of the sonorous wave depends on the rate of vibration of the body which produces it: the slower the rate of vibration, the longer will be the wave, and the more grave the tone.

All vibrations which are performed at the same rate produced waves of equal length and sounds of the same pitch.

827. *Intensity or loudness.*— The intensity of a sound, or its degree of loudness, depends on the force with which the vibrations of the sounding body are made, and consequently upon the degree of condensation produced at the middle of the sonorous wave. Waves of equal length, but having different degrees of condensation at their centres, will produce notes of the same pitch, but of different degrees of loudness, in proportion to such degrees of condensation.

828. *Timbre of a sound.* — The timbre of a sound is not easily explained, and still less easily can the physical conditions on which it depends be ascertained. If we hear the same musical note produced with the same degree of loudness in an adjacent room successively upon a flute, a clarionet, and a hautboy, we shall, without the least hesitation, distinguish the one instrument from the other. Now this distinction is made by observing some peculiarity in the notes produced, yet the notes shall be the same, and be produced with equal loudness.

This property, by which the one sound is distinguished from the other, is called the *timbre*.

829. *All sounds propagated with the same velocity.* — All sounds, whatever be their pitch, intensity, or timbre, are propagated through the same medium with the same velocity. That this is the case, is manifest from the absence of all confusion in the effects of music, at whatever distance it may be heard. If the different notes simulta-

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neously produced by the various instruments of an orchestra moved with different velocities through the air, they would be heard by a distant auditor at different moments, the consequence of which would be, that a musical performance would, to the auditors, save those in immediate proximity with the performers, produce the most intolerable confusion and cacophony; for different notes produced simultaneously, and which, when heard together, form harmony, would at a distance be heard in succession; and sounds produced in succession will be heard as if produced together, according to the different velocities with which each note would pass through the air.

830. *Experiments on the velocity of sound.*—The velocity of sound varies with the elasticity of the medium by which it is propagated. Its velocity, therefore, through the air will vary, more or less, with the barometer and thermometer.

The experimental methods which have been adopted to ascertain the velocity of sound are similar in principle to those which have been briefly noticed by way of illustration. The most extensive and accurate system of experiments which have been made with this object, were those made at Paris by the Board of Longitude in the year 1822. The sounding bodies used on this occasion, were pieces of artillery charged with from two to three pounds of powder, which were placed at Villejuif and Montlhéry. The experiments were made at midnight, in order that the flash might be more easily and accurately noticed. They were conducted by MM. Prony, Arago, Mathieu, Humboldt, Gay Lussac, and Bouvard. The result of these experiments was, that when the barometer was at 29·8 inches, and the thermometer at 61°, the velocity of sound was 1118·39 feet per second.

By calculation it is ascertained, that at the temperature of 50°, the velocity would be 1106·58 feet per second; and at 32°, the velocity would be 1086·37 feet per second.

Thus it appears, that between 50° and 61°, the velocity of sound increases about 1·07 feet per second for every degree which the thermometer rises; and between 50° and 32° it increases at the mean rate of 1·12 feet per second for each degree in the rise of the thermometer.

831. *Method of estimating the distance of a sounding body by velocity of sound.*—The velocity of sound being known, the distance of a sounding body can always be computed by comparing the moment the sound is produced with the moment at which it is heard.

The production of sound is in many cases attended with the evolution of light, as, for example, in fire-arms and explosions generally, and in the case of atmospheric electricity. In these cases, by noting the interval between the flash and the report, and multiplying the number of seconds in each interval by the number of feet per second

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in the velocity of sound, the distance can be ascertained with great precision. Thus, if a flash of lightning be seen ten seconds before the thunder which attends it is heard, and the atmosphere be in such condition that the velocity of sound is 1120 feet per second, it is evident that the distance of the cloud in which the electricity is evolved must be 11,200 feet.

Among the numerous discoveries bequeathed to the world by Newton, was a calculation, by theory, of the velocity with which sound was propagated through the air. This calculation, based upon the elasticity and temperature of the air, gave as a result about one-sixth less than that which resulted from experiments.

This discrepancy remained without satisfactory explanation until it was solved by Laplace, who showed that it arose from the fact that Newton had neglected to take into account, in his computation, the effects of the heat developed and absorbed by the alternate compression and rarefaction of the air produced in the sonorous undulations. Laplace, taking account of these, gave a formula for the velocity of sound which corresponds in its results exactly with experiment.

832. *All gases and vapours conduct sound — experimental illustration.*—As all elastic fluids are, in common with air, susceptible of undulation, they are equally capable of transmitting sound.

This may be rendered experimentally evident by the following means. Let the alarum be placed under the receiver of an air-pump, as already described, and let the receiver be exhausted. If, instead of introducing atmospheric air into the receiver, we introduce any other elastic fluid, the sound of the alarum will become gradually audible, according to the quantity of such fluid which is introduced under the receiver. If a drop of any liquid which is easily evaporated be introduced, the atmosphere of vapour which is thus produced will also render the alarum audible.

833. *The intensity of a sound increases with the density of the propagating medium.*—The same sounding body will produce a louder or lower sound, according as the density of the air which surrounds it is increased or diminished. In the experiment already explained, in which the alarum was placed under an exhausted receiver, the sound increased in loudness as more and more air was admitted within the receiver. If the alarum had been placed under a condenser, and highly compressed air collected round it, the sound would be still further increased.

When persons descend to any considerable depth in a diving-bell, the atmosphere around them is compressed by the weight of the column of water above them. In such circumstances, a whisper is almost as loud as the common voice in the open air, and when one speaks with the ordinary force it produces an effect so loud as to be painful.

On the summit of lofty mountains, where the barometric column falls to one-half its usual elevation, and where therefore the air is highly rarefied, sounds are greatly diminished in intensity. Persons who ascend in balloons find it necessary to speak with much greater exertion, and as would be said louder, in order to render themselves audible. When Saussure ascended Mont Blanc, he found that the report of a pistol was not louder than a common cracker.

834. *Calm air favourable to the propagation of sound.*—Violent winds and other atmospheric agitations affect the transmission of sound.

When a strong wind blows from the hearer towards the sounding body, a sound often ceases to be heard which would be distinctly audible in a calm. A tranquil and frosty atmosphere placed over a smooth and level surface is favourable to the transmission of sound. Lieutenant Forster held a conversation with a person on the opposite side of the harbour of Port Bowen, in the third polar expedition of Sir Edward Parry, the distance between the speakers being more than a mile.

It is said that the sound of the cannon at the battle of Waterloo was heard at Dover, and that the cannon in naval engagements in the English Channel have been heard in the centre of England.

835. *Transmission of sound through liquids and solids.*—Liquids are also capable of propagating sound. Divers can render themselves audible at the surface of the water; and stones or other objects struck together at the bottom produce a sound audible at the surface.

It appears from the experiments of M. Colladon, made at Geneva, that sounds are transmitted through water to great distances with greater force than through air. A blow struck under the water of the Lake of Geneva, was distinctly heard across the whole breadth of the lake, a distance of nine miles.

Solid bodies, such as walls or buildings, interposed between the sounding body and the hearer, diminish the loudness of the sound, but do not obstruct it when the sound is made in air; but it appears from the experiments of M. Colladon, that the interposition of such obstacles almost destroys the transmission of sound in water.

836. *Interference of sonorous waves,—two sounds may produce silence.*—When two series of sonorous undulations propagated from different sounding bodies intersect each other, the phenomena of interference explained in the theory of undulation are produced, and an ear placed at such a point of interference will not be affected by any sense of sound, so long as the two sounding bodies continue to vibrate; but the moment the vibration of either of the two is discontinued, the other will become audible. Thus, it appears that two sounds reaching the ear together, instead of producing, as might be expected, a louder sound than either would produce alone, may altogether destroy each other and produce silence.

This phenomenon is precisely analogous to the case of two series of

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waves formed upon the surface of the same liquid, at a point where the elevation of a wave of one series coincides with the depression of a wave of the other.

If two sounding bodies were placed in the foci of an ellipse, as represented in *fig.* 236., an ear placed on any of the lines of interference there indicated would be conscious of no sound; but the moment that either of the two sounding bodies became silent, the other would be heard; or if the ear of the listener were removed to a position midway between two lines of interference, then both sounds would be heard simultaneously, and combined would be louder than either alone.

837. *Experimental illustration of interference of sound.* — This phenomenon of interference may be produced in a striking manner by means of the common tuning-fork, used to regulate the pitch of musical instruments.

Let A and B, *fig.* 243., be two cylindrical glass vessels, held at right angles to each other, and let the tuning-fork, after it has been put in vibration, be held in the middle of the angle formed by their mouths. Although, under such circumstances, the vibration of the tuning-fork will be imparted to the columns of air included within the two cylinders, no sound will be heard; but if either cylinder be removed, the sound will be distinctly audible in the other. In this case, the silence produced by the combined sounds is the consequence of interference.

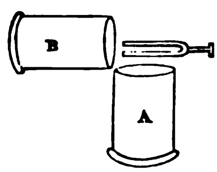

Fig. 243.

Another example of this phenomenon may be produced by the tuning-fork itself. If this instrument, after being put into vibration, be held at a great distance from the ear, and slowly turned round its axis, a position of the prongs will be found at which the sound will become inaudible. This position will correspond to the points of interference of the two systems of undulation propagated from the two prongs.

838. *Examples of sound propagated by solids.* — Solids which possess elasticity have likewise the power of propagating sound. If the end of a beam composed of any solid possessing elasticity be lightly scratched or rubbed, the sound will be distinct to an ear placed at the other end, although the same sound would not be audible to the ear of the person who produces it, and who is contiguous to the place of its origin.

The earth itself conducts sound, so as to render it sensible to the ear when the air fails to do so. It is well known, that the approach of a troop of horse can be heard at a distance by putting the ear to the ground. In volcanic countries, it is said that the rumbling noise which is usually the prognostic of an eruption is first heard by the

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beasts of the field, because their ears are generally near the ground, and they then by their agitation and alarm give warning to the inhabitants of the approaching catastrophe. Savage tribes are well known to practise this method of ascertaining the approach of persons from a great distance.

The velocity with which sound is propagated through different media varies with their different physical conditions.

In the following table are given the velocities with which sound is propagated through the several liquids therein named, the temperature being 50°.

TABLE.

839. Velocities of Sound in Liquids at 50° Fahr.

Names.	Specific Gravity.	Compressibility under one Atmosphere in Millionths of primitive Volume.	Velocity of Sound in Feet per Second.
Sulphuric ether	·712	131·35	3,409
Alcohol	·795	94·95	3,796
Hydrochloric ether	·874	84·25	3,834
Essence of turpentine	·870	71·35	4,186
Water	1·000	47·85	4,767
Mercury	13·544	3·38	4,869
Nitric acid	1·403	30·55	5,036
Water of ammonia (saturated)	·900	33·05	6,044

840. Table showing the Velocities of Sound as propagated by various solid Substances.

Names.	Velocities (that through Air being 1.)
Whalebone	6·66
Tin	7·50
Silver	9·00
Walnut Yew Brass Oak Plum-tree	10·66
Tobacco-pipes	10·00 12.00
Copper	12·00
Pear-tree Red-beech	12·50
Maple	13·33

Names.	Velocities (that through Air being 1).
Mahogany-wood	14·40
Ebony	
Horn-beam	
Elm	
Alder	
Birch	
Lime	15·00
Cherry	
Willow	16·00
Pine	
Glass	16·66
Iron	
Steel	

841. *Effects of elasticity of air.*—The velocity with which sound is transmitted through the air varies with its elasticity; and where different strata are rendered differently elastic by the unequal radiation of heat, the agency of electricity or other causes, the transmission of sound will be irregular. In passing from stratum to stratum differing in elasticity, the speed with which sound is propagated is not only varied, but the force of the intensity of the undulations is diminished by the combined effects of reflection and interference, so that the sound, on reaching the ear, after passing through such varying media, is often very much diminished.

The fact, that distant sounds are more distinctly heard by night than by day, may be in part accounted for by this circumstance, the strata of the atmosphere being during the day exposed to vicissitudes of temperature more varying than during the night.

842. *Biot's experiment on the relative velocities of sound in air and metal.*—The relative velocities of sound, as transmitted by air and by metal, are illustrated by the following remarkable experiment of Biot:—A bell was suspended at the centre of the mouth of a metal tube 3000 feet long, and a ring of metal was at the same time placed close to the metal forming the mouth of the tube, so that when the ring was sounded its vibrations might affect the metal of the tube, and when the bell was sounded its vibrations might affect only the air included within the tube. A hammer was so adapted as to strike the ring and the bell simultaneously. When this was done, an ear placed at the remote end of the tube heard the sound of the ring, and after a considerable interval heard the sound of the bell.

843. *Chladni's experiment on hearing.*—The solids composing the body of an animal are capable of transmitting the sonorous undulations to the organ of hearing, even though the air surrounding that organ be excluded from communicating with the origin of the sound.

Chladni showed that two persons stopping their ears could converse with each other by holding the same stick between their teeth, or by

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resting their teeth upon the same solid. The same effect was produced when the stick was pressed against the breast or the throat, and other parts of the body.

If a person speak, directing his mouth into a vessel composed of any vibratory substance, such as glass or porcelain, the other stopping his ears, and touching such vessel with a stick held between his teeth, he will hear the words spoken.

The same effect will take place with vessels composed of metal o wood.

If two persons hold between their teeth the same thread, stopping their ears, they would hear each other speak, provided the thread be stretched tight.

844. *Loudness dependent on distance.* — It has been shown that while the pitch of a sound depends upon the length of the sonorous wave, or, what is the same, the number of waves which strike the ear per second, the loudness depends on the degree of condensation or rarefaction produced in each such wave; but the loudness is also dependent on the distance of the hearer from the sounding body; and therefore, when it is stated that it is proportional to the condensation and rarefaction of the sonorous waves, the estimate must be understood to be applied to sounds heard at the same distance from their origin.

In explaining the general theory of undulations, it has been shown that as the undulation spreads round the centre from which it emanates, its intensity diminishes as the square of the distance is augmented; and this general principle consequently becomes applicable to sonorous undulations; and therefore, when other things are the same, the intensity or loudness of the sound diminishes in the same proportion as the square of the distance of the hearer from the sounding body is augmented. Thus in a theatre, if the linear dimensions be doubled, other arrangements being the same, the loudness of the performers' voices, as heard at any part of its circumference, will be diminished in a fourfold proportion.

CHAP. II.

VIBRATIONS OF MUSICAL SOUNDS.

845. *The monochord.* — Of the various forms of apparatus which have been contrived for the production of musical sounds with a view to the experimental illustration of their theory, that which is best adapted for this purpose are those which, under various denominations,

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consist of strings submitted to tension over a sounding-board. An instrument of this form, consisting of a single string, and called a *monochord* or *sonometer*, is represented in *fig.* 244. It consists of a

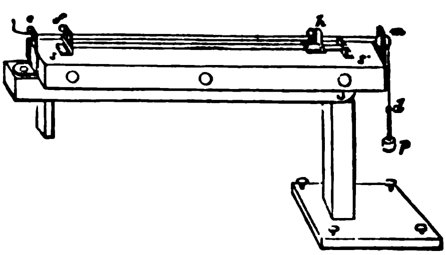
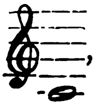


Fig. 244.

string of catgut or wire attached to a fixed point at *c*, carried over a pulley at *m*, and stretched by a known weight *p*. Under the string is a hollow box or sounding-board, to the frame of which the pulley *m* is attached. The string rests upon two bridges, one of which *f* is fixed, and the other *h* can be moved with a sliding motion to or from *f*, so as to vary at pleasure the length of the part of the string included between the two bridges.

A divided scale is placed under them, so that the length of the vibrating part of the string may be regulated at pleasure. By varying the weight *p*, the tension of the string may be increased or diminished in any desired proportion. This may be accomplished with facility by circular weights which are provided for the purpose, and which may be slipped upon the stem *d* of the weight *p*. By means of this apparatus, the relation between the various notes of the musical scale and the rate of vibration by which they are respectively produced, have been ascertained.

846. *Its application to determine the rates of vibrations of musical notes.* — It has been shown (794.) that the rate of vibration of a string such as that of the monochord is inversely as its length, other things being the same. Thus, if its length be halved, its rate of vibration is doubled; if its length be diminished or increased in a three-fold proportion, its rate of vibration will be increased or diminished in the same proportion; and so forth.

Let the bridges be placed at a distance from each other as great as the apparatus admits, and let the weight which stretches the string be so adjusted, that the note produced by vibrating the string shall correspond with any proposed note of the musical scale; such, for example, as , the low C of the treble clef. This being done, let the

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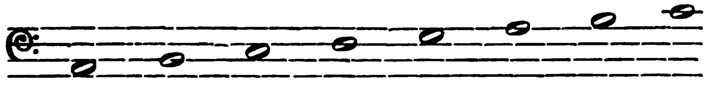
moveable bridge be moved towards the fixed bridge, continually sounding the string until it produces the octave above the note first sounded, that is, until it produces the middle C of the treble.

If the length of the string be now ascertained by reference to the scale of the monochord, it will be found to be precisely one-half its original length.

847. *A double rate of vibration produces an octave.* — Hence it follows, that the same string will sound an octave higher if the length is halved. But it has already been shown that the rate of vibration will be doubled when the length of the string is halved. Hence it follows, that two sounds, one of which is an octave higher than the other, will be produced by vibrations, the rate of which will be in the proportion of 2 to 1; and, consequently, the length of the undulation producing the lower note will be double that of the undulation producing the higher note.

848. *Rates of vibration for other intervals.* — If, instead of moving the bridge *h* to the point necessary to produce the octave to the fundamental note C, it be moved successively to such positions that the string shall produce the successive notes of the scale between it and its octave, the lengths of the string being noted by reference to the scale, it will be found that they will be respectively those which are inscribed below the annexed scale under the notes severally. The length of the string producing the fundamental note C is assumed to be 1, the fractions expressing, with reference to this length, the lengths which are found to produce the successive notes of the scale severally.

Let the seven successive notes of the gamut be expressed as follows: —



ut	re	mi	fa	sol	la	si	ut
C	D	E	F	G	A	B	C
1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

The names given by continental writers to these seven notes are those written beneath them in the upper line—ut, re, mi, fa, sol, la, si, ut; but those by which they are most generally known in England are the letters of the alphabet inscribed in the lower line, the fundamental note being C, and the succeeding ones designated by the letters inscribed beneath them.

Let us suppose, then, that the monochord produces this fundamental note C, and that the moveable bridge be then advanced towards the fixed bridge so as to shorten the string until it produces the note D.

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It will be found that its length will be reduced $\frac{1}{9}$th, and that, consequently, the length necessary to produce the note D will be $\frac{8}{9}$ths of that which produces the note C. Let the bridge be now advanced until the string sound the note E; its length will then be $\frac{4}{5}$ths of that which produces the fundamental note. In the same manner, being further shortened, let it produce the note F; its length will be $\frac{3}{4}$ths of its original length. In the same manner, the lengths of the string corresponding to each of the successive notes of the gamut, will be found to be expressed by the fractions which are written in the above diagram under the notes severally.

But since the number of vibrations per second is, by the principles already established, in the inverse ratio of the length of the string, it follows, that if the number of vibrations per second corresponding to the fundamental note C be expressed by 1, the number of vibrations per second corresponding to the other notes successively will be as follows:—

ut	re	mi	fa	sol	la	si	ut
C	D	E	F	G	A	B	C
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

The meaning of which is, that in producing the note D, nine vibrations will be made in the same time that eight are made by the note C. In like manner, when the note E is sounded, five of its vibrations correspond to four of C, four vibrations of F correspond to three of C, three vibrations of G correspond to two of C, five vibrations of A correspond to three of C, fifteen vibrations of B correspond to eight of C, and, in fine, two vibrations of the octave C correspond to one of the fundamental C.

The relative numbers corresponding to the notes of one octave being known, those of the octaves higher or lower in the musical scale can be easily calculated.

It appears from what has been already proved that the note which is an octave higher than the fundamental note is produced by a rate of vibration twice as rapid; and this principle would equally apply to any other note. We shall, therefore, always find the rate of vibration of a note which is an octave above a given note by multiplying the rate of vibration of the given note by 2; and consequently, to find the rate of vibration of a note an octave lower, it will only be necessary to divide the rate of vibration of the given note by 2. If, therefore, it be desired to find the rate of vibration of the series of notes continued upwards beyond the series given in the preceding diagram, it will only be necessary to multiply the numbers in the preceding series by 2.

849. *Physical cause of harmony.* — If these results be compared *with the* effect produced upon the ear by the combination of these

musical notes sounded in pairs, we shall discover the physical cause of those agreeable sensations denominated harmony, and the opposite sensations denominated discord.

The most perfect harmony is that of the octave, which is so complete as to be nearly equivalent to unison. Now the fundamental note C produced simultaneously with its octave is attended by two series of vibrations, of which two of the octave correspond to one of the fundamental note. It follows, therefore, that the commencement of every alternate vibration of the upper note coincides with the commencement of a vibration of the lower.

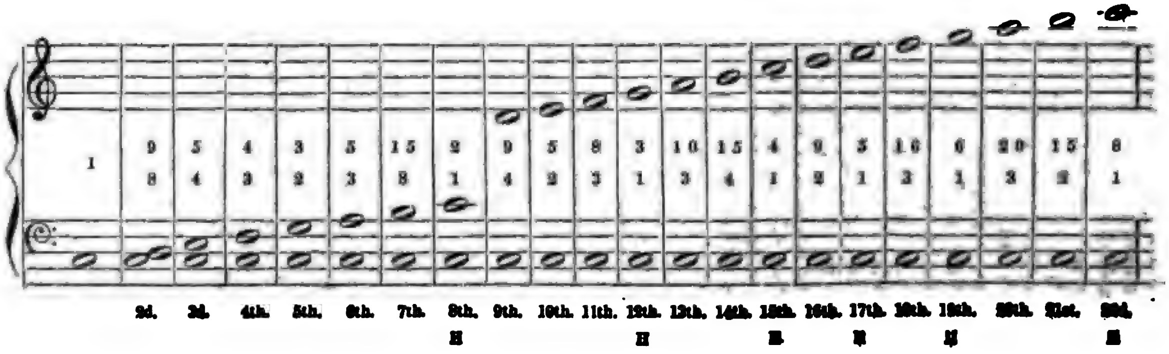
Next to the octave, the most agreeable harmony is that of the fifth, which is produced when the fundamental note C is sounded simultaneously with G. Now it appears by the preceding results that three vibrations of G are simultaneous with two of C. It follows, therefore, that every third vibration of G commences simultaneously with every second vibration of C. The coincident vibrations, therefore, are marked by the commencement of every second vibration of the fundamental C, whereas, in the octave, a coincidence takes place at the commencement of every vibration.

The coincidences, therefore, are more frequent in the octave than in the fifth, in the proportion of 1 to 2.

The next harmony to that of the fifth is the fourth, which is produced when the fundamental note C is sounded simultaneously with F. Now it appears from the preceding results that four vibrations of F are simultaneous with three of the fundamental note, and consequently that there is a coincident vibration at the commencement of every third vibration of the fundamental note. The coincident vibrations are therefore less frequent than in the fifth in the proportion of 3 to 2; and less frequent than in the octave in the proportion of 3 to 1.

The harmony which comes next in order to the fourth is that of the third, produced when the fundamental note C is sounded simultaneously with E. Now it appears from the preceding results that five vibrations of E are made simultaneously with four of C; and that consequently there is a coincidence at every fourth vibration of the fundamental note. The coincidences, therefore, in this case are less frequent than in the fourth, in the ratio of 3 to 4, less frequent than in the fifth in the proportion of 2 to 4, and less frequent than in the octave in the proportion of 1 to 4.

850. *Scale exhibiting the Effect of Binary Combinations of the Fundamental Note with a Series of Three Octaves continued severally upwards and downwards.*



The figures which are placed over each combination express the number of vibrations which in each case take place simultaneously, and the name of the interval, as it is technically called in music, is written under the lower line. Thus, the interval between the fundamental note C and the note B is a seventh; and the figures above indicate that fifteen vibrations of B are made in the same time as eight vibrations of C. In the same way, the interval between C and F in the treble is called an eleventh; and the figures indicate that eight vibrations of F are made while three of C take place.

851. *Physical cause of the harmonics of the harp or violin.* — On inspecting the numbers which in the preceding scale indicate the relative rates of vibration of these pairs of musical sounds, it will be observed that there are certain combinations in which a complete number of vibrations of the upper note are made in the time of a single vibration of the lower note. These are distinguished by the letter H written under the interval. The first is the octave, in which two vibrations of the upper note correspond to one of the lower; the second is the twelfth, in which three vibrations of the upper note correspond to one of the lower; the third is the fifteenth, in which four vibrations of the upper note correspond to one of the lower; the fifth is the nineteenth, in which six vibrations of the upper correspond to one vibration of the lower; and, in fine, the seventh is the twenty-second, in which eight vibrations of the upper correspond to one vibration of the lower.

These combinations (which possess other and important properties) are called *harmonics*.

One of the most remarkable properties of the harmonics is, that if the fundamental note be produced by sounding the open string, a practised ear will detect in the sound mingled with the fundamental, the several harmonics to it, and more especially those which are in nearest accord with the fundamental note. Thus the octaves will be produced; but these are so nearly in unison with the fundamental note that the ear cannot distinguish them. The twelfth, or that which has three vibrations for one of the fundamental note, is distinctly perceptible to common ears. The more practised can distinguish the seventeenth, or that which vibrates five times more rapidly than the fundamental; and some pretend to be able to distinguish the vibrations of the nineteenth, which vibrates six times for one of the fundamental note.

852. *Experimental verification by Sauveur.* — These phenomena have been explained and verified in a satisfactory manner by Sauveur, who showed that when a string is put into vibration it undergoes subordinate vibrations, which take place in its aliquot parts. Thus, if an edge touch the string gently, when in vibration, at its middle point, as represented in *fig.* 245., each half will continue to vibrate independently.

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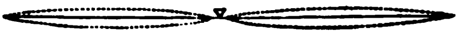

Fig. 245.

If the edge be in like manner applied at one-third of the length, the vibration will still continue, each third part vibrating independently of the other; and in fine, the condition of the entire string, when left to vibrate freely, is represented in *fig.* 246., where the subordinate vibrations produced in the aliquot parts of the string are represented.

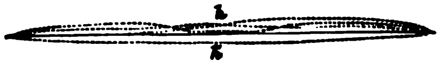

Fig. 246.

853. *Limit of the musical sensibility of the ear.*—Since the pitch of a musical note depends on the number of vibrations produced per second, it follows that whenever two notes are produced by a different number of vibrations per second, they will have a corresponding musical difference. Now a question arises as to the limits of the power of the ear to distinguish minute differences of this kind. For example, it may be asked whether two musical notes produced by vibrations differing from each other by only one in a million, that is to say, if, while one string make a million of vibrations, another string shall make a million and one, is the ear capable of perceiving that one note is more acute than the other? It is certain that no ear could discover such a difference, although it is equally certain that such a difference would exist. The question then is, what is the limit of sensibility of the ear?

If two strings of the same wire were extended by equal weights on the monochord, and the moveable bridges brought to coincide, so that the strings would be of precisely equal length, then it is certain that when struck they would produce the same note, since all the conditions affecting the vibration of the string would be identical. Now, if one of the bridges be moved slowly, so as gradually to lengthen the vibrating part of the string, the limit may be found at which the ear will begin to be sensible of the dissonance of the notes. The point thus determined may fix the limit of the sensibility of the ear.

The comparative lengths of the two strings in such a case would indicate the different rates of vibration of which the ear is sensible.

854. *Sensibility of practised organists.*—The result of such an experiment would of course be different for different ears, according

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to their natural sensibility, and to the effects of cultivation in improving their musical perception. Practised organists are able to distinguish between notes which differ in their vibrations to the extent of one in eighty.

Thus, if a string of the monochord have 20 inches between the bridges, and the other 20¼ inches, their rates of vibration being then in the proportion of 80 to 81, the difference would be distinguishable. Such an interval between two musical sounds is called a *comma.*

But when the differences of the rates of vibration are much less than this, they cannot be distinguished by the ear. The notes on common square pianos are each produced by two strings, and on grand pianos by three strings struck simultaneously by the same hammer. In tuning the instrument, these strings are tuned separately, until they are brought as nearly to the same pitch as the ear can determine. When struck together, however, a slight dissonance will in general be perceptible, which is adjusted by tuning one or the other until the sounds are brought into unison.

Since, however, such unison is only determined by the ear, and since the sensibility of that organ is limited, it follows that the unison thus obtained can never be perfect otherwise than by chance.

855. ***Methods of determining the absolute number of vibrations producing musical notes.*** — We have hitherto noticed only the relative rates of vibration of different musical notes. If the absolute number of vibrations per second, corresponding to any one note of the scale, were known, the absolute number of vibrations of all others could be computed. Thus, the note which is an octave higher than the note proposed, would be produced by double the number of vibrations per second; a note one-fifth above it would be produced by a number of vibrations per second found by multiplying the given number by 3 divided by 2, and so on. In a word, the number of vibrations per second necessary to produce any given note would be found by multiplying the number of vibrations per second necessary to produce the fundamental note by the fractions given in (849.) corresponding to the proposed note.

856. ***The Sirene of Cagniard de la Tour.*** — An instrument of great ingenuity and beauty, called the *Sirene,* has been supplied by the invention of M. Cagniard de la Tour, for the purpose of ascertaining the whole number of vibrations which correspond to any proposed musical sound. A tube of about four inches in diameter, represented at *f f'*, *fig.* 247., to which wind can be supplied by means of a bellows or otherwise through a pipe *y y'*, is terminated in a smooth circular plate *v v'*, stopping its end. In this plate, and near its edge, a number of small holes are pierced very close together, and disposed in a circular form, as represented in *fig.* 248., the perforations being made, not perpendicular to the plate, but in an oblique direction

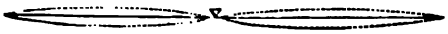


Fig. 245.

If the edge be in like manner applied at one-third of the length, the vibration will still continue, each third part vibrating independently of the other: and in fine, the condition of the entire string, when left to vibrate freely, is represented in *fig*. 246., where the subordinate vibrations produced in the aliquot parts of the string are represented.

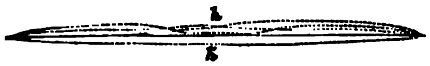


Fig. 246.

853. ***Limit of the musical sensibility of the ear.***—Since the pitch of a musical note depends on the number of vibrations produced per second, it follows that whenever two notes are produced by a different number of vibrations per second, they will have a corresponding musical difference. Now a question arises as to the limits of the power of the ear to distinguish minute differences of this kind. For example, it may be asked whether two musical notes produced by vibrations differing from each other by only one in a million, that is to say, if, while one string make a million of vibrations, another string shall make a million and one, is the ear capable of perceiving that one note is more acute than the other? It is certain that no ear could discover such a difference, although it is equally certain that such a difference would exist. The question then is, what is the limit of sensibility of the ear?

If two strings of the same wire were extended by equal weights on the monochord, and the moveable bridges brought to coincide, so that the strings would be of precisely equal length, then it is certain that when struck they would produce the same note, since all the conditions affecting the vibration of the strin [illegible] identical. Now, if one of the bridges be moved [illegible] to lengthen the vibrating part of the string, [illegible] at which the ear will begin to be sensible of [illegible] The point thus determined may fix [illegible] the ear.

The comparative length [illegible] indicate the different [illegible]

854. *Sensibility* [illegible] experiments [illegible]

to their natural [illegible] ing their musical [illegible] guish between [illegible] one in eighty [illegible]

Thus, if a string [illegible] bridges, and the [illegible] in the proportion [illegible] Such an interval [illegible]

But when the difference [illegible] than this, they [illegible] common [illegible] grand pianos [illegible] mer. In tuning the [illegible] until they are [illegible] termine. [illegible] general be perceptible [illegible] until the [illegible]

Since, however, [illegible] since the sensibility [illegible] a thus obtained [illegible] to

855. *Methods of* [illegible] p- *producing* [illegible] tive rates of [illegible] nd. number of vibrations per second [illegible] the the scale, were known, the [illegible] h is could be computed. Thus, [illegible] wn so the note proposed, [illegible] tions per second; [illegible] number of vibrations per second [illegible] ber by 3 divided by 2, and [illegible] tions per second necessary to produce [illegible] by multiplying the number of [illegible] duce the fundamental note by [illegible] sponding to the proposed note.

856. *The Siren of Cagniard* [illegible] great ingenuity and [illegible] the invention of M. C[illegible] [illegible] the whole [illegible]

[illegible] 5, consists of a [illegible] supporting a large [illegible] small grooved wheel [illegible] which is formed into

[redacted]

through it. Another plate of equal magnitude *u u'*, *fig.* 247., and having a circle of holes precisely similar, is fixed upon this so as to be capable of revolving with any required velocity round its centre. As it revolves, the holes in the upper plate *u u'* correspond in certain positions with the holes in the lower plate *v v'*; but in intermediate positions, the holes in the lower plate not corresponding with those in

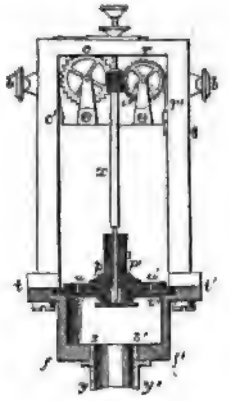


Fig. 247.

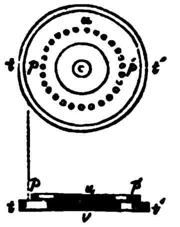


Fig. 248.

the other plate, the exit of the air from the tube *f f'* is stopped. If, then, we suppose the upper of these two plates to revolve upon the lower, a current of air being supplied to the tube *f f'* through *y y'*, the air will escape when the holes in the superior plate correspond in position with those in the lower plate, but in intermediate positions it will be intercepted. The effect will be, that when the superior plate moves with a uniform velocity, there will be a series of puffs of wind allowed to escape from the holes of the inferior plate through those of the superior plate in uniform succession with equal intervals of time between them. This succession of puffs will produce undulations in the air surrounding the instrument, and when their velocity is sufficiently increased, these undulations will produce a sound. If the motion be uniform, this sound will be maintained at a uniform pitch; but as the motion of the plate is increased, the pitch will become more elevated; and, in short, such a velocity may be given to the superior plate as to make the instrument produce a sound of any desired pitch, acute or grave.

A small apparatus is connected with the superior plate, by which its revolutions are counted and indicated. This apparatus consists of a *spindle*, *x*, *fig.* 247., which carries upon it a worm or endless screw,

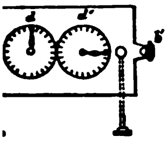


Fig. 249.

which drives the teeth of a small wheel *r*, which is connected by pinions and wheelwork with another wheel *c*. These wheels govern the motion of hands upon small dials *d d'*, *fig.* 249. These hands being brought to their respective zeros at the commencement of the experiment, their position at the end of any known interval will indicate the number of of air which have escaped from the holes of the revolving plate ı the interval, and will consequently determine the number of ations of the air which correspond to the sound produced.

7. *Experiments made with this instrument.* — Various series of sting experiments have been performed with this instrument by entor, which have shown that it not only indicates the pitch of ote produced, but also that the *timbre* of the sound has a rela-o the thickness of the revolving plate, and of the fixed plate which it turns, and with the space between the holes pierced in plates. These conditions, however, have not been investigated sufficient precision to supply any general principles. M. Cag-de la Tour thinks, nevertheless, that when the intervals between oles pierced in the plates are very small, the sound approaches to f the human voice, and when they are very considerable it ap-aes to that of a trumpet.

3. *Savart's apparatus for measuring the vibrations of sound.* nother instrument for the experimental determination of the er of vibrations corresponding to a note of any proposed pitch is o M. Savart, whose experimental investigations have thrown so light upon the physics of sound.

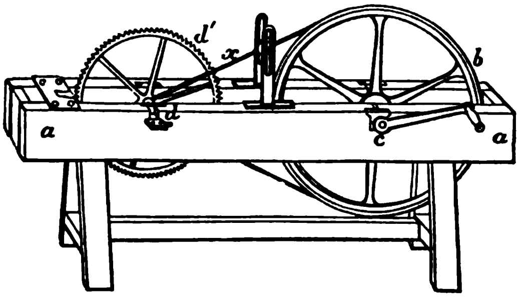


Fig. 250.

is apparatus, which is represented in *fig.* 250., consists of a ı *a a*, constructed in a very solid manner, supporting a large l *b*, connected by an endless band *x* with a small grooved wheel upon the axis of another large wheel *d'*, which is formed into

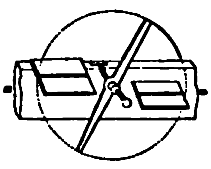


Fig. 251.

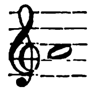
teeth at its edge. These teeth strike successively a piece of card or other thin elastic plate presented to them, and fixed upon the frame *a a*, as represented in *fig.* 250. The successive impulses given to the card produce corresponding undulations in the air, the effect of which is a musical sound.

The number of undulations per second thus produced in the air will correspond with the number of teeth of the wheel *d'* which pass the edge of the card in a second. Now, if the number of turns per second given to the primary wheel *b* be known, the relative magnitudes of this wheel and the small wheel attached to the axis of *d'* will determine the number of revolutions per second given to the wheel *d'*, and consequently the number of teeth of the latter, which, in a second, will strike the edge of the card. In this way, undulations of the air can be produced at the rate of 25,000 per second.

The sounds produced by this means are said to be clear, continued and distinct, and easily brought into unison with any musical instrument, since they can be produced at a uniform pitch for any desired interval of time.

Since by the stroke of each tooth of the wheel *d'*, the card is made to move first downwards and then upwards, or *vice versâ*, it is clear from what has been explained that, for each tooth of the wheel *d'* which passes the card, a condensed and a rarefied wave of air will be produced.

In the sound, therefore, which results there will be as many double vibrations, that is to say, undulations, including each a condensed and rarefied wave, as there are teeth of the wheel *d'* which pass the card; and to ascertain the number of such double vibrations corresponding to any note, it will be only necessary to observe the number of teeth of the wheel *d'* which pass the card when the sound produced by the instrument is brought into unison with the proposed note.

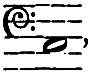
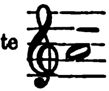
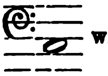
859. *The absolute rates of vibration of musical notes ascertained.* — By accurate experiment, made both with the Sirene and with the instrument of M. Savart, it has been found that the lower A of the treble clef or  is produced by imparting undulations to the air at the rate of 880 single vibrations, or 440 double vibrations, per second. By single vibration is here to be understood condensed waves only, or rarefied waves only; and by double vibration, the combination of a condensed and rarefied wave. It is more usual to count the vibrations, taking the latter or the double vibration, as the unit, and we shall therefore here adopt this nomenclature; and it

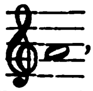
[redacted]

may therefore be stated, in this sense, that the A of the diapason, the note usually produced by the sounding-fork for determining the pitch of musical instruments, is produced by imparting to the air 440 undulations per second.

It must be stated, however, that some slight departure from this standard prevails in different established orchestras. Thus, it is estimated that the pitch of this note in the under-mentioned orchestras is produced by the number of vibrations per second exhibited below :—

Orchestra of Berlin Opera - - - - -	- 437·32
" Académie de la Musique, Paris -	- 431·34
" Opéra Comique, Paris - - -	- 427·61
" Italian Opera, Paris - - - -	- 424·14

The number of vibrations corresponding to all the other notes of the musical scale may be computed by the result here obtained, combined with the relative numbers of vibrations given in 850. Thus, if it be desired to determine the number of vibrations per second corresponding to the fundamental note , it will be only necessary to divide 440, the number of vibrations of the note , by the fraction $\frac{10}{3}$, or, what is the same, to divide it by 10, and multiply the quotient by 3. The number of vibrations, therefore, per second which will produce the note  will be 44 × 3 = 132.

860. *Range of musical sensibility of the ear.* — On a seven octave pianoforte the highest note in the treble is three octaves above , and the lowest note in the bass is four octaves below it. The number of complete vibrations corresponding to the former must be, therefore,

$$440 \times 2 \times 2 \times 2 = 3520;$$

and the number of vibrations per second corresponding to the latter is

$$\frac{440}{2 \times 2 \times 2 \times 2} = \frac{440}{16} = 27\tfrac{1}{2}$$

Now, since all ordinary ears are capable of appreciating the musical sounds contained between these limits, it is clear that the range of perception of the human ear is greater than that of such an

[redacted]

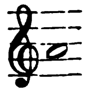
instrument, and that, consequently, this organ is capable of distinguishing sounds produced by vibrations varying from 27 to 3520 per second.

861. *The most grave note of which the ear is sensible.*—It has been generally assumed that the lowest of these notes constituted the most grave musical sound of which the ear is sensible; but Savart has shown, by a series of experiments remarkable for their conclusiveness, that the organ of hearing has a wider range of sensibility. For this purpose he substituted for the toothed wheel *d'*, *fig.* 250, a simple bar of iron or wood, which was made to revolve round its centre in the same manner as the toothed wheel. Two plates of wood were placed on each side of the bench, as represented in *fig.* 251, and were so adjusted that the revolving bar passed nearly in contact with them. At each transit of the bar near their edges an explosive sound was produced, of deafening loudness. The loudness was found to be a maximum when the distance of the bar from the edges of the plates was from the 4000th to the 8000th of an inch. When the bar was moved very slowly, these were recognized by the ear as distinct and successive sounds; but when a velocity was imparted to it which produced from seven to eight sounds per second, the sound became continuous, and was recognized as a musical note of great depth in the scale. It was rendered evident, therefore, from this experiment, that the ear is capable of appreciating musical sounds produced by from seven to eight complete vibrations per second.

862. *The most acute note.*—To determine, on the other hand, the limit of the sensibility of the ear for acute musical sounds, Savart increased the diameter of the wheel *d'*, *fig.* 250, so as thus to impart a more rapid motion to the teeth. In this way he found that musical sounds were distinctly recognized produced by 24,000 complete undulations per second.

By this experiment it was, therefore, established that the range of sensibility of the ear for musical sounds extended from 7 vibrations to 24,000 per second.

Savart, however, maintains that these limits are not the extreme ones of the susceptibility of the ear.

863. *Calculation of the length of the waves corresponding to musical notes.*—It has been already shown, that by the combination of the velocity of sound with the rate of undulation, the length of the sonorous waves corresponding to any given note can be determined. Thus, if we know that 440 undulations of the note  strike the ear in a second, and also that the velocity with which this undulation passes through the air is at the rate of 1120 feet per

second, we may conclude that in 1120 feet there are 440 complete undulations; consequently, that the length of each such undulation is

$$\frac{1120}{440} = 2{\cdot}54 \text{ feet.}$$

By a like calculation, the length of the sonorous waves corresponding to all the musical notes can be determined.

To find the length of the sonorous waves corresponding to the highest and lowest notes of a seven octave pianoforte, we are to consider that the highest note has been shown to be produced by 3520 vibrations per second; the length of each variation will, therefore, be

$$\frac{1120}{3520} = 0{\cdot}3181.$$

The number of vibrations corresponding to the lowest note is 27·5; the length, therefore, of the sonorous undulation will be

$$\frac{1120}{27{\cdot}5} = 40{\cdot}73 \text{ feet.}$$

To find the length of the vibrations corresponding to the gravest note produced in Savart's experiments, we must divide 1120 by 7; the quotient will be 160 feet, which is the length of the undulation required.

864. *Application of the Sirene to count the rate at which the wings of insects move.*—The buzzing and humming noises produced by winged insects are not, as might be supposed, vocal sounds. They result from sonorous undulations imparted to the air by the flapping of their wings. This may be rendered evident by observing, that the noise always ceases when the insect alights on any object.

The Sirene has been ingeniously applied for the purpose of ascertaining the rate at which the wings of such creatures flap. The instrument being brought into unison with the sound produced by the insect indicates, as in the case of any other musical sound, the rate of vibration. In this way it has been ascertained that the wings of a gnat flap at the rate of 15,000 times per second. The pitch of the note produced by this insect in the act of flying is, therefore, more than two octaves above the highest note of a seven octave pianoforte.

CHAP. III.

SOUNDS PRODUCED BY THIN RODS AND PLATES.

865. *Vibration of rods, longitudinal and transverse.* — **Among the numerous results of the labours of contemporary philosophers, some of the most beautiful and interesting are those which have attended the experimental researches of Savart, made with a view to determine the phenomena of the vibration of sonorous bodies, some of which we have already briefly adverted to. Although these researches are too complicated, and the reasoning and hypotheses raised upon them are not sufficiently elementary to be introduced with any detail into this volume, there are nevertheless some sufficiently simple to admit of brief exposition, and so interesting that their omission, even in the most elementary treatise, would be unpardonable.**

The vibration of thin rods, whether they have the form of a cylinder or a prism, or that of a narrow thin plate, may be considered as made transversely or longitudinally. If they are made transversely, that is to say, at right angles to the length, they will be governed by nearly the same principles as those which have been already explained as applicable to elastic strings.

866. *Longitudinal vibrations.*—Let us suppose a glass tube, about seven feet long, and from an inch to an inch and a half in diameter, to be suspended in equilibrium at its middle point. Let one half of it be rubbed upon its surface, in the direction of its length, with a piece of damp cloth. The friction will excite longitudinal vibration, that, with a little practice, may be made to produce a musical sound, which will be more or less acute according to the force and rapidity of the friction.

It will be found that the several sounds which will be successively produced by thus increasing the force of the friction, will correspond with the harmonics already explained in 849.; that is to say, that the rate of vibration of the lowest of these tones being expressed by 1, that of the next above it will be expressed by 2, and will therefore be the octave; the next will be expressed by 3, and will therefore be the twelfth, and the next by 4, which will therefore be the fifteenth.

If the same experiment be performed with long rods of any form and of any material whatever, the same result will be noticed. When rods of wood are used, instead of a moistened cloth, a cloth coated with resin may be employed. It is found that rods, composed of the same material, will always emit the same notes, provided they are of the same length, whatever be their depth, thickness, or form, provided only that their length be considerable compared with their other dimensions

[redacted]

867. ***Nodal points.*** — Were it possible to render visible the state of vibration of each point of the surface of these rods, it would be found that the degree of vibration would vary from point to point, and that at certain points distributed over the surface of these rods, there would be no vibration. These nodal points, as they have been called, are distributed according to certain lines surrounding the rods.

But it is evident that motions so minute and so rapid as these vibrations, cannot be rendered directly evident to the senses.

868. ***Method of determining nodal points and lines.***—The following ingenious method of *feeling* the surface while in vibration, and ascertaining the position of the nodal lines, was practised with signal success by Savart. A light ring of paper was formed, having a diameter considerably greater than that of the tube or rod. This ring was suspended on the tube, as represented in *fig.* 252.

The tube, which we shall suppose here, as before, to be formed of glass, and of the same dimensions as already explained, being suspended on its central point, and put in vibration, as already described, by friction produced upon that half of the tube on which the ring is not suspended, it will be found that the vibration of the tube will give the ring a jumping motion which will throw it aside, and cause it to move to the right or left, as the case may be, until it shall arrive at a point where it shall remain at rest, its motion as it approaches this point being gradually diminished. At this point it is evident that there is no vibration, and it is, consequently, a nodal point.

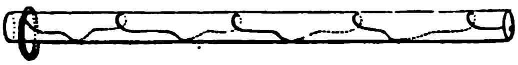


Fig. 252.

Let this point be marked upon the glass with ink, and let the tube be then turned a little round on its axis, so as to bring the point thus marked a little aside from the highest position which it held when the ring rested upon it. Let the tube be now again put in vibration, so as to produce the same note as before. The ring will be again moved, and will find another point of rest.

Let this point be marked as before, and let the tube be again turned, and let the same process be repeated, so that a third nodal point shall be determined. By continuing this process, a succession of nodal points will be found following each other round the tube, and thus a nodal line will be determined.

This process may be continued until the entire course of the nodal line shall be discovered.

Experiments conducted in this way have led to the discovery that the nodal lines surrounding the tube have a sort of spiral or screw-like form, represented in *fig.* 252. The course is not that of a regu-

[redacted]

lar helix, since it forms at different points of the surface of the tube different angles with its axis, whereas a regular helix will at every point form the same angle; but this variation of the inclination of the nodal line to the axis is not irregular, but undergoes a succession of changes which are constantly repeated, so that each revolution of the nodal line is a repetition in form of the last, as represented in *fig.* 252.

If the ring be now suspended on the other half of the tube, a similar nodal curve is formed, which is not, however, a continuation of the former. The two spirals seem to have a common origin at the end, and to proceed from that point, either in the same or contrary directions, towards the other end of the tube.

869. *Nodal lines on the inside of a tube.* — Savart examined also the position of the nodal line on the inner surface of the tube, by spreading upon it grains of sand, or a small bit of cork. These were put in motion in the same manner as the ring of paper by the vibration, and were brought to rest on arriving at a nodal point. A series of nodal lines similar to the exterior system was discovered.

When the friction is increased so as to make the tube sound the harmonics to the fundamental note, the spirals formed by the nodal line are reversed two, three, or four times, according to the order of the harmonic produced.

870. *Nodal lines on rods and thin plates.* — In the case of prismatic rods or flat laminæ, the nodal curves are still spirals, but more irregular and complicated than in the case of tubes or cylinders.

The vibrations of thin plates were produced and examined by the following expedients: — An apparatus was provided, represented in *fig.* 253. A small piece of metal *a*, having a form slightly conical, is fixed in the bottom of a frame, and at its upper surface a piece of cork, or buffalo skin, is fixed to intercept vibration. A corresponding cylinder is moved vertically, directly above it, by a screw, which plays in the frame *b*, and which is also covered at its extremity with a piece of cork.

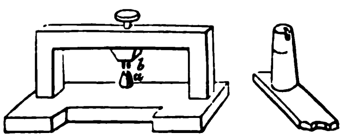


Fig. 253.

When the screw is turned, the two extremities can be brought into contact, so as to press between them with any desired force any plate which may be interposed.

An elastic plate, the vibration of which it is desired to observe, is inserted between them, and held compressed at any desired point by turning the screw. The plate thus held can be put in vibration by means of a violin bow, which being drawn upon its edge, clear musical sounds may be produced, and brought into unison with those of a pianoforte, or other musical instrument.

[redacted]

To ascertain the state of vibration of the different points of the surface of the plate, sand or other light dust is spread upon it, to which motion is imparted by the vibrating points. Those points which are at rest, and which are therefore nodal points, impart no motion to the grains of sand which lie upon them, and those which are upon the vibrating points are successively thrown aside, until they reach the lines of repose or nodal lines, where at length they settle themselves.

When a musical sound of a uniform pitch has, therefore, been continued for any length of time, the disposition of the grains of sand upon the plate will indicate the position and direction of the nodal lines.

871. *Method of delineating the nodal lines practised by Savart.* — When experiments of this kind were multiplied to some extent, it became apparent that the nodal lines assumed such varied and complicated forms that it was difficult to delineate them with accuracy by the common methods of drawing.

An ingenious expedient suggested itself to Savart, by which facsimiles of all these figures were obtained. Instead of sand, he used litmus mixed with gum, dried, reduced to a fine powder, and passed through a sieve, so as to obtain grains of equal and suitable magnitude. This coloured and hygrometric powder he spread upon the vibrating plates, and when it had assumed the form of the nodal lines, he applied to the plates with gentle pressure damp paper, to which the coloured powder adhered, and which, therefore, gave an exact impression of the form of the nodal lines.

In this manner he was enabled to feel, as it were, the state of vibration of the different parts of the plate, and to ascertain with precision the lines of no vibration, or the nodal lines, which separated from each other those parts of the plate which vibrated independently.

In this way many hundred experiments were made, and exact diagrams obtained representing the condition of the vibrating plates.

872. *Nodal lines multiplied as sounds become more acute.* — One of the consequences which most obviously followed from these experiments was, that the nodal lines became more and more multiplied the more acute the sound was which the plate produced. This consequence was one which might have been anticipated from the analogy of the nodal lines of the plate to nodal points of the elastic string. It has been already shown, that with a single nodal point in the middle of the string, the octave to the fundamental note is produced; that when two nodal points divide the string into three equal parts, the twelfth is produced; that when three nodal points divide the string into four equal parts, the fifteenth is produced, and so on. What the subdivisions of the string are to the notes produced by its vibrations, the subdivisions of the surface of the vibrating plate by the nodal lines is to the note which it produces; and it was consequently natural to expect, that the higher the note produced, the more multiplied would be the divisions of the plate.

873. *Curious forms of the nodal lines.* — But a circumstance attending these divisions not less curious than their number was their form, for which no analogy existed in the vibration of strings. It would be impossible here to give any definite notion of the infinite variety of which these nodal figures are susceptible; they change not only with the pitch of the note produced, but also with the form and material of the plate and the position of the point at which it is held in the instrument, represented in *fig.* 253. It will not, however, be without interest to give an example of the variety of figures presented by the nodal lines produced upon the same square plate. These are represented in the series of *figures* 254.

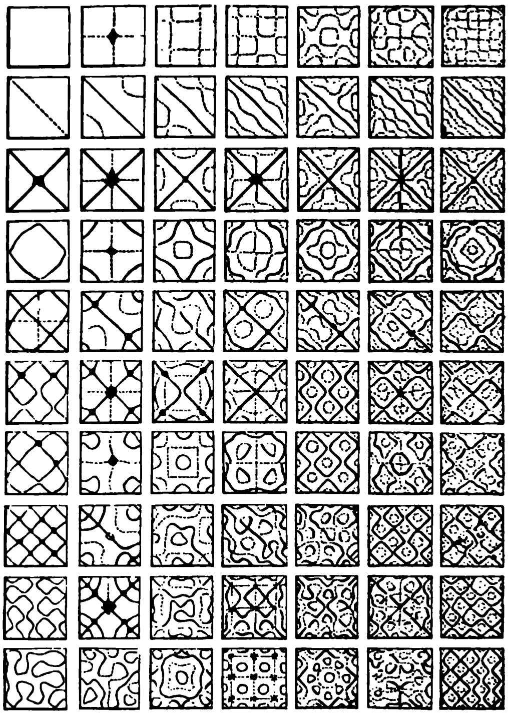


Fig. 254.

Similar experiments, made on circular plates, showed that the nodal lines distributed themselves either in the direction of the diameter *dividing* the circle into an equal number of parts, or in circular forms,

[redacted]

more or less regular, having the centre of the plate as their common centre, or, in fine, in both of these combined. In the annexed *figure* 255. are represented some of the varieties of form thus obtained.

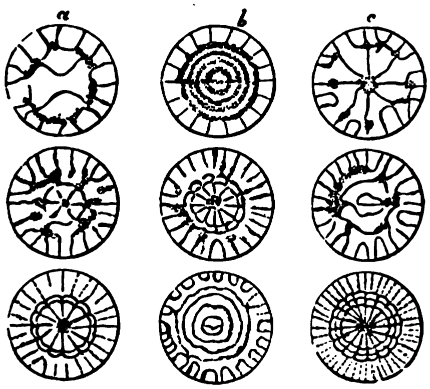


Fig. 255.

CHAP. IV.

SOUNDS PRODUCED BY THE VIBRATORY MOVEMENTS OF FLUIDS.

874 *Fluid bodies sonorous.* — Fluids, whether in the liquid or gaseous state, have been hitherto considered merely as conductors of sound, their sonorous undulations having been derived from the vibratory impulses of solid bodies acting upon them.

Fluids themselves, however, are capable of originating their own undulations, and consequently must be considered not merely as conductors of sound, but likewise as sonorous bodies.

If the Sirene of Cagniard de la Tour, already described, be submerged in water, and made to act as it has been described already to act in air, the pulsations of the water will produce a sound. In this case, the origin of the sound is the action of the liquid upon itself. The successive movements of the liquid through the holes in the circular plate of the Sirene are the origin of the sonorous undulations which are transmitted through the liquid.

875. *Wind instruments.* — Innumerable examples might be found of sonorous undulations produced by air upon air. The Sirene itself, which has been already explained, forms an example of this, and at the same time indicates the manner in which the pulsations are im-

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parted to the air. All wind instruments whatever are also examples of this. The air, by the impulses of which the sonorous undulations are produced, proceeds either from a bellows, as in the case of organs, or from the lungs, as in the case of ordinary wind instruments. The pitch of the sound produced depends partly upon the manner of imparting the first movement to the air, and partly on varying the length of the tube containing the column of air to which the first impulse is given.

When the tube, as is generally the case in instruments of music, has a length which is considerable in proportion to its diameter, the gravest note which it is capable of producing is determined by a sonorous undulation of its own length. By varying the embouchure, and otherwise managing the action of the air on entering the tube, notes may be produced which are harmonics to the fundamental notes determined by the length of the tube.

When these harmonics are produced, nodal points will be formed in the column of air included in the tube; and if the tube were divided and capable of being detached at these nodal points, the removal of a part of the tube would not alter the pitch of the note produced.

In wind instruments in which various notes are produced by the opening and closing of holes in their sides by means of the fingers or keys, there is a virtual variation in the length of the sounding part of the tube which determines the pitch of the various notes produced. In some cases, the length of the tube is varied, not by apertures opened and closed at will, but by an actual change of length in the tube itself. Examples of this are presented in some brass instruments, and more particularly in the trombone.

876. *Effect of the material composing a wind instrument.*—Although the length of the column of the air included in the tube of a wind instrument alone determines the pitch of the note, its *timbre* depends in a striking and important manner upon the material of which the tube is composed.

877. *Example in organ building.*—It is well known that organ builders find that the quality of tone is so materially connected with the quality of the material composing the tube, that a very slight change in the alloy composing a metal tube would produce a total change in the quality of the tone produced. The excellence of an organ depends in a great degree upon the skill with which the material of the tubes, whether wood or metal, is selected.

878. *Sound produced by a gas flame in a glass tube.*—The sound produced by a jet of hydrogen, directed into a glass tube, forms a remarkable example of the manner in which the sonorous undulations of air would be produced by movements originating in air itself.

This apparatus, the explanation of the principle of which is due to M. de la Rive, consists of a small glass vessel in which hydrogen is generated in the usual way, by the action of acid on zinc or iron. A

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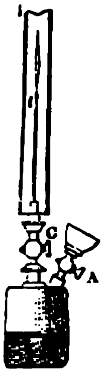

Fig. 256.

funnel and stop-cock A, *fig.* 256., are provided, by which the supply of the acid may be renewed. A pipe proceeds from the centre of the top of the vessel furnished with a stop-cock C, in which a small tube is inserted terminating in a very small aperture, from which a fine jet of the gas escapes when the stop-cock is opened, and a sufficient pressure produced by the accumulation of gas within the vessel. The jet proceeding from *t* in this manner being inflamed, a glass tube of considerable length and having a diameter of about two inches is held over it, so that the jet is made to burn at some distance above the lower end of the tube. A musical sound will thus proceed from the air within the tube, the intensity of which will depend upon the length of the tube.

This effect is explained as follows:—The vapour, which is the first product of the combustion of the hydrogen, fills a portion of the tube above the flame, and excludes from it the air, but the cold of the tube soon condenses this vapour, and a vacuum is produced, into which the air rushes with a rapid motion. This effect being repeated by the continuance of the combustion of the hydrogen, a corresponding undulation is produced in the column of air in the tube, and a musical sound is the result.

879. *Reflection of sound.—Explanation of echoes.*—It has been already shown, that when undulations propagated through a fluid encounter a solid surface, they will be reflected from it, and will proceed as though they had originally moved from a different centre of undulation.

Now, if this take place with the sonorous waves of air, such waves encountering the ear will produce the same effect as if they proceeded, not from the surrounding body which originally produced them, but from a sounding body placed at that centre from which the waves thus reflected move. Upon these principles, echoes are explained.

If a body, placed at a certain distance from the hearer, produce a sound, this sound would be heard first by means of the sonorous undulations which produced it proceeding directly and uninterruptedly from the sonorous body to the hearer, and afterwards by sonorous undulations which, after striking on reflecting surfaces, return to the ear. The repetition of the sound thus produced is called an *echo*.

To produce an echo it will be necessary, therefore, that there shall be a sufficient magnitude of reflecting surface, so placed with respect to the ear, that the waves of sound reflected from it shall arrive at the ear at the same moment, and that their combined effect shall be sufficiently energetic to affect the organ in a sensible manner.

If, for example, the sounding body be placed in a focus F of an ellipse, as represented in *fig.* 257., the hearer being at the other focus

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F', the sound will be first heard by the effect of the undulations, which are produced directly along the line F F', from one focus to the other. But it will be heard a little later by the effect of the waves, which, diverging from the sounding body at F, strike upon the elliptic surface, and are reflected to the other focus F', where the hearer is placed. The interval which elapses between the sound and the echo in this case will be the time which sound takes to move through the difference between the direct distance F F', and the sum of the two distances at any point in the ellipse from the foci F F'. It has been already explained that the sum of these two distances is always the same wherever the point of reflection may be, being equal to the major axis of the ellipse. It is for this reason that all the reflected rays of sound from every part of the ellipse will meet the ear placed at F' at the same moment, since they will take the same time to move over the same distance. If the reflected surface were not elliptical, or if, being elliptical, the hearer were not placed at the focus F', then the sum of the distances of the different points of the reflecting surface from the ear would be different, and the reflected rays of sound arriving from different points of the surface, would reach the ear at different moments of time. In this case, each ray of sound would be too feeble to produce sensation, or a confused effect would be produced.

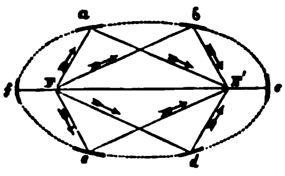

Fig. 257.

It is not necessary that the elliptic surface reflecting the sound should be complete. If different portions of the reflecting surface, *a*, *b*, *c*, *d*, *e*, *f*, *fig.* 257., be so placed that they would form part of the same ellipse, they will still reflect the rays of the sound to the other focus of the ellipse; and if they are so numerous or extensive as to reflect rays of sound to the ear in sufficient quantity to affect the sense, an echo will be heard.

880. *Repeated echoes.* — If surfaces lie in such a position round the points F and F', that these points shall be at the same time the foci of different ellipses, one greater than the other, a succession of echoes will ensue, the sounds reflected from the greater elliptic surface arriving at the ear later than those reflected from the lesser. The interval between the successive echoes in such a case would be the time which the sound takes to move over a space equal to the difference between the major axes of the ellipses.

If a person who utters a sound stand in the centre S of a circle, *fig.* 258., the circumference of which is either wholly or partly composed of surfaces, such as *a*, *b*, *c*, *d*, *e*, which reflect sound, he will hear the echo of his own voice; as in this case, the sonorous undulation which proceeds from the speaker encountering the reflecting

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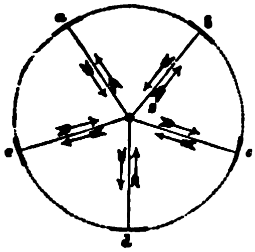

Fig. 258.

surfaces in a direction perpendicular to them, will be reflected by them back to the speaker, as represented by the arrows, and will reach his ear after an interval corresponding to that which sound requires to move over twice the radius of the circle. If the speaker in such a case be surrounded by surfaces composing either wholly or partly two or more circles, of which he is the common centre, then he will hear a succession of echoes of his own voice, the interval between them corresponding to the time which sound would take to move over twice the difference between the successive radii of the circles.

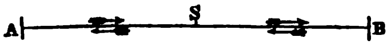

Fig. 259.

If a speaker stand at S, *fig.* 259., midway between two parallel walls A and B, these walls may be considered as forming part of a circle of which he is the centre, and they will reflect to his ear the sounds of his own voice, producing an echo. In this case, the position of the speaker S being equally distant from A and B, the sounds reflected from these surfaces will return to his ear simultaneously, and produce a single perception. But a part of the undulation reflected from B, not intercepted by the speaker at S, will arrive at A, and will be reflected from A and again arrive at S, where it will affect the ear. The same may be said of the sounds reflected from A, which, proceeding to B, will be again reflected to S, and as the distance moved over by the sounds thus twice reflected are equal, they will arrive simultaneously at S, and would then produce a second echo. This second echo, therefore, will proceed from the successive reflections of the sound by the two walls A and B, and the interval between it and the first echo will be the time which sound takes to move over twice the distance S A, or the whole distance between the two walls.

Thus, if the two surfaces A and B were distant from each other 1120 feet, then the interval between the utterance of the sound and the first echo would be one second, and the same interval would take place between the successive echoes.

If the speaker, however, be placed at a point S, *fig.* 260., which is

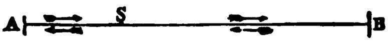

Fig. 260.

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not midway between the two walls A and B, the echo proceeding from the first reflection by the wall A will be heard before the echo which proceeds from the reflection by the wall B, and in this case a single reflection from each wall will produce two echoes.

If we suppose a second reflection from each wall to take place, two echoes will be again produced. Thus in case there are two reflections from each wall, four echoes will be heard; and in general the number of echoes which will be heard will be double the number of reflections.

881. *Each succeeding echo less loud.*—It may be asked, why the number of reflections, in such case, should have any limit? The answer is, that the reflected waves are always more feeble than the direct waves; and that consequently intensity, or loudness, is lost by each reflection, until at length the waves become so feeble as to be incapable of affecting the ear. A speaker can articulate so as to be distinctly audible at the average rate of four syllables per second. If, therefore, the reflecting surface be at the distance of 1120 feet, the echo of his own voice will be perceived by him at the end of two seconds after each syllable is uttered; and since, in two seconds, he can utter eight syllables, it follows that he can hear, successively, the echo of these eight syllables; if he continue to speak, the sounds he utters will be confused with those of the echo.

The more distant the reflecting surfaces are, the greater will be the number of syllables which can be rendered audible by the ear.

It is not necessary that the surface producing an echo should be either hard or polished. It is often observed at sea, that an echo proceeds from the surface of the clouds. The sails of a distant ship have been found also to return very distinct echoes.

882. *Remarkable cases of multiplied echoes.*—Numerous examples are recorded of multiplied repetitions of sound by echoes. An echo is produced near Verdun by the walls of two towers, which repeats twelve or thirteen times the same word. At Adernach, in Bohemia, there is an echo which repeats seven syllables three times distinctly. At Lurleyfels, on the Rhine, there is an echo which repeats seventeen times. The echo of the Capo di Bouve, as well as that of the Metelli of Rome, was celebrated among the ancients. It is matter of tradition that the latter was capable of repeating the first line of the Æneid, which contains fifteen syllables, eight times distinctly. An echo in the Villa Simonetta, near Milan, is said to repeat a loud sound thirty times audibly. An echo in a building at Pavia, is said to have answered a question by repeating its last syllable thirty times.

883. *Whispering galleries.*—Whispering galleries are formed by smooth walls having a continuous curved form. The mouth of the speaker is presented at one point of the wall, and the ear of the hearer at another and distant point. In this case the sound is successively reflected from one point of the wall to another until it reaches the ear.

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884. *Speaking tubes.* — Speaking tubes, by which words spoken in one place are rendered audible at another distant place, depend on the same principle. The rays of sound proceeding from the mouth at one end of the tube, instead of diverging, and being scattered through the surrounding atmosphere, are confined within the tube, being successively reflected from its sides, as represented in *fig.* 261.; so that a much greater number of rays of sound reach the ear at the remote end, than could have reached it if they had proceeded without reflection.

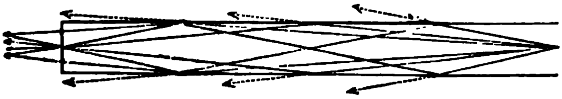

Fig. 261.

Speaking tubes, constructed on this principle, are used in large buildings where numerous persons are employed, to save the time which would be necessary in dispatching messages from one part of the building to another. A speaking tube is sometimes used on shipboard, being carried from the captain's cabin to the topmast. A like effect is produced by the shafts of mines, walls, and chimneys, as well as by pipes used to convey heated air or water.

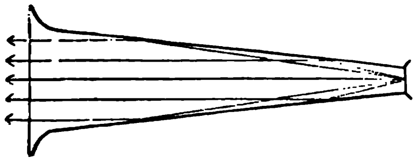

Fig. 262.

885. *Speaking trumpet.* — The speaking trumpet is another example of the practical application of this principle. A longitudinal section of this instrument is represented in *fig.* 262. The form of the trumpet is such, that the rays of sound which diverge from the mouth of the speaker are reflected parallel to the axis of the instrument. The trumpet being directed to any point, a collection of parallel rays of sound moves towards such point, and they reach the ear in much greater number than would the diverging rays which would proceed from a speaker without such instrument.

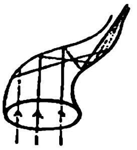

Fig. 263.

886. *Hearing trumpet.* — A hearing trumpet represented in *fig.* 263., is, in form and application, the reverse of the speaking trumpet, but in principle the same. The rays of sound proceeding from a speaker more or less distant, enter the hearing trumpet nearly parallel; and the form of the inner surface of such instrument is such that, after one or more

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reflections, they are made to converge upon the tympanum of the ear.

If a sounding body be placed in the focus of a parabola formed of any material capable of reflecting sound, the rays which issue from it will, after reflection, proceed in a direction parallel to the axis of the parabola. This will be apparent from what has been explained in 807.; and if, on the other hand, rays parallel to the axis strike on such a surface, they will be reflected converging towards the focus. Hence it appears that a parabola, in the focus of which the mouth of the speaker is placed, would be a good form for a speaking trumpet.

If a watch be placed in the focus of a parabolic surface, such as a metallic speculum of that form, an ear placed in the direction of its axis will distinctly hear the ticking, though at a considerable distance; but if the parabolic reflector be removed, the ticking will be no longer heard.

CHAP. V.

THE ORGANS OF SPEAKING AND HEARING.

887. *Organs of voice.* — The organ of voice in the human species is an apparatus consisting of a pipe extending in a vertical direction through the throat, having an air-chest at its lower end, and a complicated apparatus adapted to impart sonorous undulations to the external air at its upper extremity. This pipe, which is called the trachea or wind-pipe, and which is very nearly of a cylindrical form, consists of a series of strong cartilaginous rings united one to another by a flexible membrane, so that the interior tube has more or less elasticity both longitudinally and laterally; that is to say, that within certain limits it is capable of varying both its length and diameter. This pipe is at its lower extremity divided into two, which being directed to the right and left, terminate in the lungs, over the tissue of which their extremities are spread.

The lungs play the part of the air-chest of an organ in relation to the *wind-pipe*. The air which is inspired filling the lungs is compressed within them by the muscular action of the chest, and is thus driven through the wind-pipe with a force proportional to the difference between the elasticity of the air thus compressed in the lungs and that of the external atmosphere.

Towards the upper extremity, the *trachea* is gradually contracted in one direction and flattened, so as to terminate in a narrow slit-like opening, which is closed by two membranes capable of being brought into contact, and which are opened to a greater or less extent by the ac-

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tion of the will exerted upon the muscles which govern them. This slit, which may be compared to the opening between the two reeds which form the mouth-piece of a hautboy, is called the *glottis*.

The length of the opening of the glottis is about an inch and a quarter. Between the glottis and the first ring of the trachea is a cartilaginous passage, gradually widening from the glottis downwards, and capable, by muscular action, of being varied within considerable limits in its transversal area.

Immediately above the glottis is a cavity called the *ventricle*, the height of which is about half an inch. The superior part of this cavity is provided with an opening, forming a sort of superior glottis.

888. ***Manner in which vocal sounds are produced.*** — The air propelled from the chest through the wind-pipe, passing rapidly through the glottis into the ventricle, and again through the superior glottis, produces sonorous undulations, and would be attended with only sound without articulation, but the mouth, the tongue, teeth, and lips, and nasal passages, supply the means of varying without limit the *timbre* of each sound, and of giving it infinitely varied articulation. The flexibility of the tongue acting on the palate, the mouth, and the teeth; that of the lips acting on each other and contracting in an infinite variety of degrees; the mobility of the jaws, and the effect of the nasal passages on the air propelled through the mouth, all combine in giving variety without limit to the sounds produced by the apparatus just described.

889. ***Articulation of vowels and consonants.***—The vowels are produced by opening or closing in different degrees the passages between the lungs, the palate, and the lips. The consonants, on the other hand, are produced by interrupting at intervals the current of air proceeding through the mouth by means of the tongue, the lips, and the teeth; and hence these have been classed as labials, palatals, linguals, gutturals, &c.

890. ***An hypothesis in which the vocal organ is regarded as similar to a hautboy.*** — Physiologists are not agreed as to the principle upon which sound is produced by the organ of voice. Two hypotheses have been advanced to explain it. In the first, and that which appears the most generally received, the organ of voice is considered to be analogous to the hautboy, the lips of the glottis performing the part of the double reed in the mouth-piece of that instrument. It is considered that the air arriving through the wind-pipe and issuing between the lips, puts them into vibration more or less rapidly, and that these vibrations impart corresponding undulations to the external air. The superior opening aids in this effect.

The varied pitch of the tones produced by the voice is explained in this system partly by the contraction and expansion of the trachea, and especially of its superior part, partly by widening and contracting

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the opening of the glottis, and partly by the power of giving more or less closeness to the lips of the glottis.

891. *Experiments of Magendie and others in support of this.* — This theory has not been allowed to rest on mere hypothesis. M. Magendie has made numerous experiments on inferior animals for the purpose of verifying it by direct observation. He has for example laid bare the trachea and its appendages in living dogs, and has observed the lips of the glottis enter into visible vibration when these animals uttered cries. He has also, in the same experiment, shown that the slit-like aperture of the glottis is contracted when the sounds uttered become acute, and is widened when they become grave.

Numerous experiments have been made by other physiologists upon the organs of voice of animals recently deprived of life. In these experiments, air has been forced through the trachea by bellows, and sounds have been thereby produced more or less analogous to those uttered by the living animal.

892. *An hypothesis in which the vocal organ is regarded as similar to a bird-call.* — The second hypothesis by which the effect of the vocal organ is explained assimilates it to a bird-call. In this system, the cavity or ventricle immediately above the glottis already described is considered as analogous to the barrel of the bird-call; the glottis and the superior opening above it represent the two holes in the circular sides of the bird-call. According to this system, the sonorous undulations are imparted to the air, not by the vibration of the lips of the glottis, but by the alternate compression and expansion of the air included in the ventricle between the upper and lower glottis.

This alternate compression and expansion is varied by the pressure of the air propelled from the lungs, and by the muscular action of a part of the trachea and its appendages.

In this system, the pitch of the sound uttered is also considered to be effected by the power of varying the opening of the glottis. The effects produced by the tongue, lips, teeth, palate, jaws, and nasal passages in varying the *timbre* and articulation of the sound are explained in the same manner as in the former hypothesis.

893. *Vocal organs of birds.* — Savart, who inclines to this theory, has made numerous experiments in support of it. In inferior animals, and more especially in birds, the arrangement of the organ of voice differs materially from the human species. The mechanism which produces sonorous undulations instead of being at the top is at the bottom of the trachea, the resemblance to wind instruments being thus still more close than in the case of the human voice.

It will be observed that in the human organ as already described, air enters at that part of the wind-pipe most distant from the reed or mouth-piece, contrary to the arrangement of a wind instrument. In the case of birds, however, the mechanism composing the mouth-piece is placed at that part of the wind-pipe which is next the lungs; and, consequently, the air is affected by it before it enters the wind-pipe. In these animals, therefore, air passes through the wind-pipe in a state of sonorous undulation, whereas in the human species undulations commence after it leaves the wind-pipe.

894. *Experiments of Cuvier.* — Cuvier made some important experiments in verification of this theory of the vocal organs of birds. He found that, having taken off the head of a duck, the voice of the animal continued to utter cries, both loud and well articulated, for a considerable time after decapitation.

895. *Organs of hearing.* — The organ of hearing in the human species consists of three distinct parts, the first and exterior of which would resemble, in its general form and construction, an ear-trumpet, if an elastic membrane, such as a piece of parchment, were stretched tight over the inner end of that instrument which is applied to the ear. The external visible part of the ear is analogous to the wide end of the ear-trumpet, and serves the purpose of collecting the rays of sound, and reflecting them inwards into the pipe which forms its continuation, contracting as it proceeds, and which enters an apparatus formed in the skull behind the external and visible ear. This pipe, which is formed of the same cartilaginous and gristly substance as the external ear, after entering the skull, and gradually contracting its dimensions, terminates in a small aperture, which is covered by a membrane tightly stretched over it, called, from its analogy to the parchment extended over a drum-head, the *membrana tympani* or *drum-membrane* of the ear. This elastic membrane is highly susceptible of vibration, and, when affected by the sonorous undulation of the air transmitted to it by the funnel and canal already described, vibrates in correspondence with them.

Behind this membrane is the second chamber of the ear, which is called the *tympanum* or drum of the ear, and corresponds to the space between the two heads of a drum.

This chamber is filled with air, which is supplied by a pipe communicating between it and the mouth. This free communication with the mouth, and through the mouth with the external atmosphere, keeps the air in the drum constantly in equilibrium with the atmospheric pressure, and its contact with the surrounding bones and flesh keeps the air within it at the constant temperature of the blood. This conservation of pressure and temperature is essential to the due sensibility and healthy action of the tympanum; for if, by any accidental cause, or organic derangement, the pipe of communication with the mouth, called the Eustachian tube, be obstructed, a humming of the ears ensues, which produces annoyance by a consciousness of its derangement.

At the inner extremity of the second chamber of the ear just described there is another opening, called the *fenestra ovalis*, covered

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by a similar elastic membrane, thus completing the analogy of the drum. Attached to the membrana tympani, or drum-membrane, is a chain of minute bones, which are carried to the inner ear, and appear to perform some functions in the conduction of sound which have not been fully explained. It seems, however, certain, that this chain of bones, which are capable of extension and contraction, have the effect of increasing or relaxing the tension of the membrane of the drum, and thereby rendering the organ more or less sensible.

Within the fenestra ovalis is the inner ear, filled with a complicated mechanism of canals and ducts, the uses of which are very imperfectly understood, and can scarcely be regarded as more than conjectural.

Three of these, which have a semicircular form, and which are disposed in planes at right angles to each other, are supposed to indicate the direction from which sounds proceed. This inner chamber is filled with a liquid in which the acoustic nerve, consisting like other nerves of a bundle of fine cords, floats. This nerve is continued to the brain, where it discharges its functions in the production of sensation.

Thus the sonorous undulations of the external atmosphere, collected and reflected by the external ear, impart corresponding vibrations to the membrana tympani. This transmits these vibrations to the air which fills the cavity within it, and probably also to the chain of small bones already described. These undulations are next imparted to the membrane which is stretched over the fenestra ovalis and other small membranes, too complicated to be described here. The vibrations of this latter membrane are imparted to the liquid which fills the inner ear, and in which is the acoustic nerve; this nerve, in fine, receives the vibrations from the fluid round it, and transmits them to the brain.

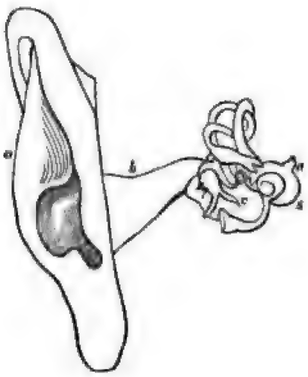

Fig. 264

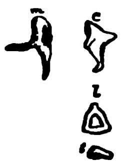

Fig. 265.

Fig. 266.

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The representation of the ear and its appendages in *fig.* 264. may render this explanation still more intelligible.

The funnel of the external ear is represented at *a*, technically called the ala or wing; the canal which leads from this funnel and enters the aperture already mentioned in the skull is represented at *b*; the membrana tympani or elastic membrane extended over the end of this canal is represented at *c*; the fenestra ovalis is placed at *v*; and the chain of small bones which connects the membrane of the tympanum with that of the fenestra ovalis is separately represented in *fig.* 265. by *m c l t*; the first and principal of these bones *m*, attached to the surface of the tympanum, forming a sort of solid radius of this membrane, extends from the centre to the air. The membrana tympani, with these bones attached, is represented in *fig.* 266. There are several muscles which act upon the chain of small bones *m c l t*, which connect the two membranes of the tympanum and the fenestra ovalis, so as to contract or relax them, thereby simultaneously increasing or diminishing the tension of the two drum-heads.

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PRACTICAL QUESTIONS FOR THE STUDENT.

1. In the hydrostatic press, represented in *fig.* 172., the diameter of the cylinder C is 1 inch, and that of the cylinder C′ 2½ feet; the arm of the power L M is 9 feet, and the arm of X M 2½ inches; with what force will the plunger A′ be urged upwards by a weight of 150 lbs. applied at L?

2. A cylindrical vessel 3 inches in diameter and 1 foot long, is let down into the sea 5,000 feet: what pressure will be exerted on the convex surface? and, supposing one end to be closed by a water-tight stopper, with what force will it be urged inwards? (636. and 640.)

3. A Greenland whale sometimes has a surface of 3,600 square feet: what pressure would he bear at the depth of 800 fathoms?

NOTE. — In the two preceding examples, as well as in articles 635. and 636., no allowance is made for the increased specific gravity of water at great depths.

4. A cubical vessel, each side of which is ten feet square, is filled with water, and a tube thirty-two feet long is fitted to an aperture in it, whose area is one square inch. If the tube be vertical, of the same diameter as the aperture, and filled with water, what is the pressure on the interior surface of the vessel, taking into consideration only the weight of the water in the tube?

5. What is the pressure on the bottom of the vessel, in Example 4, when the weight of the water in the vessel is taken into account: 1st. without the vertical tube, and 2d, with it?

6. What is the pressure on each vertical side of the vessel, the weight of the water both in the tube and vessel being considered?

7. A sphere 15 feet in diameter is filled with water: what is the entire pressure on the interior surface? and what is the weight of the water? (633.)

8. If the sphere, in the preceding example, were filled with mercury whose sp. gr. is 13·598, what would be the pressure and the weight? (637.)

9. A solid weighs 1,500 lbs. in air and 1,325 lbs. in water: what is its specific gravity?

10. If the air-weight of a substance soluble in water be 960 grains, and its weight in mercury 128 grains, what is its specific gravity?

[redacted]

11. An island of ice rises 50 feet out of water, and its upper surface is a circular plane, containing $\frac{3}{4}$ths of a square acre. On the supposition that the mass is cylindrical, what is its weight and depth below the water, the sp. gr. of sea-water being 1·026, and that of ice, ·865?

12. A cylindrical vessel 3 feet in diameter and 15 feet high is kept constantly filled with water; a circular aperture 1 inch in diameter being made in the bottom, with what velocity will the liquid escape? and what quantity will escape in 2 hours? (682., 685., and 687.)

13. If the vessel in the previous example be allowed to empty itself, what will be the velocity of efflux at the commencement? and what time will be required for the complete exhaustion?

NOTE. — In article 685., the sectional area of the contracted vein is stated to be about two-thirds that of the orifice. Measurements of the vein made of late by many, and especially by Weisbach, show that the vein at a distance equal to about half the width of the orifice, has the greatest contraction, and a diameter ·8 that of the orifice. Hence, since the areas are as the squares of the diameters, if O′ be the sectional area of the contracted vein, and O that of the orifice, we shall have

$$O' = \cdot 64\ O.$$

The factor ·64 is called the *coefficient of contraction*. But experiments made by Michelotti, by Eytelwein, and by Weisbach, with smoothly polished metallic orifices, have shown that the effective discharge, or that which actually flows out, amounts to from 96 to 98 per cent. of the theoretical discharge, taking into account the effect of the contracted vein. Since the area of the orifice remains the same, the theoretical velocity of the escaping liquid must be diminished in the same proportion as the theoretical discharge or efflux. This loss of velocity is caused by the friction of the water with the sides of the orifice, and perhaps by the imperfect fluidity of the water itself. Hence, calling the real or effective velocity of escape V′, and taking the mean per-centage, we have

$$V' = \cdot 97\ V = \cdot 97 \times 2\sqrt{193\ H}.$$

The factor ·97 is called the *coefficient of velocity*. Hence, also, we shall have for the real or effective efflux per second,

$$\begin{aligned} E &= O' \times V' \\ &= \cdot 64\ O \times \cdot 97 \times 2\sqrt{193\ H} \\ &= \cdot 62\ O \times 2\sqrt{193\ H}. \end{aligned}$$

The factor ·62, which is the product of the coefficients of contraction and velocity, is called the *coefficient of efflux*.

This gives us for the quantity actually discharged in any time t,

$$Q = t \times \cdot 62\ O \times 2\sqrt{193\ H};$$

the liquid being understood to be kept constantly at the same level.

Similarly, we shall have for the actual time of exhaustion in seconds,

$$t = \frac{Q}{\cdot 62\ O\sqrt{193\ H}}.$$

The above, however, must be regarded only as a *mean* value for the coefficient of efflux. Multiplied observations have shown that it is not constant; being greater for small orifices and for small velocities than for large ori

[redacted]

fices and great velocities; and being considerably greater for elongated and small orifices than for orifices which have a regular form, or which approximate to the circle.

14. The horizontal section of a cylindrical vessel is 100 square inches, its altitude is 50 inches, and it has an orifice whose sectional area is one-tenth of a square inch. What will be the *actual* time of exhaustion?

15. If a piece of stone weigh 10 lbs. in air, but only 6$\frac{3}{4}$ lbs. in water, what is its specific gravity?

16. Suppose a piece of elm weighs 15 lbs. in air, and that a piece of copper, which weighs 18 lbs. in air and 16 lbs. in water, is affixed to it, and that the compound weighs 6 lbs. in water; what is the specific gravity of the elm?

17. A piece of cast-iron weighed 34$\frac{61}{108}$ oz. in a liquid, and 40 oz. out of it: of what sp. gr. is that fluid?

18. A compound weighing 112 lbs. is made of tin and copper; the sp. gr. of the compound is 8·784, that of tin 7·291, and that of copper 8·850: what is the quantity of each ingredient?

19. How many cubic feet are there in a ton of zinc? (778.)

20. What is the weight of a block of Parian marble, whose length is 63 feet, and its breadth and thickness each 12 feet?

21. A tube 30 inches long, closed at one end and open at the other, was let down into the sea with the open end downwards, until the inclosed air occupied only one inch of the tube. Assuming the truth of Mariotte's law, how far did it descend? (708., 635. and 640.)

22. A spherical air-bubble, having risen from a depth of 3,000 feet in sea-water, was nine inches in diameter when it reached the surface. What was its diameter in its original position?

23. A cylindrical tube 40 inches long is half filled with mercury and then inverted in a vessel of mercury. How high will the mercury stand in the tube, the pressure of the external air being taken at 30 inches?

24. With what velocity per second does air rush into a vacuum?

NOTE.—The reasoning of articles 679., 680., 681., and 682. is as applicable to this case as to that of liquids. Hence, *air rushes into a vacuum with the velocity which a heavy body would acquire by falling from the top of a homogeneous atmosphere.* This height varies with the temperature and other circumstances. At the temperature of 32°, the barometer standing at 30 inches, the height of the homogeneous atmosphere is 26,146 feet. Hence, denoting the velocity by v, we have

$$v = 2\sqrt{16\tfrac{1}{12} \times 26146}$$
$$= 1{,}296 \text{ feet per second nearly.}$$

25. In a thunder storm, I saw the flash of the lightning 9 seconds before hearing the thunder, the thermometer at the time standing at 84°: at what distance was the cloud? (830.)

NOTE.—In practice, the velocity of sound is taken at 1,090 feet per second, when the thermometer is at 32°; and one foot is added for each additional degree of temperature.

26. A stone, being dropped into a vertical pit, is heard to strike the bottom

[redacted]

after 10 seconds have elapsed: how deep is the pit, if the thermometer stands at 32°? and how deep, if it stands at 90°?

27. A board 3 feet square is moved through a mass of water with the velocity of 50 feet per second: what resistance does it encounter? (692.)

28. If the board in the preceding example were at rest, with what force would it be struck by a stream of water moving 90 feet per second? (693.)

29. A cord of a certain length and diameter makes 50 vibrations per second, when stretched with a force of 50 lbs.: with what force must the same cord be stretched in order that it may vibrate 75 times per second? (794.)

30. If a cord whose diameter is $\frac{1}{14}$th of an inch, length 7 feet, and tension 144 lbs., make 36 vibrations per second, how many vibrations per second will be made by a cord of the same material whose diameter is $\frac{1}{18}$th of an inch, length 9 feet, and tension 324 lbs.?

31. A cube whose side is 4 feet, rests, with two of its surfaces horizontal, 150 feet below the surface of a lake: in what time will it be filled through a circular aperture, 1 inch in diameter, in its upper surface?

32. A cylindrical vessel whose base is 1 foot in diameter and whose altitude is 16 feet, empties itself in 4 hours by a circular aperture in the bottom: what is the diameter of the aperture?

33. A lump of lime weighing 8 lbs. is balanced in air by the same weight of cast-iron in the opposite scale: how will the equilibrium be affected when the bodies are plunged in water? and by what weight of lime, properly disposed, may the equilibrium be restored?

34. A cubical vessel, whose side is 3 feet, is filled half with mercury and half with water: what is the ratio of the pressure on the vertical sides to the pressure on the base?

35. A lump of gold and a lump of silver are found to balance each other when weighed in water; what is the ratio of their weights?

36. The tube of a barometer is 33 inches long. A quarter of an inch of air was left in it at the time of inversion. When the mercury in this barometer stands at 28 inches, what is the true altitude?

37. The orifices in the equal bases of two upright prismatic vessels are in the ratio of 2 to 1, and the vessels are emptied in equal times; what is the ratio of their altitudes?

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A

HAND-BOOK

OF

OPTICS.

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IN UNIVERSITY COLLEGE, LONDON.

ILLUSTRATED BY ONE HUNDRED AND FIFTY-EIGHT
ENGRAVINGS ON WOOD.

PHILADELPHIA:
BLANCHARD AND LEA
1859.

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BOOK THE NINTH.

LIGHT.

CHAPTER I.

LUMINOUS AND NON-LUMINOUS BODIES. — TRANSPARENCY. — OPACITY.

896. *Physical nature of light.* — Light is the physical agent by which the external world is rendered manifest to the sense of sight.

Opinion has long been divided as to its nature; one party has regarded it as a specific fluid, another as the effect of undulation.

The former consider that the eye is affected by light as the sense of smell is affected by the odoriferous effluvia; the latter maintain that light is to the eye what sound is to the ear. Before these theories, however, can be understood, or their claims to adoption be appreciated, it will be necessary that the chief properties of light, and the phenomena consequent upon them, be explained.

897. *Bodies luminous and non-luminous.* — In relation to the production of light, bodies are considered as luminous and non-luminous.

Luminous bodies, or luminaries, are those which are original sources of light, such, for example, as the sun, the flame of a lamp or candle, metal rendered red-hot, the electric spark, lightning, and so forth.

Luminaries are necessarily always visible when present, provided the light they emit be strong enough to excite the eye.

Non-luminous bodies are those which themselves produce no light, but which may be rendered temporarily luminous when placed in the presence of luminous bodies. These cease, however, to be luminous, and therefore visible, the moment the luminary from which they borrow their light is removed. Thus the sun, placed in the midst of the planets, satellites, and comets, renders these bodies luminous and visible; but when any of them is removed from the solar influence by the interposition of any object not pervious to light, they cease to be visible, as is manifest in the case of lunar eclipses, when the globe of the earth is interposed between the sun and moon, and the latter object is therefore deprived of light. A candle or lamp placed in the room renders the walls, furniture, and surrounding objects temporarily luminous, and therefore visible; but if the candle be screened by any object not pervious to light, those parts of the room from which light is intercepted would become invisible, did they not receive some light from the other parts of the room still illuminated. If, however, the candle or lamp be completely covered, all the objects in the room become invisible.

898. *Transparency and opacity.* — In relation to the propagation of light, bodies are considered as transparent and opaque. Bodies through which light passes freely are called transparent, because the eye placed behind them will see such light through them. Bodies, on the contrary, which do not admit light to pass through them, are called opaque; and such bodies consequently render a luminary invisible if interposed between it and the eye.

Transparency and opacity exist in various bodies in different degrees. Glass, air, and water are examples of very transparent bodies. The metals, stone, earth, wood, &c. are examples of opaque bodies.

Correctly speaking, no body is perfectly transparent or perfectly opaque.

899. *No body perfectly transparent.* — There is no substance, however transparent, which does not intercept some portion of light, however small. The light is thus intercepted in two ways; first, when the light falls upon the surface of any body or medium, a portion of it is arrested, and either absorbed upon the surface, or reflected back from it; the remainder passes through the body or medium, but in so passing more or less of it is absorbed, and this increases according to the extent of the medium through which the light passes. Analogy, therefore, justifies the conclusion that there is no transparent medium which, if sufficiently extensive, would not absorb all the light which passes into it.

A very thin plate of glass is almost perfectly transparent, a thicker is less so, and according as the thickness is increased the transparency will be diminished. The distinctness with which objects are seen through the air diminishes as their distance increases, because more or less of the light transmitted from them is absorbed in its progress through the atmosphere. This is the case with the sun, moon, and other celestial objects, which when seen near the horizon are more dim, however clear the atmosphere may be, than when seen in the zenith. In the former case, the light transmitted from them passes through a greater mass of atmosphere, and more of it is absorbed. According to Bouguer, sea-water at about the depth of 700 feet would lose all its transparency, and the atmosphere would be impervious to the sun's light if it had a depth of 700 miles.

900. *Various degrees of transparency.* — The transparency of the same substance varies according to the density of its structure, the transparency generally increasing with the density. Thus, charcoal

[redacted]

is opaque; but if the same charcoal be converted into a diamond, which it may be, without any change of the matter of which it is composed, it will become transparent.

Bodies are said to be imperfectly transparent, or semi-transparent, when light passes through them so imperfectly, that the forms and colours of the objects behind them cannot be distinguished. Ground glass, paper, and thin tissues in general, foggy air, the clouds, horn and various species of shell, such as tortoise-shell, are examples of this.

The degrees of this imperfect transparency are infinitely various; some substances, such as horn, being so nearly transparent as to render the form of a luminous object behind it indistinctly visible. Porous bodies, which are imperfectly transparent, usually have their transparency increased by filling their pores with some transparent liquid. Thus paper, which is imperfectly transparent, is rendered much more transparent by saturating it with oil, or by wetting it with any liquid. The variety of opal called hydrophane is white and opaque when dry, but when saturated with water it becomes transparent. Ground glass is rendered more transparent by pouring oil upon it. Two plates of ground glass placed one upon the other are very imperfectly transparent; but if the space between them be filled with oil, and their external surfaces be rubbed with the same liquid, they will be rendered nearly transparent.

901. *Opaque bodies become transparent when sufficiently attenuated.* — Bodies, however opaque, lose their perfect opacity when reduced to the form of extremely attenuated laminæ. Gold, one of the most dense of metals, is, in a state of ordinary thickness, perfectly opaque; but if it be reduced to the form of leaf-gold by the process of the gold-beater, and attached to a plate of glass, light will pass partially through it, and to an eye placed behind it it will appear of a greenish colour. Other metals, when equally attenuated, show the same imperfect opacity.

CHAP. II.

RECTILINEAR PROPAGATION OF LIGHT. — RADIATION. — SHADOWS AND PENUMBRÆ — PHOTOMETRY.

902. *Light transmitted in straight lines.* — One of the first properties recognised in light by universal observation and experience is, that when transmitted through a uniform medium, it maintains a rectilinear course.

A luminous point is a centre from which light issues in every direction through the surrounding space in straight lines. This effect of rectilinear propagation in all directions from a common centre is called *radiation.*

Any straight line along which light is transmitted is called a *ray of light.*

Any point from which rays of light radiate through the surrounding space, is called a *luminous point.*

The rectilinear propagation of light is established by numerous examples, and by a vast variety of effects, of which it affords the explanation. If any opaque object be interposed in a right line between the eye and a luminous point, the luminous point will cease to be visible; but if the opaque object be removed in the slightest degree from the direct line between the eye and the luminous point, the latter will become immediately visible.

This law, in its strictest sense, may be verified by the following experiment. Let three disks be pierced, each with a small hole, and let them be attached to a straight rod, in such a manner that the three holes shall be precisely in the same straight line, and consequently, at the same distance from the rod. If a light be placed behind one of the extreme disks, and the eye behind the others, the light will be visible. The ray, therefore, which renders it visible, must pass successively through the holes in the two extreme disks, and in the intermediate disk; but if the intermediate disk be slightly moved on either side, or upwards or downwards, or, in a word, have its position deranged in any manner, so that a thread stretched between the holes in the extreme disks would not pass through the hole in the intermediate disk, then the light will be no longer visible.

903. *Pencil of rays.* — Any collection of rays having a luminous point as their common origin, and included within the surface of a cone, or any other regular limit, is called a *pencil of rays.* The point from which such rays diverge, and which is the apex of the cone, is called the focus of the pencil.

When the surface of any object receives light from a luminous point, it is customary to consider each portion of such surface as the base of a pencil of rays, the focus of which is the luminous point, so that the illuminated surface of any body is considered as composed of the bases of a number of pencils of rays having the luminous point as their common focus.

When rays radiate from a luminous point in this manner, they are called *divergent.*

But cases will be shown hereafter, in which such rays may be so changed in their direction, that, instead of diverging from the same point, they will converge to a common point. In this case the rays are called *converging* rays, the pencils *converging* pencils, and the point towards which the rays converge, and at which they would meet, if not intercepted, is called the *focus of the pencil.*

[redacted]

904. *Shadow of a body.* — When light radiating from a luminous point through the surrounding space encounters an opaque body, it will be excluded from the space behind such body. The space from which it is thus excluded is called the *shadow* of the opaque body.

This term *shadow* is sometimes applied, not to the space from which the light is thus excluded, but to a section of such space formed upon the surface of some body placed behind the opaque body which intercepts the light. Thus, the floor or wall of a room intersecting the space from which light is excluded by an opaque body placed between such wall or floor and a luminary, will exhibit a dark figure, resembling more or less in outline the body which intercepts the light.

If a straight line be imagined to be drawn from the luminous point to the boundary of the opaque body, and to be continued beyond it indefinitely, such line being imagined to be moved round the opaque body following its limits and its form, that part of the line which is beyond the body will pass through a surface which will form the limits of the shadow of such body, or of the space from which it excludes the light. If such line, however, encounter a wall, screen, or other surface, it will trace upon such surface the limits of the shadow, in the common acceptation of that term.

If the opaque object be a sphere, or any other figure whose section taken at right angles to the direction of the luminous pencil is a circle, the shadow will be a truncated cone, as represented in *fig.* 267.

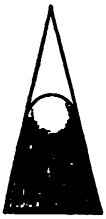

Fig. 267.

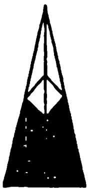

Fig. 268.

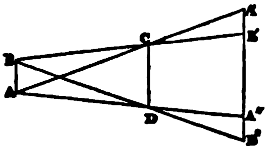

Fig. 269.

If the section be of a rectilinear figure, such, for example, as a square, the shadow will be what in geometry is called a truncated pyramid, as represented in *fig.* 268.

There is, however, no luminary which, strictly speaking, is a luminous point. All luminous objects have a certain definite surface of more or less extent, and consist therefore of an infinite number of luminous points. Now each luminous point of such a body is the focus of an independent pencil of luminous rays, and each such pencil encountering an opaque object will produce an independent shadow

905. *Cause of penumbræ.*—This gives rise to phenomena which it is necessary here more fully to explain.

Let C D, *fig.* 269, represent the section of an opaque object, and let B A represent the section of a luminary. B A will then consist of a line of luminous points, from each of which a pencil of rays will issue. The pencil which issues from the point B, will encounter the object C D, and the extreme rays of the pencil grazing the edge of the object, will proceed in the direction C B′ and D B″, being the continuation of the lines B C and B D. Now it is evident that the light proceeding from the point B will be excluded from the space included between the lines C B′ and D B″.

In like manner it may be shown that the light issuing from the point A will be excluded from the space included between the lines C A′ and D A″. It will also be easily perceived that the light proceeding from all the luminous points from A to B will be excluded from the space included between the lines C B′ and D A″; while more or less of such light, according to the position of the luminous points, will enter the space included between the lines C A′ and C B′, and the lines D A″ and D B″ respectively. The space, therefore, included between the lines C B′ and D A″, from which the entire light of the luminary A B is excluded, is called the umbra or absolute shadow; while the spaces included between C A′ and C B′ and between D A″ and D B″, from which the light of the luminary A B is only partially excluded, is called the *penumbra*, or imperfect shadow.

If a screen be fixed behind the body C D, the shadow and penumbra will be cast upon it, and will be perceptible. At B′ and A″, the boundaries between the shadow and the penumbra, the limit of shadow will be scarcely discernible, and the shadow will become gradually less dark, proceeding from such points to the points A′ and B″, which are the limits of the penumbra. The points A′ and B″ respectively receive light from all the points between A and B, but a point below A′ receives no light from the point A, or from the points immediately above it.

In like manner the points immediately above B″ receive no light from the point B, or the points immediately below it; and as we proceed onwards along the penumbra, the nearer we approach to the limits B′ and A″, the less will be the number of luminous points of the luminary A B from which light will be received. Hence it is, that the obscurity of the penumbra augments by degrees in proceeding from its outward limits to the limits of the umbra, where the obscurity becomes complete.

906. *Forms and dimensions of shadow.*—When an object is placed with its principal plane parallel to the plane of a screen, both being at right angles to the pencil of rays which proceeds from the luminary, the outline of the shadow will resemble the outline of the

object; but if the pencil fall obliquely on the object, or if the screen be not parallel to it, then the form and dimensions of the shadow will be distorted, the relative proportions and directions being different from those of the object.

When the sun is near the horizon, the shadow of an object standing vertically, which is cast upon a vertical wall, will present the form of the object with but little distortion, but the shadow which is cast upon the level ground will be disproportionally elongated in relation to its breadth.

907. *Light diminished in brightness by distance.—The intensity of light which issues from a luminous point diminishes in the same proportion as the square of the distance from such point increases.*

This is a common property of radiation, and has been already explained in the case of the radiation of sound. The intensity of the light at any point is in the direct proportion of the number of rays which fall upon a surface of given magnitude, or in the inverse proportion of the surface over which a given number of rays are diffused.

Now let us suppose a luminous point radiating in all directions round it to be the centre of a sphere. Let two spheres be imagined, having the luminous point as a common centre, and the radius of one being double the radius of the other. The surface of the greater sphere will be therefore twice as far from the luminous point as the surface of the lesser sphere; and since the surfaces of spheres are in the ratio of the squares of their radii, the surface of the greater sphere will be four times that of the lesser. Now since all the light issuing from the luminous point is diffused over the surface of such sphere, it is clear that its density on the surface of the lesser sphere will be greater than its density on the surface of the greater sphere, in the exact proportion of the magnitude of the surface of the greater sphere to the magnitude of the surface of the lesser sphere; that is, in the present example, as 4 to 1. In general it is evident, therefore, that the superficial space over which the rays issuing from a luminous point are diffused, is in the inverse proportion of the squares of the distances from the luminous point.

If, therefore, any opaque surface be presented at right angles to the rays proceeding from a luminous point, the intensity of the illumination which it receives will be increased in the same proportion as the square of its distance from its luminous point is diminished.

Since, then, the intensity of the light proceeding from each luminous point is inversely as the square of the distance from such point, it follows that the intensity of the light proceeding from any luminary will depend conjointly on, first, the number of luminous points upon the luminary, or, what is the same, the magnitude of the luminous surface; secondly, on the intensity of the light of each luminous point composing such surface; and thirdly, upon the distance from the luminary at which the illuminated object is placed.

908. *Absolute brilliancy depends conjointly on absolute intensity and distance.* — The absolute brilliancy of each luminous point composing any luminous object is called the absolute intensity of its light. Let this be expressed by I. Let the number of luminous points composing it, or the magnitude of its luminous surface, be expressed by S, and let the distance of the illuminated object from the luminary be expressed by D. The brilliancy of the illumination will then be expressed by

$$B = \frac{I \times S}{D^2}.$$

In other words, the brilliancy of the illumination is proportional to the absolute intensity of the luminary multiplied by the magnitude of its illuminating surface, and divided by the square of the distance of the illuminated object from it.

909. *Effect of obliquity of the light.* — It is here supposed, however, that the illuminated surface is placed at right angles to the rays of light, as would be the case with the surface of a sphere surrounding a luminous centre; but as it seldom happens that the illuminated surface has exactly this position, it is necessary to inquire in what manner the brightness of the illumination will be affected by its obliquity to the rays of light falling upon it.

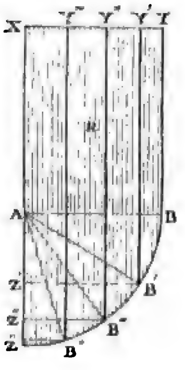

Fig. 270.

Let R, *fig.* 270., be a pencil of rays which we shall here suppose to be parallel; and let A B be a surface on which these rays fall. Let this surface be supposed to be capable of being turned upon the point A as a centre or hinge, so as to assume different obliquities in relation to the rays. If it were in the position A B, at right angles to the direction of the rays, it would receive upon it all the rays included between the lines A X and B Y. If it be in the position A B', it will receive upon it only the rays included between the lines A X and B' Y'. If it be in the position A B'', it will receive upon it only the rays included between the lines A X and B'' Y''. Again, if it be in the position A B''', it will receive upon it only the rays which are included between the lines A X and B''' Y'''.

Thus it is quite apparent that as the obliquity of the surface upon which the rays fall to the direction of the rays is increased, the number of rays incident upon such surface will be diminished, and that this diminution will be in the proportion of the distances B' Z', B'' Z'', B''' Z''', &c.

These lines are called in geometry the *sines* of the angles formed by the surfaces B′ A, B″ A, &c., with the direction of the rays.

It follows, therefore, that the intensity of the illumination produced upon a given surface by a given pencil of rays will diminish in the same proportion as the sine of the angle of obliquity of such surface to the direction of the rays is diminished.

It follows, therefore, evidently from this that the illumination is greatest when the surface is at right angles to the rays, and gradually diminishes until the surface is in the direction of the rays, when it ceases altogether to be illuminated.

910. *Methods of comparing the illuminating power of lights.* — If two luminaries, having equal luminous surfaces at equal distances from the same white opaque surface, placed at the same angle with the rays, shed lights of equal brightness on such surface, it follows that their absolute intensities must be equal.

In that case, the distances and the luminous surfaces being respectively equal, there is no other condition which can affect the illumination, except the intensity of the light proceeding from each luminous point; and since, therefore, the illuminations are equal, these intensities must be equal.

If, on the contrary, two such luminaries so placed produce different degrees of illumination on the same surface, their absolute intensities must be different, and must be in the proportion of the illuminations they produce. If in this case that luminary which produces the more feeble illumination be moved towards the illuminated object, until its proximity is increased, so that it produces an illumination equal to that of the other luminary, then the absolute intensity of the two luminaries will be as the squares of their distances. This may be demonstrated as follows:—

Let B express the brilliancy of the illumination produced by the two luminaries. Let S express the common magnitude of their luminous surfaces. Let I and I′ express their intensities, and let D and D′ express those distances which render their illuminations equal; we shall then have for the one

$$\mathrm{B} = \frac{\mathrm{I} \times \mathrm{S}}{\mathrm{D}^2},$$

and for the other,

$$\mathrm{B} = \frac{\mathrm{I}' \times \mathrm{S}}{\mathrm{D}'^2};$$

consequently, we shall have

$$\frac{\mathrm{I}}{\mathrm{D}^2} = \frac{\mathrm{I}'}{\mathrm{D}'^2};$$

and consequently,

$$\mathrm{I} : \mathrm{I}' :: \mathrm{D}^2 : \mathrm{D}'^2.$$

[redacted]

911. *Photometry.*—The art of measuring the intensity of light by observation is called *photometry*, and the instruments or expedients serving this purpose are called *photometers.*

The most simple form of photometer is that which may be called the method of shadows, and which is founded upon the principle which has just been demonstrated,—that with equal illumination the intensity of the light is directly as the square of the distance of the luminary.

912. *Photometer by shadows.*—This photometric apparatus, the invention of which is due to Count Rumford, consists of a white screen fixed in a vertical position, having a small opaque rod placed at a short distance from it, also in a vertical position. The screen, rod, and the two lights whose powers are to be compared, are so placed relatively to each other, that the two shadows of the rod formed by the two luminaries on the screen shall just touch without overlaying each other. Under these circumstances, it is evident that the space on the screen occupied by the shadow proceeding from each luminary will be illuminated by the other luminary. Thus, two spaces on the screen are exhibited in juxtaposition, each of which is illuminated by one of the luminaries independent of the other. It will at first be found that these two spaces will be unequally bright. The position of the luminaries, or of the screen or rod, must then, one or all, be changed until the two shadows, being still kept in juxtaposition, appear to be equally bright, so as to present a uniform shadow. Let the distance of the two luminaries from the shadows be then measured, and it will follow, according to the principle that has been already established, that the intensities of the two luminaries will be as the squares of these distances.

If in this case the two luminaries have equal luminous surfaces, their absolute intensities will be in the ratio of the squares of their distances; but if either luminous surface be unequal, the squares of the distances will represent the proportion, not of their absolute intensities, but of the products of their absolute intensity multiplied by their luminous surface.

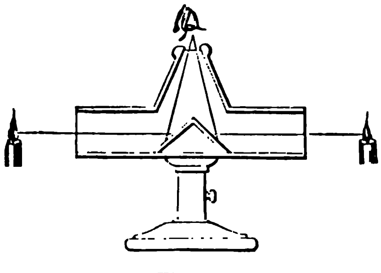

Fig. 271.

913. *Ritchie's photometer.*—Another photometer, on a simple and beautiful principle, proposed by the late Professor Ritchie, and represented in *fig.* 271., consists of a rectangular box about an inch and a half or two inches wide, and eight inches long, open at both ends, and blackened in the middle. In the centre of its length are two

surfaces placed at right angles with each other, and at an angle of 45° with the bottom of the box. Upon these surfaces, white paper is pasted. A round hole is made in the top of the box immediately over the line formed by the edges of the paper, so that an eye looking in at this hole may see equally the two surfaces of paper. To compare two lights, the instrument is placed in such a manner before them that each may illuminate one of the pieces of paper. The distance of the lights from the surfaces of the paper are then to be so adjusted by successive trials that the two surfaces of paper shall appear to the eye of uniform brightness. In that case, the illumination of the surfaces being the same, the illuminating powers of the luminaries will be in the same proportion as the squares of their distances from the paper, the principle of this being the same as that of the photometer of Count Rumford.

In this and all similar experiments, the colour of the light exercises a material influence on the results; and the comparative brilliancy cannot be ascertained with any precision, unless the two luminaries give light of nearly the same colour.

914. *Method of comparing the absolute intensity of light.*—When it is desired to ascertain the absolute intensities of the lights, it is, as has been stated, necessary to expose equal illuminating surfaces to the photometric apparatus; but as it is not always easy to produce luminaries having surfaces exactly equal, this object may be attained by the following expedient:—Let two opaque screens, having holes in them of exactly equal magnitude, be placed near and exactly opposite to the middle of each luminous surface. The rays of light which pass through the two apertures will in such case proceed from equal portions of the surfaces of the two luminaries, and the result of the experiment will therefore show the absolute intensities.

915. *Intensity of solar light.*—The sun produces the most intense illumination with which we are acquainted. This arises partly from the absolute intensity of that luminary, and partly from the vast extent of his luminous surface. The diameter of the sun is very near a million of miles, and consequently, being a sphere, the superficial extent of his surface is about three millions of square miles; but as one-half the surface only is presented to us at any one time, the magnitude of it will be a million and a half of square miles.

916. *Electric light.*—The most brilliant artificial light yet produced is inferior to the splendour of solar light in an incredible proportion. The brightest artificial lights are those produced by the contact of charcoal points, through which a galvanic current passes, and by lime submitted to the heating power of the oxyhydrogen blow-pipe. These lights, however, when projected on the disk of the sun, appear nevertheless as a blank spot.

CHAP. III.

REFLECTION OF LIGHT.

917. *Reflection varies according to the quality of the surface.*— When rays of light encounter the surface of an opaque body, they are arrested in their progress, such surfaces not being penetrable by them. A certain part of them, more or less according to the quality of the surface and the nature of the body, is absorbed, and the remaining part is driven back into the medium from which the rays proceed. This recoil of the rays from the surface on which they strike is called *reflection*, and the light thus returning into the same medium from which it had arrived, is said to be *reflected.*

The manner in which the light is reflected from such a surface varies according as the surface is polished or unpolished, and according to the degree to which it is polished.

We shall consider three cases: 1st, that of a surface absolutely unpolished; 2dly, that of a surface perfectly polished; and 3dly, that of a surface imperfectly polished.

CHAP. IV.

REFLECTION FROM UNPOLISHED SURFACES.

If light fall upon a uniformly rough surface of an opaque body, each point of such surface becomes the focus of a pencil of reflected light, the rays of such pencil diverging equally in all directions from such focus.

The pencils which thus radiate from the various points are those which render the surface visible. If the light were not thus reflected indifferently in all directions from each point of the surface, the surface would not be visible, as it is, from whatever point it may be viewed.

The light which is thus reflected from the various points upon the surface of any opaque body, has the colour which is commonly imputed to the body. The conditions, however, which determine the colour of bodies will be fully explained hereafter; for the present, it will be sufficient to establish the fact, that each point of the surface of an opaque body which is illuminated is an independent focus from which light radiates, having the colour proper to such point, by which light each such point is rendered visible.

918 *Irregular reflection.* — This mode of reflection, by which the forms and qualities of all external objects are rendered manifest to sight, has been generally denominated, though not as it should seem with strict propriety, the irregular reflection of light.

There is, nevertheless, nothing irregular in the character of the phenomena. The direction of the reflected rays is independent of each of the incident rays; but, nevertheless, such direction obeys the common law of radiation.

The existence of these radiant pencils proceeding from the surface of any illuminated object, and their independent propagation through the surrounding space, may be rendered still more manifest by the following experiment.

Let A B, *fig.* 272., be an illuminated object, placed before the window-shutter of a darkened room. Let *c* be a small hole made in the window-shutter, opposite the centre of the object. If a screen be held parallel to the window-shutter, and the object at some distance from the hole, an inverted picture of the object will be seen upon it, in which the form and colour of the object will be preserved; the magnitude, however, of such picture will vary according to the distance of the screen from the aperture. The less such distance, the less will be the magnitude of the picture.

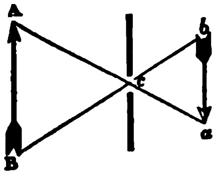

Fig. 272.

This effect is easily explained. According to what has been already stated, each point of the surface of the illuminated object A B is a focus of a pencil of rays of light having the colour peculiar to such point. Thus, each portion of the pencil of rays which radiates from the point B, and has for its base the area of the aperture *c*, will pass through the aperture, and will continue its rectilinear course until it arrives at the point *b* upon the screen, where it will produce an illuminated point corresponding in colour to the point B.

In the same manner, the pencil diverging from A, and passing through the aperture *c*, will produce an illuminated point on the screen at *a*, corresponding to the point A.

Each intermediate point of the object will produce a corresponding illuminated point on the screen. It is evident, therefore, that a series of illuminated points corresponding in arrangement and colour to those of the object will be formed upon the screen between *a* and *b*; their position, however, being inverted, the points which are highest in the object being lowest in the picture.

919. *Picture formed on wall by light admitted through small aperture.* — These effects may be witnessed in an interesting manner in any room which is exposed to a public thoroughfare frequented by moving objects. Let the window-shutters be closed and the interstices stopped so as to exclude all light except that which enters

through a small hole made for the purpose, and if no hole be found in the shutters sufficiently small, a piece of paper or card may be pasted over any convenient aperture, and a hole of the required magnitude pierced in it. Coloured inverted images of all the objects passing before the window will thus be depicted on a screen conveniently placed. They will be exhibited on the opposite wall of the room; but unless the wall be white, the colours will not be distinctly perceptible. The smaller the hole admitting the light is, the more distinct but the less bright the pictures will be. As the hole is enlarged the brightness increases, but the distinctness diminishes. The want of distinctness arises from the spots of light on the screen, produced by each point of the object overlaying each other, so as to produce a confused effect.

920. *Different reflecting powers of surfaces.* — Surfaces differ from each other in the proportion of light which they reflect and absorb. In general, the lighter the colour, other things being the same, the more light will be reflected and the less absorbed, and the darker the colour the less will be reflected and the more absorbed; but even the most intense black reflects some light. A surface of black velvet, or one blackened with lamp-black, are among the darkest brown, yet each of these reflects a certain quantity of rays. That they do so we perceive by the fact that they are visible. The eye recognizes such surfaces as differing from a dark aperture not occupied by any material surface, and it can only thus recognize the appearance of the material surface by the light which it reflects. The following experiment, however, will render this more evident.

921. *The deepest black reflects some light.*—Blacken the inside of a tube, and fasten upon the extremity remote from the eye a plate of glass. To the centre of this plate of glass attach a circular opaque disk, somewhat less in diameter than the tube, so that in looking through the tube a transparent ring will be visible, as represented in *fig.* 273. In the centre of this transparent ring will appear an intensely dark circular space, being that occupied by the disk attached to the glass.

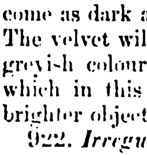

Fig. 273.

Now, let a piece of black velvet be held opposite the end of the tube, so as to be visible through the transparent ring. If the velvet reflected no light, then the transparent ring would become as dark as the disk in the centre; but that will not be the case. The velvet will appear by contrast with the disk, not black, but of a greyish colour, proving that a certain portion of light is reflected, which in this case is rendered perceptible by the removal of the brighter objects from the eye.

922. *Irregular reflection necessary to vision.*—Irregular reflection, as it has been so improperly called, is one of the properties of light which is most essential to the efficiency of vision.

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thout irregular reflection, light must be either absorbed by the es on which it falls, or it must be regularly reflected. If the which proceeds from luminous objects, natural or artificial, were ›ed by the surfaces of objects not luminous, then the only ɜ objects in the universe would be the sun, the stars, and artilights such as flames.

ese luminaries would, however, render nothing visible but them-

the light radiating from luminous objects were only reflected ırly from the surface of non-luminous objects, these latter l still be invisible. They would have the effect of so many rs, in which the images of the luminous objects only could be Thus, in the day-time, the image of the sun would be reflect- ɔm the surface of all objects around us, as if they were com- of looking-glass, but the objects themselves would be invisible. noon would be as though it were a spherical mirror, in which nage of the sun only would be seen. A room in which artilights were placed would reflect these lights from the walls and objects around as if they were specula, and all that would be e would be the multiplied reflections of the artificial lights.

egular reflection, then, alone renders the forms and qualities of s visible. It is not, however, merely by the first irregular re- ɪn of light proceeding from luminaries by which this is effected. ts illuminated and reflecting irregularly the light from their es, become themselves, so to speak, secondary luminaries, by ı other objects not within the direct influence of any luminary nlightened, and these in their turn reflecting light irregularly their surfaces, illuminate others, which again perform the same to another series of objects. Thus light is reverberated from t to object through an infinite series of reflections, so as to render nerable objects visible which are altogether removed from the ; influence of any natural or artificial source of light.

3. *Use of the atmosphere in diffusing light.*—The globe of the is surrounded with a mass of atmosphere extending forty or miles above the surface.

ıe mass of air which thus envelopes the hemisphere of the earth nted towards the sun, is strongly illuminated by the solar light, like all other bodies, reflects irregularly this light. Each par- of air thus becomes a luminous centre, from which light radiates 'ery direction. In this manner, the atmosphere diffuses in all tions the light of the sun by irregular reflection. Were it not ıis, the sun's light could only penetrate those spaces which are tly accessible to his rays. Thus, the sun shining upon the win- of an apartment would illuminate just so much of that apartment ›uld be exposed to his direct rays, the remainder remaining in ıess. But we find, on the contrary, that although that part of

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the room upon which the sun directly shines is more brilliantly illuminated than the surrounding parts, these latter are nevertheless strongly illuminated. All this light proceeds from the irregular reflection of the mass of atmosphere just mentioned.

924. *Diffusion of solar light by all opaque objects.* — But the solar light is further diffused by being again irregularly reflected from the surface of all the natural objects upon which it falls. The light thus irregularly reflected from the air falling upon all natural objects, is again reciprocally reflected from one to another of these through an indefinite series of multiplied reflections, so as to produce that diffused and general illumination which is necessary for the purposes of vision.

Light and shade are relative terms, signifying only different degrees of illumination. There is no shade so dark into which some light does not penetrate.

It is the same with artificial lights. A lamp placed in a room illuminates directly all those objects accessible to its rays. These objects reflect irregularly the light incident upon them, and illuminate thus more faintly others which are removed from the direct influence of the lamp, and thus, these again reflecting the light, illuminate a third series still more faintly; and so on.

925. *Effect of the irregular reflection of lamp-shades.* — When it is desired to diffuse uniformly by reflection the light which radiates from a luminary, the object is often more effectually attained by means of an unpolished opaque reflector than by a polished one. White paper or card answers this purpose very effectually. Shades formed into conical surfaces placed over lamps are thus found to diffuse by reflection the light in particular directions, as in the case of billiard-tables or dinner-tables, where a uniformly diffused light is required. A polished reflector, in a like case, is found to diffuse light much more unequally.

In case of white paper or card, each point becomes a centre of radiation, and a general and uniform illumination is the consequence. The light obtained by reflection in such cases is always augmented by rendering the reflector perfectly opaque; for if it be in any degree transparent, as is sometimes the case with paper shades put over lamps, the light which passes through them is necessarily subtracted from that which is reflected.

CHAP. V.

REFLECTION FROM PERFECTLY POLISHED SURFACES.

926. *Regular reflection.* — By what has been just explained, it appears that light reflected from rough and unpolished surfaces radi

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ates from all the parts composing them, as from so many foci of divergent pencils. If, however, the surface were absolutely smooth and perfectly polished, then totally different phenomena would ensue, which have been denominated *regular reflection.*

927. *Mirrors and specula.*—Surfaces which possess this reflecting power in the highest degree are called *mirrors* or *specula.*

The most perfect specula are those composed of the metals, the best being produced by various alloys of copper, silver, and zinc. If a glass plate be blackened on one side, the surface of the other will form for certain purposes a good reflector.

928. *Law of regular reflection.* — To explain the law of regular reflection, let C, *fig.* 274., be a point upon a reflecting surface A B, upon which a ray of light D C is incident. Draw the line C E perpendicular to the reflecting surface at C; the angle formed by this perpendicular and the incident ray D C, is called the *angle of incidence.*

From the point C, draw a line C D′ in the plane of the angle of incidence D C E, and forming with the perpendicular C E an angle D′ C E equal to the angle of incidence, but lying on the other side of the perpendicular. This line C D′ will be the direction in which the ray will be reflected from the point C. The angle D′ C E is called the *angle of reflection.*

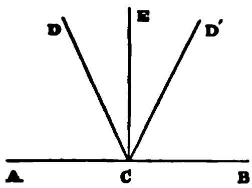

Fig. 274.

The plane of the angles of incidence and reflection which passes through the two rays C D and C D′, and through the perpendicular C E, and which is therefore at right angles to the reflecting surface, is called the *plane of reflection.*

This law of regular reflection from perfectly polished surfaces, which is of great importance in the theory of light and vision, is expressed as follows:—

When light is reflected from a perfectly polished surface, the angle of incidence is equal to the angle of reflection, in the same plane with it, and on the opposite side of the perpendicular to the reflecting surface.

From this law it follows, that if a ray of light fall perpendicularly on a reflecting surface, it will be reflected back perpendicularly, and will return upon its path; for in this case, the angle of incidence and the angle of reflection being both nothing, the reflected and incident rays must both coincide with the perpendicular. If the point C be upon a concave or convex surface, the same conditions will prevail; the line C E which is perpendicular to the surface, being then what is called in geometry, the normal.

929. *Experimental verification of this law.* — This law of reflection may be experimentally verified as follows:—

Let *c d c'*, *fig.* 275., be a graduated semicircle, placed with its diameter *c c'* horizontal. Let a plumb-line *b d* be suspended from its centre *b*, and let the graduated arc be so adjusted that the plumb-line shall intersect it at the zero point of the division, the divisions being numbered from that point in each direction towards *c* and *c'*. Let a small reflector (a piece of looking-glass will answer the purpose) be placed upon the horizontal diameter at the centre with its reflecting surface downwards, and let any convenient and well-defined object be placed upon the graduated arc at any point, such as *a*, between *d* and *c*. Now, if the point *a'* be taken upon the arc *d c'* at a distance from *d* equal to *d a*, the eye placed at *a'* and directed to *b* will perceive the object *a* as if it were placed in the direction *a' b*. It follows, therefore, that the light issuing from the point of the object *a* in the direction *a b*, is reflected to the eye in the direction *b a'*. In this case, the angle *a b d* is the angle of incidence, and the angle *d b a'* is the angle of reflection; and, whatever position may be given to the object *a*, it will be found that in order to see it in the reflector *b*, the eye must be placed upon the arc *d c'* at a distance from *d* equal to the distance at which the object is placed from *d* upon the arc *d c*.

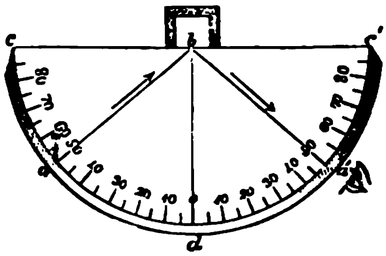

Fig. 275.

The same principle may also be experimentally illustrated as follows:—

If a ray of sun-light admitted into a dark room through a small hole in a window-shutter strike upon the surface of a mirror, it will be reflected from it, and both the incident and reflected rays will be rendered visible by the particles of dust floating in the room. By comparing the direction of these two visible rays with the direction of the plane of the mirror and the position of the point of incidence, it will be found that the law of reflection which has been announced is verified.

930. *Plane reflectors — parallel rays.* — If parallel rays be incident upon a polished plane reflecting surface, they will be reflected parallel; for since they are parallel, they will make equal angles with the perpendiculars to the surface at their points of incidence, and the planes of these angles will also be parallel.

The reflected rays will, therefore, also make equal angles with the perpendiculars, and the planes of reflection will be parallel; consequently the reflected rays will be parallel.

This may also be experimentally verified by admitting rays of solar

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light into a dark room through two small apertures. Such rays will always be parallel; and if they are received upon a plane mirror, their reflections will be found to be parallel, the rays and the reflections being rendered visible, as already explained.

931. *Divergent rays.* — If a pencil of divergent rays fall upon a plane mirror, the reflected rays will also be divergent, and their focus will be a point behind the mirror similarly placed, and at the same distance as the focus of incident rays is before it.

To demonstrate this, let A B, *fig.* 276., be the reflecting surface. Let F be the focus of the incident pencil from which the rays F A, F B, F C, &c. diverge, and let F A be perpendicular to the reflecting surface A B. If we take A F′ on the continuation of F A equal to A F, and draw the lines F′ B B′ and F′ C C′, then it can easily be perceived that the lines B B′ and C C′ make angles with the reflecting surface, and therefore with the perpendicular to it, equal to the angles which the incident rays F B and F C make with it respectively; for since A F is equal to A F′, F B will be equal to F′ B, and F C will be equal to F′ C; consequently the angles B F A and B F′ A will be equal, as will also the angles C F A and C F′ A. But the angles B F A and C F A are the angles of incidence of the two rays F B and F C; and since the angles B F′ A and C F′ A are respectively equal to them, and lie on opposite sides of the perpendicular, they will be the angles of reflection; consequently, the ray F B will be reflected in the direction B B′, and the ray F C in the direction C C′. These two rays, therefore, will be reflected from the points B and C as if they had originally radiated from F′ as a focus; and in the same way it may be shown that the other rays of a pencil diverging from F will be reflected from the mirror as if they had diverged from F′. But F′ is the point on the other side of the mirror which is placed similarly and at the same distance from the mirror as the point F is in front of it.

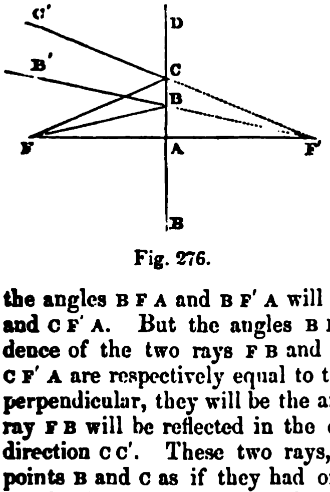

Fig. 276.

932. *Image of an object formed by a plane reflector.* — It follows from what has been just explained, that an object placed before a plane reflector will have an image at the same distance behind the reflector as the object is before it, for the rays which diverge from each point of the object will after reflection, according to what has been shown, diverge from a point holding a corresponding position behind the reflector, and if received after reflection by the eye of an observer will produce the same effect as if they had actually diverged from such point. All the rays, therefore, proceeding from the object, will after reflection follow those directions which they would follow had

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they proceeded from a series of points, on the surface of a similar object placed behind the reflector at the same distance as the object itself is before it, and consequently they will produce the same effect on the organs of vision as would be produced by a similar object placed as far behind the mirror as the object itself is before it.

The position of the different parts of the image formed in a plane reflector will be exactly determined by supposing perpendiculars drawn from every point on the object to the reflector, and these perpendiculars to be continued beyond the reflector to distances equal to those of the points from which they are drawn before it. The extremities of the perpendiculars so continued will then determine the corresponding points of the image.

It follows from this, that the images of objects in a plane reflector appear *erect*, that is to say, the top of the image corresponds with the top of the object, and the bottom of the image with the bottom of the object. But considered laterally with regard to the object itself, they will be *inverted*, that is to say, the left will become the right, and the right the left. This will be easily understood by considering that if a person stand with his face to a plane reflector, in a vertical position, his image will be presented with the face towards him, and the image of his right hand will be on the right side of his image as he views it, but will be on the left side of the image itself, and the same will apply to every other part of the image in reference to the object. There is, therefore, *lateral inversion.*

This effect is rendered strikingly manifest by holding before a reflector a printed book. On the image of the book all the letters will be reversed.

It follows also, from what has been explained, that if an object be not parallel to a reflector, but forms an angle with it, the image will form a like angle with it, and will form double that angle with the direction of the object.

Let A B, *fig.* 277., be a plane reflector, before which an object C D is placed. From C draw the perpendicular C O, and continue it from O to C', so that O C' shall be equal to O C. In like manner, draw the perpendicular D P, and continue it so that P D' shall be equal to P D. Then the image of C will be at C', and the image of D at D', and the image of all the intervening points between C and D will be at points intermediate between C' and D', so that C' D' shall be inclined to the reflector at the same angle as C D is inclined to it, and the object and the image will be inclined to each other at twice the angle at which either is inclined to the reflector.

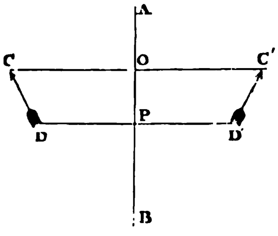

Fig. 277.

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Hence, if an object in a horizontal position be reflected by a reflector forming an angle of 45° with the horizon, its image will be in a vertical position; and if the object being in a vertical position be reflected by such a mirror, its image will be in a horizontal position.

If a reflector be placed at an angle of 45° with a wall, the image of the wall will be at right angles with the wall itself.

If a reflector be horizontal, the image of any vertical object seen in it will be inverted. Examples of this are rendered familiar by the effect of the calm surface of water. The country on the bank of a calm river or lake is seen inverted on its surface.

933. *Series of images formed by two plane reflectors.* — If an object be placed between two parallel plane reflectors, a series of images will be produced lying on the straight line drawn through the object perpendicular to the reflector. This effect is seen in rooms where mirrors are placed on opposite and parallel walls, with a lustre or other object suspended between them. An interminable range of lustres is seen in each mirror, which lose themselves in the distance and by reason of their faintness. This increased faintness by multiplied reflection arises from the loss of light caused in each successive reflection, and also from the increased apparent distance of the image.

Let A B and C D, *fig.* 278., be two parallel reflectors; let O be an object placed midway between them. An image of O will be formed at O' as far behind C D as O is before it, and another image will be formed at *o'* as far behind A B as O is before it. The image O' becoming an object to the mirror A B will form in it another image *o''* as far behind A B as O' is before it, and in like manner the image *o'* becoming an object to the mirror C D will form an image O'' as far behind C D as *o'* is before it. The images O'' and *o''* will again become objects to the mirrors A B and C D respectively; and two other images will be formed at equal distances beyond these latter. In the same way we shall have, by each pair of images becoming objects to the respective mirrors, an indefinite series of equidistant images.

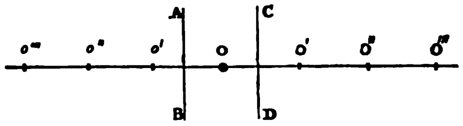

Fig. 278.

The distance between each successive pair of images will be equal to the distance of the object O from either of the images O' or *o'*, and consequently to the distance between the mirrors.

934. *Images repeated by inclined reflectors.* — A variety of interesting optical phenomena are produced by the multiplied reflection

of plane mirrors inclined to each other at different angles. As all these phenomena may be explained upon the same principle, it will suffice here to give a single example.

Let A B, A C, *fig.* 279., be two reflectors, inclined to each other at a right angle, and let O be an object placed at a point between them, equally distant from each. From O draw O M and O N perpendicular to A C and A B, and produce O M to O′ so that M O′ will be equal to M O; and produce O N to O″, so that N O″ shall be equal to N O. Two images of the point O will be formed at O′ and O″. The image O′ becoming an object to the mirror A B will have an image at O‴ just as far behind A B as O′ is before it; and, in like manner, the image O″ becoming an object to the reflector A C will have an image just as far behind A C as O″ is before it; but, in the present case, this latter image of O″ in the reflector A C will coincide with the image of O′ in the reflector A B, and will appear at O‴. Thus, the mirrors will present three images of the object O, which are placed at the angles of a square, of which the point A is the centre.

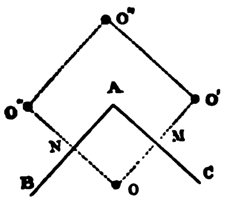

Fig. 279.

In the same manner, if the reflectors A B and A C be placed at an angle which is the eighth part of 360°, there will be formed seven images of the point O, which, with the point O, will be placed at the eight angles of a regular octagon of which the point A, where the mirrors meet, will be the centre; and like results will be found by giving the mirrors other inclinations.

935. *The kaleidoscope.* — The optical effects of the kaleidoscope depend upon this principle. Two plates of common looking-glass are fixed in a tube forming an angle of 45°, or some other aliquot part of 360°, with each other; semi-transparent objects of various colours are loosely thrown between them, and shut in by means of plates of glass at the ends, one of which is ground, so as to be semi-transparent. The images of the coloured fragments between the mirrors are multiplied so as to form a polygon as just described, and thus a regularity is given to their arrangement, however irregular their disposition may be between the mirrors. The effect of this instrument may be varied by a provision for varying the inclination of the mirrors.

936. *Optical toy by multiplied reflection.* — An amusing optical toy is represented in *fig.* 280., by means of which objects may be seen, notwithstanding the interposition of any opaque screen between them and the eye. The rays proceeding from the object P entering the tube *d* strike on the mirror *l* placed at an angle of 45°, and are reflected downwards vertically to the mirror *h*, also placed at 45°, from which they are reflected horizontally to the mirror *g* placed at 45°, from which they are again reflected vertically to the mirror *k*

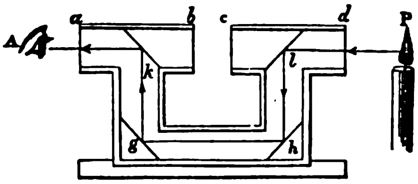

Fig. 280.

placed at 45°, from which they are reflected horizontally to it at A. The eye thus sees the object after four reflections, the rays which render it visible having travelled round the rectangular tube *l h g k a*.

937. *Formation of images by reflecting surfaces in general.* — In order that a reflector should produce a distinct image of an object placed before it, it is necessary that the rays diverging from each point of the object should, after reflection, diverge from, or converge to, some common point.

Thus, the surface of the object may be considered as an assemblage of foci of an infinite number of pencils of incident rays. Each of these pencils will, by reflection, be converted into other pencils having other foci, the assemblage of which will determine the form and magnitude of the image of the object produced by the reflector. In the case of a plane reflector, it has been shown that the assemblage of these foci corresponds in form and magnitude to the object, and therefore, in this case, the image is equal, and in all respects similar to the object; but this does not always happen.

938. *Magnified, diminished, or distorted images.* — The pencils of incident rays may be converted by reflection into pencils of reflected rays having different foci, but the assemblage of these foci may not correspond with the points forming the surface of the object. They may be similar to it in form, but greater in magnitude, in which case the reflector is said to magnify the object; or they may be similar to it in form and less in magnitude, in which case the reflector is said to diminish the object. In fine, they may assume such a form as to present the object in altered proportions. Thus, while the proportion of the vertical dimensions is preserved, that of the horizontal dimensions may be increased or diminished, or *vice versâ*; or either of these dimensions may be generally increased at various points of the image. In such case, the reflector is said to present a distorted image.

939. *Cases in which no image is formed.* — Since to produce a distinct image of any point in an object, it is necessary that the rays diverging from that point should be reflected, so as to diverge from some other point, if after reflection they have no common point of in-

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tersection, the point of the object from which they originally diverged can have no distinct image.

In this case the effect of the reflection will be to produce upon the vision a confused impression of the colour of the object, without any distinct form.

940. ***Conditions under which the reflected rays shall have a common focus.*** — In order, therefore, that a **polished surface** should reflect the rays which diverging from any point are incident upon it exactly to or from another point, it is necessary that the surface should be of such a nature that lines drawn from the two points in question to any one point on the surface shall make equal angles with the surface. No surface possesses this property except one whose section made by a plane passing through the two points is an ellipse, the two points being its foci. It follows, therefore, that if a pencil of light have its focus at one of the foci of an ellipse, the rays which diverging from such focus strike upon the ellipse, or upon any surface with which the ellipse would coincide, will be reflected to the other focus.

941. *Elliptic reflector.* — To render this more clear, let A C B D, *fig.* 281., be an ellipse whose foci are F and F′. Then, according to what has been explained, if two lines be drawn from F and F′ to any one point, such as P, in the ellipse, they will make equal angles with the ellipse; and, consequently, if F P be a ray of light forming part of a pencil of rays whose focus is F, it will be reflected along the line P F′ to the other focus.

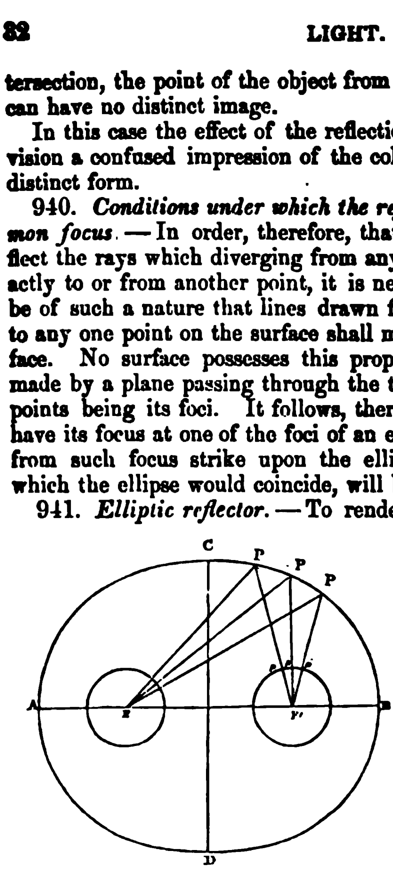

Fig. 281.

Now if we suppose a reflecting surface so formed that the ellipse by turning round the line A B as an axis will everywhere coincide with it, this surface is called an *ellipsoid;* and if it were a polished and reflecting surface, it would be called an *elliptic reflector.*

It is evident that it is not necessary that such a surface should form a complete ellipsoid. Any portion of it upon which a pencil of rays passing from one of the foci would fall, would reflect such pencil so as to make it converge to the other focus. In this case the pencil proceeding from the focus in which the luminous point is placed, would be a diverging pencil, and that which is reflected to the other focus would be a converging pencil.

942. *Parabolic reflectors.* — It has been shown (807.) that a parabola has a property in virtue of which a line drawn from any point in it. such as P, *fig.* 282., to a point F called its focus, and another,

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P M, parallel to its axis, make equal angles with the curve. It follows from this, that if the parabola possessed the power of reflecting light, rays diverging from its focus **F** would be reflected parallel to its axis **V M**; and, on the other hand, if rays directed along lines parallel to its axis were incident on the parabola, they would be reflected in the form of a pencil converging to its focus.

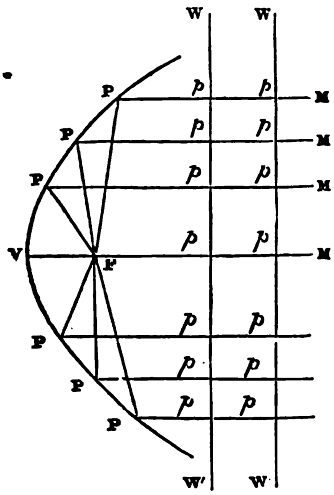

Fig. 282.

If we suppose the parabola to revolve round its axis **V M**, a surface with which it would everywhere coincide as it revolves is called a *paraboloid*; and if such a surface were polished so as to reflect light regularly, it would form a parabolic reflector. It follows, therefore, that if a luminous point be placed in the focus of such a reflector, its rays after reflection will be parallel to the axis; and, on the other hand, if rays strike upon the reflector in directions parallel to its axis, they will be reflected to its focus.

943. ***Experimental verification of these properties in the case of an elliptic reflector.*** — These remarkable properties of elliptic and parabolic reflectors may be easily verified by experiment. Let **A B C**, *fig.* 283., be the section of an elliptic reflector made by a plane passing through its focus **F**, the other focus being at **F'**. Let a luminous point, such as a small flame, or still better the light produced by two charcoal points when a galvanic current passes through them, be placed at the focus **F**.

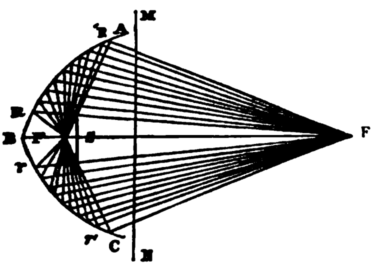

Fig. 283.

Let straight lines be imagined to be drawn from **F'** through the extremities of a circular screen **S**, meeting the reflector at **R** and *r*, and from the luminous point **F** draw the lines **F R** and **F** *r*. It is clear from what has been stated that a ray of light passing from **F** to **R** will be reflected from **R** to **F'**; and one passing from **F** to *r* will be reflected from *r* to **F'**, both grazing the edge of the screen **S**; and the same will be true for all rays passing from **F** which are incident upon

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a circle traced on the reflector whose diameter would be a line joining R and *r*.

The rays proceeding from F, and incident between the points R and *r*, will after reflection strike upon the screen S, and will thus be prevented from proceeding towards the point F'.

From the point F draw the lines F R' and F *r'* passing the extremities of the screen S. It is clear that the rays passing from F between the lines F R' and F *r'* will be intercepted by the screen.

Thus it follows that all the rays which strike upon the reflector, and which are not intercepted by the screen S, are included on the one side by the lines F R and F R', and on the other by the lines F *r* and F *r'*.

Now, according to what has been explained, all the rays incident upon the surface of the reflector would, after reflection, converge to the point F', as represented in the figure. To verify this fact, let a white screen M N be placed between F' and S, at right angles to the line F' S. The reflected light will appear upon this screen when held near to S, as an illuminated disk with a small circular dark spot in its centre, this dark spot corresponding to the space from which the light both direct and reflected is excluded by the small screen S. If the screen M N be now gradually moved towards F', being kept perpendicular to the line S F', the illuminated disk will gradually diminish in diameter, as will also the dark circular spot in its centre, and this diminution will continue until the screen arrives at the point F', when the illuminated disk will be reduced to a small light spot, and the dark spot in its centre will disappear.

This experiment may be further varied by placing the screen M N as near the reflector as possible, and piercing several holes in it within the area of the illuminated disk. The rays of light passing through these holes will severally converge to the point F', as may be shown by holding another screen beyond M N, by means of which the course of the rays may be traced, since their light will produce light spots upon this screen. As it is moved towards F', these light spots will gradually approach each other, and when it arrives at F' they will coalesce and form a single spot.

944. *Case of a parabolic reflector.* — The reflecting property of a parabolic reflector may be experimentally exhibited by a like expedient. Let A B C, *fig.* 284., represent a section of the reflector, the focus being F. Let a luminous point be placed at F, and a small circular screen S, as before, be placed perpendicular to the axis, and near the point F. It may be shown, as in the case of the elliptic re-

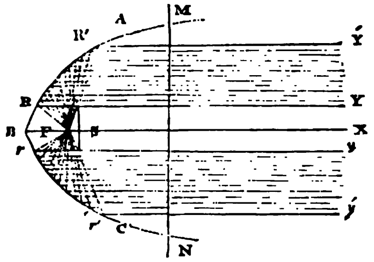

Fig. 284.

flector, that the rays F R′ and F *r*′, which graze the screen, will be reflected in the direction R′ Y′ and *r*′ *y*′, parallel to the axis B X; and, in like manner, that the rays F R and F *r*, which, after reflection, graze the screen, will also be reflected in the direction R Y and *r y* parallel to the axis.

Hence it follows that the reflected light will be excluded from a cylindrical space, of which the screen S is the circular base, and whose axis coincides with the axis B X of the reflector.

It also appears that no light diverging from the focus F will strike the reflector beyond the points R′ and *r*′. The light reflected will therefore be included between two cylindrical surfaces, having the axis of the parabola as their common axis, the sides of the exterior cylinder being R′ Y′ and *r*′ *y*′, and those of the interior cylinder being R Y and *r y*.

It is easy to verify these phenomena. Let a white screen M N be held as before at right angles to the axis B X, an illuminated disk will appear upon it, whose diameter will be equal to the line R′ *r*′, having a small dark spot in the centre, equal in magnitude to the screen S. If the screen M N be moved towards or from the screen S, this illuminated disk will continue of the same magnitude, having a dark spot in the centre constantly of the same magnitude also. Thus it appears that the reflected rays must follow the course already described.

The experiment may be further varied, as in the case of the ellipse, by piercing several holes in the screen M N, through which distinct rays shall pass. These rays being received upon another screen behind M N, will produce upon it luminous spots, and if then either screen be moved towards or from M N, these spots will maintain always the same relative position.

If, in the case of the elliptic reflector, the luminous point be placed at F′, *fig.* 283, instead of F, then the effects will take place in an inverse order, the incident rays being in this case what the reflected rays were in the former, and *vice versâ;* and the phenomena may be verified by a like expedient. If a small circular screen be held between S and B at right angles to the axis, it will be found that the rays reflected from the elliptic surface will be inclosed between two conical surfaces, one of which is bounded by F R′ and F *r*′, and the other by F R and F *r*. The light will be excluded from the cone whose base is the screen S, and whose vertex is at F; and also from the cone whose base is R *r*, and whose vertex is also at F.

In the same manner, all the effects will be inverted if a cylinder of rays parallel to the axis be directed upon a parabolic reflector. In this case, the reflected rays will be included between the conical surface bounded by the lines F R′ and F *r*′, *fig.* 284., and the conical surface bounded by the lines F R and F *r*.

This may be in like manner experimentally verified by means of

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a white screen moved before the screen s in the vertex B of the reflector.

945. *Parabolic reflectors useful as burning reflectors.*—In consequence of this property, parabolic reflectors are well adapted for collecting the rays of the sun or moon into a focus. Owing to the enormous distance of these objects, compared with any magnitudes which can be subject to experiment, all pencils proceeding from them may be considered as parallel.

If, then, a parabolic reflector be placed so that its axis shall be directed towards the sun, the rays of the sun reflected by it will be collected in its focus; and as their heating power will then be proportionally augmented, the apparatus may be used as a burning reflector.

946. *Experiment with two parabolic reflectors.*—If two parabolic reflectors be placed at any distance asunder, their axes coinciding, the rays proceeding from a luminous point placed in the focus of one will, after two reflections, be collected into the focus of the other.

Thus, if A B and A′ B′, *fig.* 285., be the two parabolic reflectors, the light proceeding from a luminous point at F will be reflected by the surface A B in lines parallel to V V′, and striking upon the reflector A′ B′ will converge to the focus F′.

This is precisely similar to, and explicable on the same principles as the phenomena of echo explained in 879.; all that has been explained above in reference to elliptic reflectors is also analogous to the phenomena of echo explained in 879.

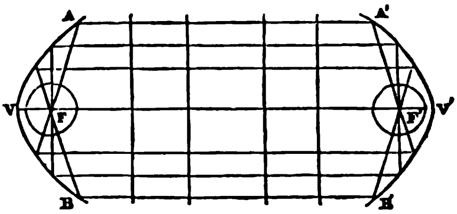

Fig. 285.

Thus the reflection of light is in all respects analogous to the reflection of sound, and subject to the same laws.

947. *Reflection by elliptic or parabolic surfaces when the luminous point is not in the focus.*—If, in the preceding experiments, the luminous point be moved from the position of the focus F, and be placed either nearer to or further from the reflector, or above or below the focus, the reflected rays will no longer converge to a common point after reflection by an elliptic surface, nor will they proceed in parallel directions after reflection by a parabolic surface. These effects may be verified experimentally by the same expedients as before.

If, when the luminous point is placed before the reflector out of the focus F, the screen M N be moved as before, the reflected rays will pro-

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duce upon it as before an illuminated disk; but this disk will not be reduced to a luminous point by moving the screen from the reflector; it will diminish in magnitude to a certain limit, and then increase, but will not in any case be reduced to a point.

In the same manner with the parabolic reflector, when the light is placed out of the focus, the illuminated disk produced upon the screen will not continue to be of the same magnitude, but will either increase or diminish, according as the luminous point is placed within or beyond the focus. In the latter case, however, although the illuminated disk will diminish, it will not be reduced to a point, but after being reduced to a certain magnitude, it will again increase, and in all these cases the disk will be much more regular in its outline than in the former case.

It appears, therefore, that an elliptical reflector will only convert rays diverging from a determinate point into rays converging to another determinate point, when the former of these points is at one of the foci; and a parabolic reflector will only convert diverging rays into parallel rays when these rays diverge from the focus, and will only convert parallel rays into rays converging to a determinate point when these parallel rays are parallel to the axis.

948. *Spherical reflectors.* — The form of reflecting surface, however, which is most easy of construction, and most convenient in practice, and consequently which is most generally used, is the *spherical reflector.*

The spherical reflector is a surface which may be conceived to be formed by the arc of a circle less in magnitude than a semicircle revolving round that diameter which passes through its middle point.

Thus, let us suppose A B C, *fig.* 286., to be such an arc, B being its middle point, and O its centre. Taking the line B O X as an axis of revolution, let the arc be imagined to rotate round it.— Now let a surface be conceived, which with the arc as it revolves would be everywhere in exact contact. Such a surface is that of a spherical reflector. If the concave side of it be the polished side, it is called a *concave reflector*, the solidity and thickness being then on the convex side; but if the solidity be included within the concavity, and the convex side be polished, then the reflector is said to be *convex.*

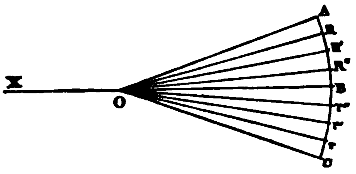

Fig. 286.

These two classes of spherical reflectors, *concave* and *convex*, have distinct properties, which will be explained in succession.

The point B, which is the middle point of the generating arc, is called the *vertex* of the reflector; and the point O, the centre of the

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generating arc, is called its *centre*. The length A C of the generating arc itself, expressed in degrees, is called the *opening of the reflector*. Consequently, the angle which the axis O B makes with the radius O A drawn to the edge of the reflector is half the opening. The right line B O X, drawn through the vertex and the centre of the reflector, is called the *axis of the reflector.*

Since all radii of a circle are at right angles to the circumference at the point where they meet it, it follows also that the radii of a spherical surface are at right angles to such surface. Hence it follows, that all radii of a spherical reflector, such as O B, O B′, O B″, &c., are respectively at right angles to the surface of the reflector.

These definitions and consequences are equally applicable to concave and convex reflectors.

When a pencil of rays proceeding from any luminous point or illuminated object is incident upon a spherical reflector, that ray of the pencil which passes through the centre O of the reflector is called the *axis of the pencil.* Thus, if a pencil of rays diverging from the point I, *fig.* 287., 288., be incident upon the reflector A B C, the axis

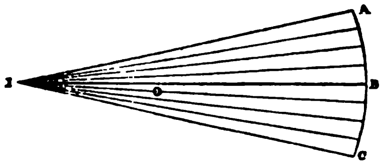

Fig. 287.

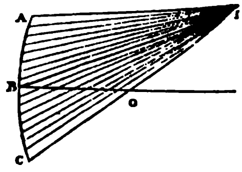

Fig. 288.

of that pencil will in such case be the line I O passing through the centre O of the reflector, and meeting the surface.

In the case represented in *fig.* 287., the axis of the pencil coincides with the axis of the reflector; but in the case represented in *fig.* 288., it is inclined to it at the angle B O C. A pencil, such as that represented in *fig.* 287., is called the *principal pencil,* and the line I B the principal axis. The pencil represented in *fig.* 288. is called the *secondary pencil,* and the axis I O a secondary axis.

It is clear, from mere inspection of the diagram, that the axis of the principal pencil is the axis of the reflector. But in the case of the secondary pencil, represented in *fig.* 288., the axis I O C of the pencil is not in the centre of the rays which strike the reflector, being more on the side B A than on the side B C.

The axis of a pencil of parallel rays is defined in the same manner; a principal pencil of parallel rays being one whose direction is parallel to, and whose axis coincides with the axis of the reflector, and a secondary pencil of parallel rays being one whose rays and axis are inclined to the axis of the reflector.

A principal pencil of parallel rays is represented in *fig.* 289., B O X being its axis; and a secondary pencil of parallel rays is represented in *fig.* 290., X O B′ being its axis.

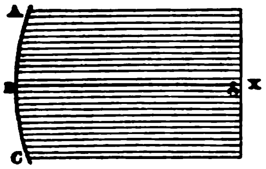

Fig. 289.

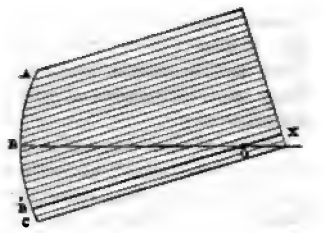

Fig. 290.

949. *Reflection of parallel rays by spherical surfaces.* — Let us first consider the case of a principal pencil of parallel rays.

Let R Y and *r y fig.* 291., be two rays of the pencil at equal distances from the axis B O X. Draw O R and O *r*. These being radii of the reflector, will be perpendicular to its surface; and since the angles of reflection are equal to the angles of incidence, the reflected rays will proceed in the direction R P, *r* P making with the lines O R and O *r* angles equal to the angles of incidence O R Y and O *r y*. But it is evident that since R Y and *r y* are parallel to B X, the angles O R Y and O *r y* are equal to the angles R O P and *r* O P. From this it follows that P R, P O, and P *r* are equal to each other.

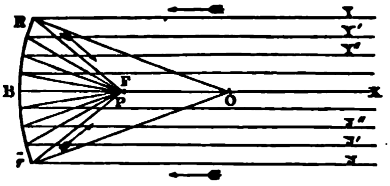

Fig. 291.

Since the two sides of a triangle taken together must be greater than its base, P R and P O taken together are greater than the radius O R of the reflector, and consequently O P must be greater than half of O B. If then F be the middle point of O B, the point P will be between F and B, and this will be the case at whatever point of the reflector the rays R Y and *r y* are incident.

Now, if two other parallel rays R′ Y′ and *r′ y′* be taken, in like manner equally distant from B X, but nearer to it than R Y and r y, it can be shown that they will be reflected to a common point in the axis O B between P and F. In the same manner, if two other parallel rays R″ Y″ and r″ y″, still equally distant from the axis B X, but nearer to it than R′ Y′ and r′ y′, be reflected, they will converge to a common point, still nearer to the middle point F of the axis O B, but still between F and B; in a word, the nearer such rays are to the axis B X, the nearer will be their common point of convergence after reflection to the middle point F; but however near they may be to B X, they cannot converge to any point beyond F in the direction of the centre O.

It is evident, therefore, from these results, that parallel rays incident upon a spherical surface do not after reflection converge to any common point, since each cylindrical surface formed by such rays converges to a different point upon the axis; nevertheless, it appears, that all these points of convergence are included within a small space P F upon the axis, provided that the reflector have not great extent; and it is found, that if the reflector do not extend to more than about 5° or 6° on each side of its vertex, all the parallel rays reflected from it will converge so nearly to the middle point F of the radius O B passing through its vertex, that, for practical purposes, the reflector may be considered as possessing the properties of a parabola already explained, and the reflected rays may be considered as vertically convergent to a common point. This common point will be F, the middle point of the radius O B, which forms the axis of the reflector, and which is parallel to the incident rays.

If a secondary pencil of parallel rays be incident on the reflector, as represented in *fig.* 292., the focus to which its rays will be reflected will be the middle point F of the radius O B′, which forms the secondary axis.

All the reasoning which has been applied to the principal pencil, *fig.* 291., will be equally applicable in this case.

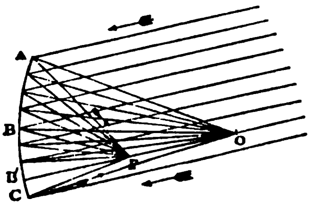

Fig. 292

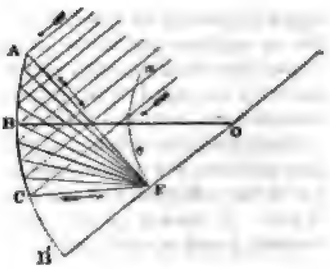

Fig. 293.

If a secondary pencil be inclined to the axis O B, at an angle greater than half the opening of the reflector, its axis will not meet the reflecting surface. This case is represented in *fig.* 293., where the line O F B′ drawn through the centre parallel to the rays of the pencil pass below the limit C of the reflector. In such a case, nevertheless, the focus of the reflected rays is determined in the same manner as it would be if the reflector extended to B′, and, accordingly, the rays reflected from A C will converge to a focus at F, the middle point of O B′.

950. *Principal focus of spherical reflector at the middle point of the radius.* — If, therefore, any number of pencils of parallel rays, principal and secondary, are incident upon the same reflector, their several foci will lie at the middle point of the radii of the reflector which coincide respectively with their several axes; and if an infinite number of such pencils fall at the same time on the reflector, their foci will form a circular arc *a c*, *fig.* 293., whose centre is the centre of the reflector O, and whose radius is O F, one-half the radius of the reflector.

951. *Experimental verification.*—All these effects may be experimentally verified by means of screens, in a manner similar in all respects to that which has been already explained in the case of a parabolic reflector. Thus it can be shown, that if the opening of a reflector be much greater than 20°, parallel rays will not be reflected converging to a common point; and, on the other hand, if a luminous point be placed at F, *fig.* 292., the reflected rays will not be parallel; but if the opening do not exceed 20° or thereabouts, parallel rays will be sensibly convergent to the point F after reflection, and rays diverging from F will be reflected in directions sensibly parallel.

The focus to which parallel rays converge after reflection is called the principal focus of the reflector.

It follows, therefore, from what has been stated, that the principal focus of a spherical reflector is the middle point of that radius which is parallel to the incident rays; and the principal foci for secondary pencils of parallel rays lie in a spherical surface *a c*, *fig.* 293., whose centre is the centre of the reflector, and whose radius is half the radius of the reflector.

952. *Aberration of sphericity.* — When the opening of a spherical reflector exceeds the limit already stated of about 20°, parallel rays falling on that part of its surface which is more than 10° from its vertex will be reflected sensibly distant from the principal focus, and consequently the entire pencil of rays whose base is the reflector will not have a common point of convergence. Those which are incident upon the reflector within a distance of 10° from its vertex will converge sensibly to the principal focus; but those beyond that limit will converge to points more or less distant from the principal focus, according as these points of incidence, more or less, exceed a distance of 10° from the vertex of the reflector.

[redacted]

This departure from correct convergence, produced by the too great magnitude of the reflecting surface, is called the *aberration of sphericity*, or spherical aberration.

To convey a more exact idea of the form and curvature of a spherical reflector which has the effect of effacing spherical aberration, such a reflector is represented in *fig.* 294., where A C is an arc 20° in length, representing the vertical section of the reflector, B being its vertex, O its centre, and F its principal focus. Rays falling on A C parallel to O B would be reflected sensibly to the point F; but if the reflector were greater in the opening, as, for example, if it extended to A′ and C′, being 20° on each side of the vertex B, then the parallel rays incident at its extreme points A′ and C′ would be reflected to *f*, a point between F and B. In such cases, the space *f* F would be that within which all the rays incident between A and A′, and between C and C′, would be collected. This space *f* F would then be the extremity of the aberration of sphericity due to a reflector 40° in magnitude.

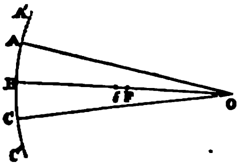

Fig. 294.

The spherical aberration of a secondary pencil will be greater than that of a principal pencil; for in the case of the secondary pencil represented in *fig.* 293, the axis of which is in the direction of O B′, the aberration will be the same as if the opening of the reflector were twice the arc A B′; and in proportion as the angle formed by the axis of the secondary pencil O B′ with the axis of the reflector O B is increased, this cause of aberration will be also increased. Thus in the secondary pencil represented in *fig.* 293, the aberration would be the same as if the opening of the reflector were twice the angle A O B′.

In fine, the aberration attending any secondary pencil will always be the same as that which would be produced with a principal pencil by a reflector whose opening would be equal to the opening of the proposed reflector, added to twice the angle formed by the axis of the reflector and the axis of the secondary pencil. Thus, in the case represented in *fig.* 293, the aberration of the secondary pencil is the same as would be produced upon a principal pencil by a reflector having an opening equal to twice A B′.

953. *Case of convex reflectors.* — In what precedes, the case of concave reflectors only has been contemplated. The same conclusions, however, will be applicable, with but little qualification, to the case of convex reflectors.

Let such a reflector be represented by A C, *fig.* 295, a pencil of rays parallel to the axis B X being incident upon it. The extreme

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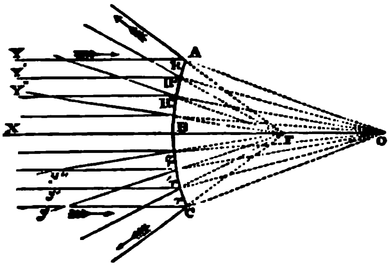

Fig. 295.

rays R Y and *r y*, equidistant from B X, will be reflected from R and *r*, as if they had diverged from F, the middle point of O B, provided R and *r* be not more distant than 10° from B. In the same manner, the rays R' Y' and *r' y'*, and also the rays R'' Y'' and *r'' y''*, and, in a word, all rays between the extreme rays and the axis, will be reflected as if they had diverged from F. This point F, being the middle point of the radius O B, is therefore, as in the case of the concave reflector, the principal focus.

A difference is presented here in the two cases, which suggests a distinction to which we shall often have occasion to recur in other instances. In the case of the concave reflector represented in *fig.* 291, the principal focus is a point to which the reflected rays do actually converge, and where, as has been shown, the light is concentrated. In the case, however, of the convex reflector represented in *fig.* 295, the rays diverging from the surface diverge as if they had originally been united at F. This point F is, therefore, in such case, not a point, as in the case of a concave reflector, where the rays do actually coalesce, but a point where they would coalesce if they had been continued backwards from the points on the surface of the reflector.

954. ***Foci real and imaginary.***—A focus like the former, where the rays do actually converge, is called a real focus, and sometimes a physical focus; whereas a focus like the latter, in which the rays do not actually converge, but which merely forms the point of convergence of their directions, is called an imaginary focus. In the case already explained of plane reflectors, the focus of reflection of a divergent pencil is an imaginary focus; and, on the other hand, of a convergent pencil is a real or physical focus.

955. ***Images formed by concave reflectors.***—If an object be placed before a concave reflector at so great a distance from it that all pencils of rays passing from such object would be considered as parallel, an image of such object will be formed at the principal focus of the reflector; that is to say, midway between its centre and its surface.

Let A C, *fig.* 296, be such a reflector, B being its vertex, O its centre, and F the principal focus. Let L M be an object, placed at so great a distance from the reflector, that the divergence of a pencil of rays passing from any point upon it, and having the reflector as their

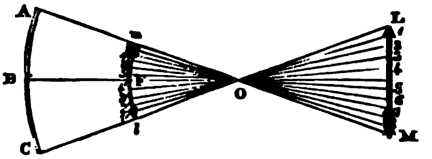

Fig. 296.

base, shall be so small that the rays may be considered as practically parallel.

Let L O *l* be the axis of the secondary pencil passing from L, and M O *m* the axis of the secondary pencil passing from M, *l* and *m* being respectively the middle points of the radii, and therefore the foci to which the pencils proceeding from L and M respectively are collected after reflection. Images, therefore, of the points L and M respectively will be produced at *l* and *m*.

In the same manner, the pencils proceeding from the several points marked 1, 2, 3, 4, 5, &c., will converge, after reflection, to the corresponding points marked 1′, 2′, 3′, 4′, 5′, &c., which are the middle points of the several radii which are in the direction of the axes of the several pencils. At these points, therefore, images will be formed of the corresponding points in the object, and the assemblage of these images will form a complete image of the object in an inverted position, midway between the centre O and the surface A B C of the reflector.

It is evident that the points forming the image *m l* will lie in a spherical surface, whose centre is O, and whose radius is half the radius of the reflector. If, therefore, the object be a straight line, its image will be the arc of a circle; and if the object be a plane surface, its image will be a spherical surface.

In the case represented in *fig.* 296., the central point of the object is placed in the direction of the axis of the reflector, and the central point of the image lies consequently also in the axis, and the image is at right angles to the axis of the reflector and is bisected by it.

It will be explained hereafter that the apparent visual magnitude of an object is determined by the angle formed by two straight lines drawn from the eye to the extremities of the object. Thus if the eye were placed at O, the centre of the reflector, the angle L O M would be the apparent magnitude of the object. The full import and propriety of this term will be explained more fully hereafter, but for the present it will be convenient to use it in the sense just explained.

It is evident, then, that the apparent magnitude of the object L M, as viewed from the centre of the reflector O, is the same as the apparent magnitude of its image *l m* viewed from the same point, since the lines drawn from the limits of the object and the image intersect each other at the point O.

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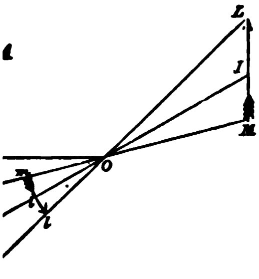

Fig. 297.

It is evident also that the linear magnitude of the image will be just so much less than the linear magnitude of the object, as one-half the radius of the reflector O F is less than the distance of the object from the centre O.

956. *Case in which the object is not placed on the axis of the reflector.* — The case in which the axis of the reflector is not directed to the centre of the object is represented in *fig.* 297.

his case the image of the object L M is produced at *l m*, be-the axes of the secondary pencils, proceeding from the extre-of the object L M, and at the middle points of the radii which with the axes.

ie case of a convex reflector, let L M, *fig.* 298., be the object, as before, at such a distance that each pencil of rays proceed-

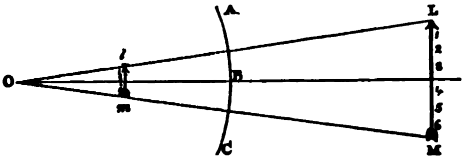

Fig. 298.

n a point in the object to the reflector may be considered as . Let L O and M O be the axes of the pencils proceeding from reme points of the object. After reflection, the rays of these will diverge as if they had proceeded from the points *l*, *m* re-ly, which are the middle points of the radius of the reflector; refore, if such rays were received by the eye of an observer, uld produce the same effect on vision as if they had proceeded ie points *l*, *m*, and consequently the points L M of the object ppear as if they were placed at *l*, *m*. In the same manner, it shown that the intermediate points 1, 2, 3, 4, 5 of the object ear as if they were at the intermediate points 1, 2, 3, 4, 5 of i, which are in the direction of their respective pencils. Thus an cted to the reflector, receiving the rays of the reflected pencils, the object as if it were on a spherical surface, of which the s O, and of which the radius is one-half the radius of the .

image *l m* in this case, though not formed by the real intersetion of the rays of light, is not the less present to vision. The eye receives the rays exactly as it would receive them if they had actually diverged from the points *l*, 1, 2, 3, 4, 5, *m*, and consequently the effect on vision is the same as if a real image of the object were placed at *l m*.

It is evident from the figure, that in this case the image is erect, and not inverted, as in the case of the concave reflector.

All that is said, however, of the relative magnitudes of the image and object in the case of the concave reflector, will be equally applicable here. Thus, to an eye placed at O, the apparent magnitude of the object L M, and of its image *l m*, will be the same; and the real linear magnitude of the image will be just so much less than that of the object, as one-half the radius of the reflector is less than the distance of the object.

957. *Experimental verification of these principles.*—The phenomena which have been just explained in the case of the reflection of very distant objects produced by concave and convex reflectors, may easily be verified experimentally.

If a concave reflector be directed towards the sun or moon, an image of either of those objects will be found at its principal focus, and such image may be rendered apparent by holding at its principal focus, and at right angles to the radius directed to the object, a small semi-transparent screen, which may be formed of ground glass o oiled paper.

A small image will be seen upon the screen, the diameter of whic will bear the same proportion to the *real* diameter of the sun or moon as half the radius of the reflector bears to the distance of one or othe of these objects.

The effects of a convex reflector can be still more easily made man fest. When a convex reflector is presented to any distant object their images will be seen in it, and will appear as if they were behin the reflector.

They will be less in magnitude than the objects in the proportio in which half the radius of the reflector is less than the distance c the objects, and they will appear as if they were painted on a sphe cal surface, having its centre at the centre of the reflector, and havi half the reflector for its radius.

958. *Geometrical principles on which the explanation of the phen mena depends.*—Before proceeding to explain the effects produced l spherical reflectors on diverging and converging pencils, it will convenient here briefly to state some principles derived from geom try, to which it will be necessary frequently to recur in explanati of the effects produced on pencils of rays by spherical surfaces which they are incident, whether these surfaces belong to opaq bodies or transparent media.

The magnitude of angles is easily explained by stating the degr and parts of degrees of which they consist. It may also be often more conveniently expressed by stating the ratio which the arc which bounds them bears to the radius. Thus an angle B A C, *fig.* 299., will be perfectly defined if the ratio which the arc B C bears to its radius A B be stated. Any other angle, such as *b a c*, the arc of which *b c* bears the same ratio to the radius *b a*, will necessarily have the same magnitude. This principle may be rendered still clearer, if, with A as a centre, several arcs, such as $B' C'$, $B'' C''$, $B''' C'''$, &c. be drawn subtending some angle A. It is demonstrated in geometry, and made evident from the figure, that the arcs $B' C'$, $B'' C''$, $B''' C'''$, bear respectively the same ratio to their radii $A B'$, $A B''$, $A B'''$, as the arc B C bears to its radius A B.

On this principle, the magnitude of an angle may with great convenience be expressed by a fraction, whose numerator is its arc, and whose denominator is its radius. Thus the angle A, *fig.* 299., may be expressed by $\frac{B C}{B A}$, or $\frac{B' C'}{B' A}$, or $\frac{B'' C''}{B'' A}$, &c.

If the angles be very small, perpendiculars drawn from the extremity of either side, including them upon the other, may be considered

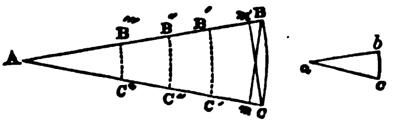

Fig. 299.

as equal to the arc. Thus, in *fig.* 299., the perpendiculars B *m* and C *m'* may be regarded as equal to the arc B C, provided the angle A do not exceed a few degrees.

In the case of such angles, therefore, their magnitude may be easily expressed by a fraction whose numerator is the perpendicular, and whose denominator is the radius.

Thus the angle A, being small, will be expressed by $\frac{B m}{B A}$ or by $\frac{C m'}{C A}$.

REFLECTION OF DIVERGENT AND CONVERGENT RAYS BY SPHERICAL SURFACES.

959. *Concave reflectors.* — Let I, *fig.* 300., be the focus of a diverging pencil of rays, incident upon a concave reflector A B C, the point I being supposed to be upon the axis of the reflector. Draw I A and I C, representing the extreme rays of the pencil. Draw O A and

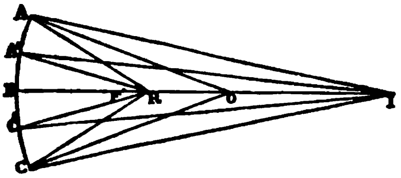

Fig. 300.

O c, the radii, to the points of incidence. The angles O A I and O c I will then be the angles of incidence; and these will evidently be equal, because the three sides of the two triangles are respectively equal.

To find the direction of the respective rays, it would be only necessary to draw from A and c lines which make with A O and c O angles equal to the angles of incidence.

Let these lines be A R and c R. The two rays I A and I c will therefore be reflected converging, and will meet at R.

By the principles of geometry,* the angle O A R of reflection is equal to the difference between the angles A R B and A O B, and the angle O A I of incidence is equal to the difference between the angles A O B and A I B.

Now, let f express the distance I B of the focus of incident rays from the vertex, and f' the distance R B of the focus of reflected rays from the same point, and let r express the radius O B of the surface We shall then have, according to what has been explained,—

$$\text{OAI} = \frac{\text{AB}}{r} - \frac{\text{AB}}{f},$$

$$\text{OAR} = \frac{\text{AB}}{f'} - \frac{\text{AB}}{r}.$$

But since the two angles are equal, we shall have

$$\frac{\text{AB}}{r} - \frac{\text{AB}}{f} = \frac{\text{AB}}{f'} - \frac{\text{AB}}{r}.$$

Omitting the common numerator A B, we shall then have

$$\frac{1}{r} - \frac{1}{f} = \frac{1}{f'} - \frac{1}{r};$$

and consequently we shall have

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{r} \ldots (\text{A}).$$

The same formula is applicable to rays incident at every point l tween A or C and the vertex B; so that rays reflected from all su points will converge to a common point on the axis, whose distar from B will be determined by the value of f', found by the precedi formula.

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* Euclid, Book 1. Prop. 32.

[redacted]

The formula (A) is of the utmost importance, and may be both understood and remembered with the greatest facility.

It may be expressed in common language as follows:—

If the fractions, whose numerator is 1, and whose denominators are the numbers expressing the distances of the foci of incident and reflected rays from the vertex, be added together, their sum will be equal to a fraction, whose numerator is 2, and whose denominator is the radius of the reflecting surface.

960. ***Rule to determine the conjugate foci in concave spherical reflectors.***—By this formula (A) the position of the focus of reflected rays can always be found when that of the incident rays is known. We have only to subtract the fraction whose numerator is 1, and whose denominator is the distance of the focus of incident rays from the vertex, that is to say, the fraction $\frac{1}{f}$ from the fraction whose numerator is 2, and whose denominator is the radius, and the remainder will be equal to a fraction whose numerator is 1, and whose denominator is the distance of the focus of reflected rays from the vertex. It is evident that by a like process the focus of incident rays can be found whenever the focus of reflected rays is known.

Since the two fractions $\frac{1}{f}$ and $\frac{1}{f'}$, added together always produce the same sum, it follows that whatever circumstances increase one must diminish the other; and hence it follows that the more distant the focus of incident rays I is from the reflector, the nearer the focus of reflected rays R will be to it, and *vice versâ*; because as I B increases, R B must diminish, and *vice versâ*, as has been just explained.

If the focus I were removed to an infinite distance, then the fraction $\frac{1}{f}$ would be infinitely small, and consequently the other fraction $\frac{1}{f'}$, would be equal to $\frac{2}{r}$, and consequently f' would be equal to $\frac{1}{2}r$; that is to say, the focus of reflected rays would then be coincident with the principal focus, which is only what might have been anticipated, because if the focus of incident rays I be removed to an infinite distance, the pencil of incident rays having the reflector for its base must be parallel.

But in order to produce this effect, it is not necessary that the focus of the pencil of incident rays should be either infinitely or even very considerably distant. Let us suppose that the distance I B, which is here expressed by f, is only one hundred times the length of the radius of the reflector, and let half the radius, or the distance of the principal focus from the vertex, be expressed by F. Then we shall have

$$f = 200\,\text{F}.$$

[redacted]

Consequently we shall have

$$\frac{1}{f}+\frac{1}{200\,\text{F}}=\frac{1}{\text{F}};$$

and therefore

$$\frac{1}{f'}=\frac{1}{\text{F}}-\frac{1}{200\,\text{F}}=\frac{199}{200\,\text{F}},$$

and therefore

$$f'=\frac{200\,\text{F}}{199}=\text{F}+\frac{1}{199}\times\text{F};$$

that is to say, the distance of the focus of reflected rays from the vertex will exceed the distance of the principal focus by the 199th part of half the radius, or nearly the 400th part of the radius of the reflector, an insignificant quantity.

It follows, therefore, that whenever the distance of an object from the reflector is not less than 100 times its radius, all pencils proceeding from it may be regarded as parallel, and therefore as coincident with the principal focus of the reflector.

It follows also from the preceding formula, that when the focus of incident rays is beyond the centre, the conjugate focus of reflected rays will be between the centre and the principal focus; and that when the focus of incident rays is between the centre and the principal focus, the conjugate focus of reflected rays will be beyond the centre.

In the preceding cases, we have supposed the focus of incident rays to be situate at some point beyond the principal focus of the reflector.

Let us now consider the case in which the focus of incident rays I, *fig.* 301., is placed between the principal focus F and the vertex.

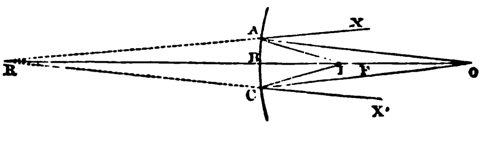

Fig. 301.

Let I A and I C, as before, be the two extreme rays of the pencil and draw the radii A O and C O. To find the direction of the reflecte rays, it is only necessary to draw from A and C two lines, which shal make with O A and O C angles equal to those which A I and C I mak with them. Let this direction be A X and C X'. It follows, therefore that in this case the reflected rays will diverge instead of converging as in the former case, and that their point of divergence will be at B

[redacted]

upon the axis behind the reflector; consequently the focus will be an imaginary focus.

By geometrical principles already referred to,* the angle of incidence I A O is equal to the difference between the angles A I B and A O B, and the angle of reflection X A O is equal to the sum of the angles A R B and A O B; and since the angles formed by O A, I A, and R A with the axis O R are so small as to come within the scope of the principles already expressed, we shall have

$$\text{I A O} = \frac{\text{A B}}{f} - \frac{\text{A B}}{r}$$

$$\text{X A O} = \frac{\text{A B}}{f'} + \frac{\text{A B}}{r},$$

where f and f' express, as in the former case, the distances of the foci of incidence and reflection respectively from the vertex B.

We shall therefore have

$$\frac{\text{A B}}{f} - \frac{\text{A B}}{r} = \frac{\text{A B}}{f'} + \frac{\text{A B}}{r};$$

and omitting the common numerator A B, we shall have

$$\frac{1}{f} - \frac{1}{f'} = \frac{2}{r} \quad . \ . \ (\text{B}),$$

a formula which is identical with formula (A), p. 48., only that $\frac{1}{f'}$ in it is negative, which indicates that the focus of reflected rays is imaginary and behind the reflector.

In the formula (B) it is not the sum of the two fractions $\frac{1}{f'}$ and $\frac{1}{f}$, but their difference, which is equal to $\frac{2}{r}$.

Analogous results, however, follow from this formula, which may be expressed in common language as follows:—

When the focus of rays incident upon a concave reflector is placed between its principal focus and the vertex, the difference between the fraction whose numerator is 1 and whose denominator is the distance of the focus of incident rays from the vertex, and the fraction whose numerator is 1 and whose denominator is the distance of the focus of reflected rays from the vertex, will be equal to the fraction whose numerator is 2 and whose denominator is the radius of the reflecting surface.

Since the difference between these two fractions is always the same, however they may separately vary, it follows, that when one increases, the other must increase to the same extent. Hence it follows, that f and f' increase and diminish together; and therefore it also follows, that as the focus of incident rays I approaches the vertex B, the focus

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* Euclid, Book 1. Prop. 32.

of reflected rays R must also approach it; and as the focus of incident rays I recedes from the vertex, the focus of reflected rays R must also recede from it.

When the focus of incident rays I arrives at the principal focus F, the focus of reflected rays R recedes to an infinite distance.

961. *Case of converging incident rays.*—If rays fall on the reflector converging to a point R behind it, they will be reflected converging to the point I. In this case, the focus of incident rays being behind the reflector will be imaginary, and the focus of reflected rays being before it will be real. The relative positions of the two foci, however, will be determined in the same manner exactly as if I were the focus of incidence, and R the focus of reflection. It may be useful to observe in general, that the conjugate foci are in all cases interchangeable.

If the focus of incidence become the focus of reflection, the focus of reflection will become the focus of incidence, and *vice versâ*.

962. *Convex reflectors.* — The effects attending diverging or converging rays incident upon convex reflectors, are in all respects analogous to those which have been just established respecting concave reflectors.

It is only necessary to observe, that converging rays upon a convex reflector are analogous to diverging rays upon a concave reflector; and diverging rays upon a convex reflector are analogous to converging rays upon a concave reflector.

Thus, if A C, *fig.* 300., instead of representing a concave, represent a convex reflector, and a pencil of rays be supposed to be incident upon it, which if not intercepted would converge upon the point I, those rays after reflection will diverge from the point R. The conjugate foci will be in this case precisely similar, and determined by similar conditions as in the former case, except that the incident rays are convergent, while the reflected rays are divergent, the contrary being the case in a concave reflector.

In like manner, if the reflector A C, *fig.* 301., be a convex instead of a concave reflector, a pencil of rays incident upon it, which if not intercepted would converge to I, will be reflected converging to R. In this case, the focus of incident rays will be imaginary, and the focus of reflected rays real, contrary to what was shown for a concave reflector; but the relative position of the two foci will be determined as before.

Images of near objects formed by spherical reflectors.—The manner has been explained in which images are formed by spherical reflectors of objects whose distance is so great that the pencils of rays proceeding from them may be considered as consisting of parallel rays. It is in this and like cases important, that the student should not confound the directions of the pencils themselves with the directions of the rays which form them. Thus, the pencils of rays proceeding from points upon the surface of the sun or moon are pencils of parallel rays, because the distance of the foci of such pencils from the observer is incomparably great compared with any surface which can form the base of the pencil. Thus, the surface of the largest reflector is as nothing compared with the distance of any point in the sun; and consequently, the rays which form a pencil, whose vertex is a point in the sun, and whose base is the surface of such a reflector, may be practically considered as parallel; but this parallelism must not be applied to the direction of the pencils themselves which proceed from different points in the sun. The directions of these pencils, or, to speak strictly, those of their respective axes, are not parallel, the axes of the extreme pencils forming an angle with each other equal to the apparent diameter of the sun; and the same observations would be applicable to any other object whose distance is so great that a pencil of rays proceeding from it may be regarded as parallel.

These observations being premised, we shall now explain the manner in which images are formed by spherical reflectors of objects which are not so distant that the rays of the pencils proceeding from points in them can be regarded as parallel.

Let A B C, *fig.* 302., be a concave reflector, whose centre is O, and whose vertex is B. Let L M be an object, whose form we shall for the present assume to be that of an arc of a circle whose centre is O. Let L L′ and M M′ be the axes of the extreme secondary pencils proceeding from this object, and let *l* and *m* be the foci of reflection conjugate to the points L and M. An image of the point L will be formed at *l*, and an image of the point M will be formed at *m*, and images of all the intermediate points between L and M will be formed at intermediate points of an arc drawn from *l* to *m*, having O as a centre.

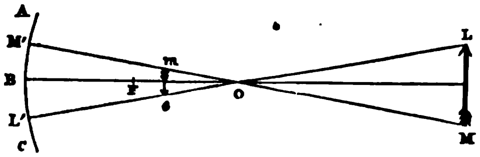

Fig. 302.

Since the lowest point of the image corresponds to the highest point of the object, and *vice versâ*, the image will in this case be inverted with respect to the object, and the linear magnitude of the image will bear to that of the object the same proportion as O *l* bears to O L.

These results follow in the same manner as in the case of the images of distant objects already explained.

The distance O *l* is determined when O L is known by the formulæ (A) and (B), p. 48. and p. 51.; that is to say, the position and magnitude of the image will be determined when the position and magnitude of the object are known.

In this case, the object L M has been supposed to have the form of a circular arc, and its image to have a similar form. If the object form part of a spherical surface whose centre is O, the image would have a like form; but if the object were a straight line or flat surface, then the image would be more or less curved, and would consequently be distorted. But as, in general, the angle O, under which the object or image would be seen from the centre, is small, this curvature may be disregarded, and we may assume that the image will be similar to the object.

963. *Spherical aberration of reflectors.* — The pencils of rays proceeding from or to the incident focus will be reflected to a common point, only on the condition that the opening of the reflector is limited, as was explained in the case of parallel rays. If it be not so limited, then the extreme rays of the pencil will converge to points sensibly different from those which are within such limit of distance of the vertex already defined, and hence will arise a spherical aberration.

If even the reflector be sufficiently limited in its opening, a sensible spherical aberration will arise from the secondary pencils which proceed from the borders of the object, and are inclined at the greatest angles to the axis of the reflector, for in this case the angle of divergence of such pencils will, as has been already explained, exceed that limit which would efface the spherical aberration. Hence it arises that images produced by spherical reflectors when the objects are too great, are indistinct towards the borders, the pencils which form each part of the image not being brought to the same focus, and consequently producing a confused effect.

964. *Case in which the object is placed between the principal focus and the reflector.* — In what precedes, the position of the object before a concave reflector has been considered as being either beyond the centre or between the centre and the principal focus F. Let us now consider the position of the object to be at L M, *fig.* 303., between the principal focus F and the reflector. In this case the image *l m* will be behind the reflector at the points which form the foci conjugate to the several points of the object L M.

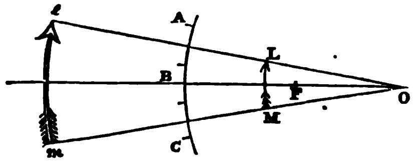

Fig. 303.

The image will in this case evidently be erect with respect to the

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object, and will be greater in magnitude than the object in the proportion of O *l* to O L.

If the reflector be convex, the object L M, *fig.* 304., will have its image at the points *l*, *m*, which are the foci conjugate to the points at L M, and those points will, according to what has been already explained, lie between the reflector and the principal focus F.

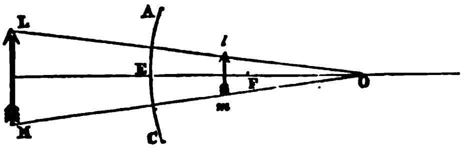

Fig. 304.

The rays proceeding from the several points of the object L M will, after reflection, diverge as if they had proceeded from the corresponding points of *l m*, and will produce upon the vision the same effects as if an object had been actually placed at *l m*.

The image in this case, therefore, will be erect, and it will be less than the object in the proportion of O *l* to O L. In this manner is explained the effect familiar to every one, that convex reflectors exhibit a diminished picture of the object placed before them.

All the preceding observations on the effect of spherical aberration, and the indistinctness incident to the borders of the image, will be equally applicable in the present case.

965. *Case in which the object is not placed in the axis of the reflector.* — In the preceding example, the object has been supposed to be placed so that its centre coincides with the axis of the reflector. The image, however, is determined on like principles, whatever other position it may have.

Thus, let L M, *fig.* 305., be the object, A B C being the reflector, O its centre, and F its principal focus. From the extremities of the object draw lines L O and M O through the centre O of the reflector to meet the continuation of the section of the reflector at P and Q. Let *l* be the focus conjugate to L, and *m* the focus conjugate to M, determined according to the principles and formulæ already established. Images, therefore, of the points L and M will be formed at *l m*, and images of all the intermediate points of the object will in like manner be formed between *l* and *m*, so that an inverted image of the object will be formed at *l m*. 525

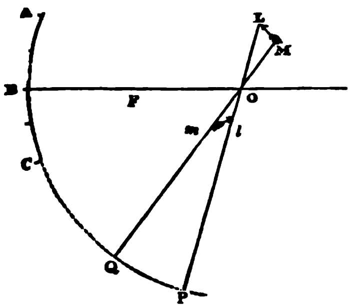

Fig. 305.

In like manner, if the object be placed at *l m*, its image will be formed at L M.

966. *Experimental verifications.* — All the preceding results may be verified experimentally by means analogous to those already explained. Thus, if the flame of a candle be placed at L M, *fig.* 302., outside the centre of a concave reflector, and a small semi-transparent screen, such as a piece of ground glass or oiled paper, be held at *l m*, an inverted image of the candle will be seen upon it; and, on the other hand, if the candle be placed at *l m*, and the screen held at L M, the image will be again seen. If any object, such as one's hand, be presented between the principal focus F and a concave reflector, as at L M, *fig.* 303., a magnified image of the hand will be seen at *l m*.

Amusing optical deceptions are often exhibited with concave reflectors founded on this principle. Thus, a hand presenting a dagger is held between O and F, *fig.* 302., when immediately a magnified image of the hand and dagger is presented outwards at L M.

If a candle be held at L M, *fig.* 305., opposite the upper edge of a concave reflector, an inverted image of the candle may be exhibited on a screen at *l m*, opposite the lower edge.

967. *Cylindrical and conical reflectors.* — A cylindrical surface is circular in one direction, and rectilinear in the other, these directions being at right angles to each other. A sheet of paper, or a plate of metal bent into the form of a circle, will be a cylindrical surface.

It may be polished either on the concave or convex side, thus presenting the varieties of a concave or convex cylindrical reflector.

If a cylindrical reflector be placed vertically before an object, its effects upon the vertical dimensions will be the same as those of a plane reflector, and its effects upon the horizontal dimensions the same as those of a spherical reflector. An image, therefore, will be presented, which will be identical in form with the object in all its vertical dimensions, but enlarged, diminished, or reversed in its horizontal dimensions in the same manner as it would be in a spherical reflector.

If a cylindrical reflector be placed with its axis horizontal before a vertical object, it will have the same effect as a plane reflector on the horizontal dimensions, and as a spherical reflector on the vertical dimensions.

The horizontal dimensions, therefore, will be preserved in the image, while the vertical dimensions will be enlarged, diminished, or reversed, in the same manner as would be the case with a spherical reflector.

A conical reflector, whether concave or convex, is circular in all sections made at right angles to its axis, and rectilinear in all sections made by planes through its axis. It will therefore, if placed with its axis vertical, have the effect of an inclined plane reflector on the vertical dimensions of an object, and will have the effect of a spheri-

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cal reflector on the horizontal dimensions; but each horizontal section will be differently magnified or diminished, according to the position of such section with reference to the axis of the cone, since the circular section of the cone will diminish in approaching the axis, and increase in receding from it. An infinite variety of amusing deceptions are thus produced.

CHAP. VI.

REFLECTION FROM IMPERFECTLY POLISHED SURFACES.

968. *A perfectly reflecting surface would be invisible.* — If the surface of an opaque body were perfectly polished, and capable of reflecting regularly all the light incident upon it, such surface would itself be invisible.

The images of all objects placed before it would appear in the position and with the form and magnitude determined in the last chapter; and an observer receiving the reflected light would perceive nothing but such images.

Thus, a plane reflector of that kind placed vertically against the wall of a room, would appear to the eye merely as an opening leading into another room, precisely similar and similarly furnished and illuminated; and an observer would only be prevented from attempting to walk through such an opening by encountering his own image as he would approach it.

969. *No such surfaces exist.* — But such a reflector as this has no practical existence, for there is no surface natural or artificial possessing the power of reflecting all the light incident upon it regularly. The absence of complete polish is one of the principal causes of this.

970. *How the surfaces of reflectors are rendered visible.* — The consequence is, that even the most polished surfaces reflect a certain portion of the light incident upon them irregularly; that is to say, the material points, the assemblage of which forms such surfaces, becoming separately illuminated form so many radiant points, from which pencils of light diverge, and render such surfaces visible exactly in the same manner, though much more faintly than is the case with unpolished surfaces. The quantity of light which is thus irregularly reflected, and which therefore renders the reflecting surface itself more or less visible, diminishes in the same proportion as the perfection of the polish of the surface increases.

The most perfectly polished surfaces, which serve as reflectors, are certain alloys of metal known as speculum metal. These are used generally for the metallic specula of telescopes, microscopes, and other optical instruments.

971. *How light incident on any opaque surface is disposed of.* — When light falls therefore on any imperfectly polished and opaque surface, it is disposed of in three ways. 1°. A part is regularly reflected, and forms the optical image of the object from which it proceeds. 2°. A part is irregularly reflected, and renders the surface of the reflector perceivable. 3°. A part is absorbed by the surface, and, consequently, not reflected. The smaller the proportion of the light subject to the two last-mentioned effects, the more perfect will be the reflector.

The quantity of light regularly reflected by a given surface also varies with the angle of incidence. When the angle of incidence is nothing, and consequently the light falls perpendicularly on such a surface, a less proportion of it is regularly reflected, and a greater proportion irregularly reflected and absorbed, than when the angle of incidence has some magnitude: and, consequently, the light falls more or less obliquely; and in general, as the angle of incidence increases, the quantity of light reflected regularly is augmented, and, consequently, the quantities reflected irregularly and absorbed are diminished.

The following is given by Bouguer as the proportion of the light regularly reflected from different reflecting surfaces, at different angles of incidence:—

972. *Table showing the proportion of light incident on reflecting surfaces which are regularly reflected at different angles of incidence.*

Species of reflecting Surface.	Angle of Incidence.	Number of Rays incident.	No. of Rays regularly reflected.	No. of Rays irregularly reflected and absorbed.
Water	89° 30′	1000	721	279
	75° 0′	1000	211	789
	60°	1000	65	935
	30° to 0°	1000	18	982
Glass	85°	1000	543	457
	75°	1000	300	700
	60°	1000	112	888
	30° to 0°	1000	25	975
Black marble polished	80° 45′	1000	600	400
	75°	1000	156	844
	60°	1000	51	949
	30° to 0°	1000	23	977
Metallic reflectors	Great angles.	1000	700	300
	Small angles.	1000	600	400

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In the preceding table, the light is understood to pass from air to the several media indicated in the first column. The law by which the quantity of light regularly reflected varies according to the density or other physical qualities of the media, has not been ascertained. It is however certain, that it depends upon the qualities of the medium from which the light passes, as well as those of the medium into which it passes.

973. *Effect of angle of incidence on the quantity of light regularly reflected.*—The angle of incidence has often so much effect upon the quantity of light regularly reflected, that it will sometimes happen that a surface which reflects no light regularly when the angle of incidence is nothing, reflects a considerable quantity when such angle has much magnitude. Thus, a surface of unpolished glass produces no image of an object by reflection when the rays fall on it nearly perpendicularly; but if the flame of a candle be held in such a position that the rays fall upon the surface at a very small angle, a distinct image of it will be seen. Similar phenomena will be observed with surfaces of wood, of common woven stuff, and of paper blackened by smoke.

974. *How light incident on the surface of a transparent body is disposed of.*—When light is incident upon the surface of a transparent body, such as glass or water, it is disposed of as follows:—1°. A part is regularly reflected, and produces an optical image of the object from which the light proceeds. 2°. A part is irregularly reflected, and renders the surface visible. 3°. A part is absorbed, and, consequently, neither reflected nor transmitted. 4°. A part is transmitted through the transparent medium.

If light be incident upon the surface of a transparent medium bounded by parallel surfaces, such as a flat plate of glass, all the circumstances above mentioned will take place both at its entrance at the one surface and its escape from the other. Light will be reflected regularly and irregularly at both surfaces; light will be absorbed at both, and light will be transmitted from both. The quantity of light, therefore, transmitted in such a case from the second surface will be less than the quantity of light incident upon the first surface by the sum of all the light regularly and irregularly reflected from the first surface, and all the light regularly and irregularly reflected from the second surface, and all the light absorbed at both surfaces in its transit through the medium.

975. *How light is affected in passing through the atmosphere.*—Even when the transparent medium consists of the same substances, these effects take place if the substance composing it varies in density. The successive strata of the atmosphere present an example of this.

It has been already explained that in ascending in the atmosphere the succeeding strata of air gradually diminish in density. The light, therefore, of the sun and other celestial bodies in passing through the

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atmosphere is transmitted through a succession of strata of increasing density, and is subject consequently to all the effects just explained. Light is gradually absorbed and reflected by the successive strata of air through which it passes, and consequently the direct solar light which arrives at the surface of the earth is less in quantity considerably than the light originally incident upon the superior surface of the atmosphere. A portion, however, of the light irregularly reflected from the successive strata of the atmosphere arrives at the earth from these strata, as has been already explained, in the same manner as light is received from the surface of any opaque illuminated body. A portion, however, of the light which enters the air is absolutely absorbed by it, and, as has been already stated, a certain depth might be assigned to the atmosphere, which would completely intercept the solar light. It is calculated that seven feet thickness of water is sufficient to intercept one-half of the light transmitted through it.

976. *Blackened glass reflectors.* — A reflecting surface convenient for certain optical purposes is produced by blackening one side of a plate of glass. By this means the light transmitted through the plate is absorbed by the blackened surface on the other side, and light is prevented from being transmitted from the opposite side by the opaque coating; consequently, the only light regularly reflected in this case will be that which is reflected from the superior surface.

977. *Effect of a common looking-glass explained.* — The effects of a common looking-glass are produced by the reflection of the metallic surface attached to the back of the glass, and not by the glass itself. The effect may be explained as follows: — A portion of the light incident upon the anterior surface is regularly reflected, and another portion irregularly. The former produces an image of the object placed before the glass visible in it; the other renders the surface of the glass itself visible. Another and much greater portion, however, of the light incident upon the anterior surface penetrates the plate, and arrives at the posterior surface. This surface, coated with an amalgam produced by the combination of tinfoil and quicksilver, has an intense metallic lustre, and possesses therefore strong reflecting power. The chief part of the light, therefore, which passes through the plate of glass is regularly reflected by this metallic surface, and returning to the eye, produces a strong image of the objects placed before the glass. There are, therefore, strictly speaking, two such images formed: first, a faint one by the light reflected regularly from the anterior surface; and, secondly, a vivid one by the light reflected regularly from the metallic surface. One of these images will be before the other, at a distance equal to twice the thickness of the glass.

In good mirrors which are well silvered, the superior brilliancy of the image produced by the metallic surface will render the faint image produced by the anterior surface of the glass invisible; but in glasses badly silvered, the two images may be easily seen.

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CHAP. VII.

REFRACTION OF LIGHT.

978. *Refraction of light explained.* — When a ray of light, after passing through a transparent medium, enters another of a different density, or possessing other physical properties, it will change its direction at the point which separates the two media, and consequently the direction it follows in the second medium will form a certain angle with that which it has followed in the first medium. The ray is as it were broken at the common surface of the two media, which has caused this phenomenon to be called *refraction.*

Let A B, *fig.* 306., be the surface which separates the two media. Let I be the point at which a ray E I is incident, and let I R be the course which this ray takes after entering the second medium. Let N I N′ be a perpendicular to the surface A B, drawn through the point of incidence I. A B is called the *refracting surface*, E I N is called the *angle of incidence*, and R I N′ is called the *angle of refraction.*

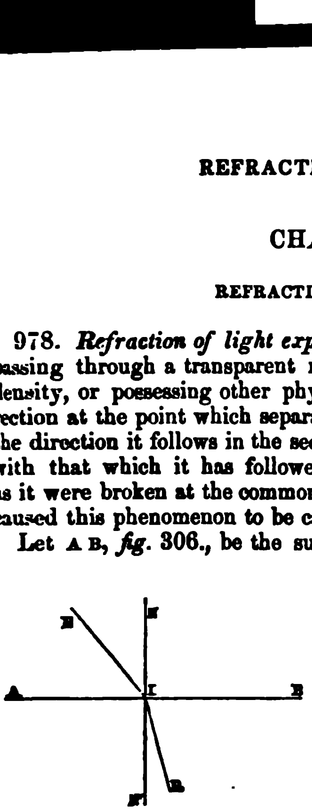

Fig. 306.

979. *Law of refraction.* — The following law of refraction has been established by experiment: —

I. The angles of refraction and incidence are in the same plane perpendicular to the refracting surface.

II. The sine of the angle of incidence has to the sine of the angle of refraction always the same ratio for the same medium.

It will appear hereafter that, under certain circumstances, a single ray of light entering a refracting medium will be divided into several, which follow different directions; but for the present we shall limit our observations to such light only as after refraction follows a single direction. To such light the above law is strictly applicable.

To explain the preceding law more fully, and to indicate the manner of verifying it by experiment, let A M B be a piece of glass, having the form of a semi-cylinder, as represented in *fig.* 307. Let O be the centre, and A B the diameter of the semi-cylinder. Let the semicircle A O B be imagined to be drawn on a vertical card, so as to complete the circle. Let O C M be the diameter perpendicular to A B, and let the surface A B be covered with an opaque card, with a small hole to admit light at C.

If the flame of a candle, or any other bright object, be held at O, it will be visible to an eye placed at M. It follows, therefore, that a

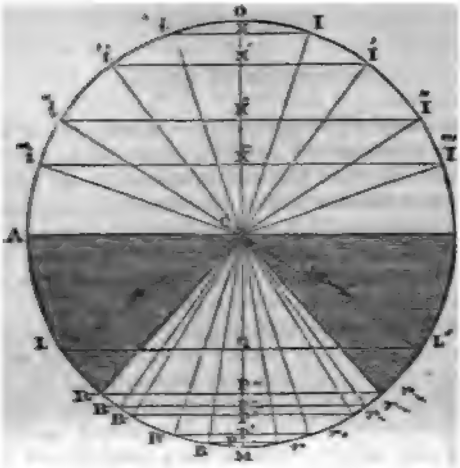

Fig. 307.

ray of light striking the refracting surface in a direction perpendicular to it, such as O C, will suffer no change of direction after it enters it, but will proceed in the same straight line C M as it would have done if it had passed through no refracting medium. Let the luminous point be now transferred to f, and let the line I N be drawn perpendicular to C O. This line I N is the sine of the angle of incidence I C O. Let the eye be now moved along the arc M A from M towards A, until it see the luminous point I.

Let R be the place at which the luminous point thus becomes visible, C R will then be the direction of the refracted ray. Draw R P perpendicular to C M. This line R P will be the sine of the angle of refraction R C M.

Now if I N and R P be respectively measured, it will be found that R P is exactly two-thirds of I N. Therefore, in this case, the sine of the angle of incidence will be to the sine of the angle of refraction as 3 to 2, that is to say, we shall have

$$\frac{\text{I N}}{\text{R P}} = \frac{3}{2}.$$

Let the luminous point be now moved to I', and let the eye be moved towards A until it see it. Let R' be the point at which it becomes visible; C R' will then be the refracted ray, I' O being the incident ray.

Draw I' N' perpendicular to C O, and R' P' perpendicular to C M; I' N' will then be the sine of the angle of incidence, and R' P' will be the sine of the angle of refraction. If these two lines be respectively

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measured, it will be found that R′ P′ will be two-thirds of I′ N′; so that we shall have, as before,

$$\frac{\text{I}'\,\text{N}'}{\text{R}'\,\text{P}'} = \frac{3}{2}.$$

In the same manner, if the luminous point be moved to any other point, such as I″, and the eye be moved towards A until it see it, the lines I″ O and O R″ will be the incident and refracted rays, I″ N″ and R″ P″ will be sines of the angles of incidence and refraction respectively; and we shall find, as before, by measurement, that

$$\frac{\text{I}''\,\text{N}''}{\text{R}''\,\text{P}''} = \frac{3}{2}.$$

Thus, in general, in whatever manner the position of the luminous point may be viewed, it will always be found that the sine of the angle of incidence will be to the sine of the angle of refraction as 3 to 2, that is to say, in one constant ratio.

In this case, the incident ray is supposed to pass through air, and the refracted ray through glass. If the semi-cylinder A M B, instead of glass, be water, then the ratio of the sine of the angle of incidence to the sine of the angle of refraction will be 4 to 3, so that we shall have

$$\frac{\text{I N}}{\text{R P}} = \frac{4}{3},$$

$$\frac{\text{I}'\,\text{N}'}{\text{R}'\,\text{P}'} = \frac{4}{3},$$

$$\frac{\text{I}''\,\text{N}''}{\text{R}''\,\text{P}''} = \frac{4}{3},$$

and so on.

Thus each transparent medium has its own particular refracting power, but for the same transparent medium the ratio of the sines of the angles of incidence and refraction is always the same.

980. *Index of refraction.*—The number which thus expresses the ratio of the sine of the angle of incidence to the sine of the angle of refraction, and which in the case of air and glass is $\frac{3}{2}$ or 1·5, and in the case of air and water is $\frac{4}{3}$ or 1·333, is called the *index of refraction.*

From what has been stated, it is evident that each transparent medium will have its own index of refraction, which constitutes one of its most important physical properties.

981. *Case of light passing from denser into rarer medium.*—If the luminous point, instead of being moved along the arc O B, be moved along the arc M A, and the eye be transferred to the arc O B, then the incident ray will pass through the denser medium, and

the refracted ray through the rarer medium. In this case it will be found that the direction of the incident and refracted rays, described in the former case, will be interchanged. Thus, if the luminous point be applied at M, it will be visible at O, showing that a ray of light incident perpendicularly on the surface of a rarer medium, will suffer no change in its direction. If the luminous point be placed at R, it will be visible at I, showing that if R C be the incident ray, C I will be the refracted ray; and in the same manner, if the luminous point be placed at R′ and R″, it will be visible at I′ and I″.

982. *Directions of incident and refracted rays interchangeable.* — Hence it follows, that if a ray of light passing from one transparent medium into another transparent medium be refracted in a particular direction, a ray of light passing from the latter into the former in the direction in which it was refracted, will, after entering the former, follow the direction in which the former ray was incident; or in general it may be stated that the direction of the incident and refracted rays passing between the media are interchangeable.

983. *Indices of refraction between two media in contrary directions reciprocals.* — It follows from this that the indices of refraction between the media are reciprocals; that is to say, if the index of refraction from air into glass be $\frac{3}{2}$, the index of refraction from glass into air will be $\frac{2}{3}$; the latter number being what is called in arithmetic the reciprocal of the former. In the same manner, the index of refraction from air into water being $\frac{4}{3}$, the index of refraction from water into air will be $\frac{3}{4}$.

It appears in the two cases which have been stated of water and glass, that when a ray passes from air into either of these media it will be bent *towards* the perpendicular; and that, on the other hand, when it passes out of either of these media into air, it will be bent *from* the perpendicular. This will be evident by reference to *fig.* 307. The rays I C, I′ C, I″ C entering water or glass are bent in the directions C R, C R′, C R″ *towards* the perpendicular C M; and, on the other hand, the rays R C, R′ C, R″ C, passing from glass or water into air, are bent in the directions C I, C I′, C I″ *from* the perpendicular C O.

984. *Rays not always bent towards perpendicular in entering a denser medium.* — This result being too hastily generalized, is sometimes announced as follows: — When a ray of light passes from a rarer into a denser medium, it is bent towards the perpendicular, and from a denser into a rarer from the perpendicular, which is by no means generally true.

Such a proposition is based upon the supposition that the refracting power always increases with the density; whereas numerous instances will be produced in which media of greater density have a less refracting power.

985. *Index of refraction increases with the refracting power.* — The refracting power is estimated by the index of refraction, one

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medium being said to have a greater or less refracting power, according as its index of refraction is greater or less than that of the other. Thus, glass is said to have a greater refracting power than water, because its index of refraction being 1·50, is greater than the index of refraction of water, which is 1·33.

The propriety of this test of the refracting power will be easily understood. If the index of refraction of one medium be greater than that of another, the angle of refraction which corresponds to a given angle of incidence will be smaller in the former than in the latter; and consequently, the same incident ray would be bent more out of its course in the one case than in the other, that is to say, it would be more refracted.

986. *But not in proportion to it.*—Although, however, the refracting power of a transparent medium increases with every increase of its index of refraction, this power does not increase in proportion to such index, but in proportion to a number found by subtracting 1 from the square of the index. Thus, in the case of glass, where the index of refraction is $\frac{3}{2}$, its square is $\frac{9}{4}$, from which 1 being subtracted leaves $\frac{5}{4}$, which represents the refracting power. In the same manner, the index of water being $\frac{4}{3}$, its square is $\frac{16}{9}$, from which 1 being subtracted leaves $\frac{7}{9}$, which represents the refracting power of water; or, in general, if n be the index, $n^2 - 1$ will represent the refracting power.

The principle upon which this number $n^2 - 1$ is shown to be proportional to the refracting power, does not admit of an explanation sufficiently elementary for this work. We must therefore adopt it as a datum without demonstration.

In the following table are given the indices of refraction of those transparent substances which are of most usual occurrence.

987. *Table of the indices of refraction for light passing from a vacuum into various media.*

SOLIDS AND LIQUIDS.

Chromate of lead (maximum)	2·974
" (minimum)	2·500
Sulphur, native	2·115
Carbonate of lead (maximum)	2·084
" (minimum)	1·813
Felspar (Spinelli)	1·764
Chrysoberyl	1·760
Nitrate of lead	1·758
Carbonate of strontia (maximum)	1·700
" (minimum)	1·543
Boracite	1·701
Aragonite (ordinary* refraction)	1·693
" (extraordinary* refraction)	1·585

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* Ordinary and extraordinary refraction will be explained in Chap XVIII.

Calcareous spar (ordinary refraction)	1·654
" (extraordinary refraction)	1·483
Sulphate of baryta	1·647
" (ordinary refraction)	1·620
" (extraordinary refraction)	1·635
Colourless topaz	1·610
Topaz of Brazil (extraordinary refraction)	1·640
" (ordinary refraction)	1·633
Anhydrite (extraordinary refraction)	1·622
" (ordinary refraction)	1·577
Euclase (extraordinary refraction)	1·663
" (ordinary refraction)	1·643
Flint-glass (maximum)	1·605
" (minimum)	1·576
Quartz (ordinary refraction)	1·548
" (extraordinary refraction)	1·558
Crown-glass (maximum)	1·534
" (minimum)	1·525
Sulphate of lime	1·525
Saltpetre (nitrate of potassa) (maximum)	1·514
" (minimum)	1·335
Sulphate of potassa	1·509
"	1·495
Sulphate of ammonia and magnesia	1·483
Carbonate of potassa	1·482
Spermaceti, melted	1·446
Albumen	1·360
Ether	1·358
Aqueous humour of eye	1·337
Vitreous do	1·339
External coating of the crystalline	1·377
Middle coating do	1·379
Central coating do	1·399
Entire crystalline	1·384
Water	1·336
Ice	1·310
Vacuum	1·000

GASES.

Atmospheric air	1·000,294
Oxygen	1·000,272
Hydrogen	1·000,138
Nitrogen	1·000,300
Ammonia	1·000,385
Carbonic acid	1·000,449
Chlorine	1·000,772
Hydrochloric acid	1·000,449
Nitrous oxide	1·000,503
Nitrous gas	1·000,303
Carbonic oxide	1·000,340
Cyanogen	1·000,834
Olefiant gas	1·000,678
Light carburetted hydrogen	1·000,443
Muriatic ether (vapour)	1·001,095
Hydrocyanic acid	1·000,451

[redacted]

Chloro-carbonic acid (phosgene gas)	1·001,159
Sulphurous acid	1·000,665
Sulphuretted hydrogen	1·000,644
Sulphuric ether (vapour)	1·001,530
Vapour of sulphuret of carbon	1·001,500
Protophosphuret of hydrogen	1·000,789

988. *How to find the index of refraction from one medium to another.* — The indices of refraction given in the preceding table relate to rays of light passing from a vacuum into the several media indicated. If it be required to find the index of refraction for a ray passing from one medium to another, it is only necessary to divide the index of the medium into which the ray is supposed to pass by the index of the medium from which it passes, and the quotient will be the required index. Thus, if it be desired to determine the index of refraction for a ray passing from atmospheric air into any medium indicated in the table, it will be only necessary to divide the index of the medium whose relative index is required by 1·000,294, the index of refraction of atmospheric air.

989. *Course of a ray passing through a succession of media with parallel surfaces.* — It follows from this, that if a ray pass from any medium successively through several transparent media with parallel surfaces, its course in the last of the series will be the same as it would be if it had been incident directly on the surface of the last without having passed through the preceding media. This is easily proved: for let I be the angle of incidence upon the surface of the first medium, and R the angle of refraction. This angle R will be the angle of incidence on the second medium, in which the angle of refraction is R′. This angle of refraction R′ will be the angle of incidence on the surface of the third medium, in which the angle of refraction is R″.

If n be the index of refraction of the original medium through which the ray passes, and n', n'', and n''' be the indices of refraction of the three successive media by which it is refracted, then the index of refraction from the original medium into the first will be $\frac{n'}{n}$ and consequently we shall have

$$\frac{\sin. \text{I}}{\sin. \text{R}} = \frac{n'}{n};$$

and in like manner we shall have

$$\frac{\sin. \text{R}}{\sin. \text{R}'} = \frac{n''}{n'}, \qquad \frac{\sin. \text{R}'}{\sin. \text{R}''} = \frac{n'''}{n''}.$$

By multiplying all these together we shall have

$$\frac{\sin. \text{I}}{\sin. \text{R}''} = \frac{n'''}{n};$$

which is the index of refraction from the original medium through which the ray passed to the last medium by which it has been refracted. The angle of refraction, therefore, R″, in this latter medium, would be the same if the original ray had been directly incident upon it with the same angle of incidence.

990. *A ray having passed through several parallel surfaces, emerges parallel to its incidence.* — It follows from this, that if a ray of light, after passing through several successive media separated by parallel surfaces, pass finally into the medium from which it was originally incident, it will issue in a direction parallel to the original ray. Thus, in the preceding example, if the original ray of light, after passing successively through the three media, issue again into the medium through which it originally passed, its direction will be parallel to its original direction; for, according to what has been already proved, its course, after passing through the three media and not the fourth, will be the same as if it passed directly from the first medium into the fourth; but in this case the first medium being the same as the fourth, the ray would not be deflected from its course. It must therefore, after passing through the parallel media, preserve its original direction.

991. *Why objects are distinctly seen through window glass.* — It is for this reason that plates of glass with parallel surfaces, such as window glass, produce no distortion in the objects seen through them; the rays from such objects, after passing through the glass, preserve their original direction.

992. *The angle of refraction in passing from a rarer into a denser medium has a limit of magnitude which it cannot exceed.* — The law of refraction which has been just explained and illustrated is attended with some remarkable consequences in the transmission of light through media of different refracting powers.

Let A B, *fig.* 308., represent, as before, the surface which separates a medium of air A O B from a medium of glass A M B. According to what has been already explained, any incident ray, such as I C, will be deflected towards the perpendicular C M, so that its angle of refraction shall have a sine equal to two-thirds of that of its angle of incidence. Now, let us suppose the angle of incidence gradually to increase, so as to approach to a right angle. It is evident that the sine of the angle of incidence I N will also gradually increase until it approach to equality with the radius C B. This will be evident on inspecting the diagram, in which I′ N′, I″ N″, I‴ N‴, &c. are the sines of the successive angles of incidence; and if we suppose the direction of the incident ray to approximate as closely as possible to that of the line B C, the sine of the angle of incidence will approach as close as possible to the magnitude of B C.

Now, let us consider what corresponding change the angles of refraction will suffer. Their sines will be respectively, in the case of

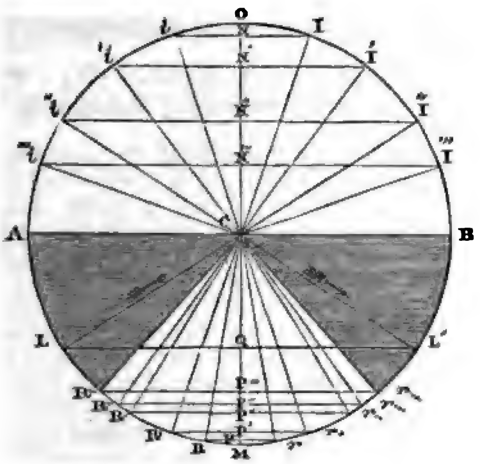

Fig. 308.

glass here supposed, two-thirds of the sines of the angles of incidence; thus the sine R P of the angle of refraction corresponding to I C will be two-thirds of I N; the sine R′ P′ of the angle of refraction corresponding to I′ C will be two-thirds of I′ N′; the sine R″ P″ of the angle of refraction corresponding to I″ C will be two-thirds of I″ N″; and so on. When the incident ray approaches to coincidence with B C, the sine of the angle of incidence will approach to equality with B C, and consequently the sine of the angle of refraction will be equal to two-thirds of B C. If, therefore, it were possible that a ray passing directly from B to C could enter the glass at C, such ray would have an angle of refraction whose sine would be two-thirds of the radius B C. Now, if we draw C R⁗ to such a point that the sine of the angle of refraction R⁗ P⁗ shall be two-thirds of the radius B C, it is evident that all the incident rays whose directions lie between O C and B C will be refracted in directions lying between C R⁗ and C M.

In like manner it may be shown, that all incident rays whose directions lie between O C and A C will be also included after refraction between the lines C M and C r⁗, corresponding in position to C R⁗.

Thus it appears that rays of light converging from all directions to the point C, will be after refraction included within a cone whose angle is R⁗ C r⁗.

Hence follows the remarkable consequence, that light entering the glass at C, from whatever direction it may proceed, will be totally excluded from the space A C R⁗ and B C r⁗, all such light being included, as has been observed, within the cone whose angle is R⁗ C r⁗.

993. ***Experimental verification of this.*** — This may be verified experimentally in the following manner. Let an opaque covering be placed on the surface A B, a small circular aperture being left uncovered at C.

Let a light be moved round the semicircle B O A. This light will enter the aperture C, and will successively illuminate the points of the arc R'''' M r''''.

Commencing from B, it will produce an illuminated spot near R''''; as it is moved successively from B to O, it will illuminate the points successively from R'''' to M; and as it is moved successively from O to A, it will illuminate successively the points from M to r''''.

In the same manner it will be found, that if the luminous point be placed at R'''', its light, after passing from the point C, will fall near B, taking the direction C B. If the light be moved successively over the parts of the arc R'''' M, it will successively illuminate the points of the arc from B to O; and being moved in like manner from M to r'''', it will successively illuminate the points of the arc from O to A.

994. *The angle of incidence at which refraction can take place from a denser to a rarer medium, has a limit which corresponds to that of the angle of refraction in the contrary direction.* — Now a question arises as to what will happen if the light be placed between R'''' and A; for since, being at R'''', the sine of the angle of incidence R'''' P'''', is two-thirds of C B, this sine will be more than two-thirds of C B if the luminous point be placed between R'''' and A; and consequently it would follow, by the law of refraction, that the sine of the corresponding angle of refraction must be greater than the radius B C.

But since no angle can have a sine greater than the radius, it would follow that there can be no angle of refraction, and consequently that there can be no refraction, for a ray which shall make with the refracting surface at C a greater angle of incidence than R'''' C M. What then, it will be asked, becomes of such a ray, as, for example, the ray L C, making an angle of incidence L C M, whose sine L Q is greater than two-thirds of the radius C B?

995. *Total reflection takes place at and beyond this limit.* — The answer is, that such a ray being incapable of refraction at C will be reflected, and that such reflection will follow the common law of regular reflection, so that the ray L C will be reflected in the direction C L', making the angle of reflection L' C M equal to the angle of incidence L C M.

Thus it follows, that all rays which meet the point C, in any direction included between R'''' C and A C, will be reflected from C in corresponding directions between r'''' C and B C, according to the common laws of reflection.

This may be verified by observation; for if the flame of a candle be moved from R'''' to A, it will be seen in corresponding positions by an eye moved in the same way from r'''' to B, and it will be seen with

[redacted]

a splendour of reflection far exceeding that produced by any artificially polished surface.

996. *Angle of total reflection determines the limit of possible transmission.* — Hence it is that this species of reflection has been called *total reflection.* The angle R'''' C M, which limits the direction of the rays capable of being transmitted from C into the superior medium, and of being reflected, is called the *limit of possible transmission.*

The rays C R''' and C r''' separate the rays which are capable of refraction at C, from those which are reflected at C.

As in the case of glass, the limit of possible transmission is one whose sine is two-thirds of the radius; so in the case of water, it would be three-fourths of the radius, and, in general, it would be an angle whose sine is the reciprocal of the index of refraction.

It follows, therefore, that this limit of possible transmission diminishes as the refracting power of the medium increases.

Since the angle whose sine is ¾ is 48° 28', and the angle whose sine ⅔ is 41° 49', it follows that these are the limits of possible transmission for water or glass into air.

997. *Table showing the limits of possible transmission, corresponding to the different transparent bodies expressed in the first column.*

Names of Media.	Index of Refraction.	Limit of Transmission.
Chromate of lead	2·926	19° 59'
Diamond	2·470	23 53
Sulphur	2·040	29 21
Zircon	2·015	29 45
Garnet	1·815	33 27
Felspar	1·812	33 30
Sapphire	1·768	34 26
Ruby	1·779	34 12
Topaz	1·610	38 24
Flint-glass	1·600	38 41
Crown-glass	1·533	40 43
Quartz	1·548	40 15
Alum	1·457	43 21
Water	1·336	48 28

The properties here described may be illustrated experimentally by the apparatus represented in *fig.* 309.; let *a b c d* represent a glass vessel filled with water or any other transparent liquid. In the bottom is inserted a glass receiver, open at the bottom, and having a tube such as a lamp-chimney carried upwards and continued above the surface of the liquid. If the flame of a lamp or candle be placed

Fig. 309.

in this receiver, as represented in the figure, rays from it penetrating the liquid, and proceeding towards the surface *d c*, will strike this surface with various obliquities. Rays which strike it under angles of incidence within the limits of transmission will issue into the air above the surface of the liquid, while those [illegible] h strike it at greater angles of in [illegible] l be reflected, and will penetrate [illegible] the glass vessel *b c*. eye placed outside *b c* w [illegible] candle reflected on that part surface *d c*, upon which the rays fall at angles of incidence ding the limit of transmission; and an eye placed above the ce will see the flame, in the direction of the reflected rays, strik- he surface with obliquities within the limit of transm sion.

CHAP. VIII.

REFRACTION OF PLANE SURFACES.

HAVING explained the principles which determine the change of direction which a single ray of light suffers when it passes from one transparent medium to another, we shall now proceed to show the effects produced by pencils of rays, whether parallel, diverging, or converging, which are incident upon plane surfaces.

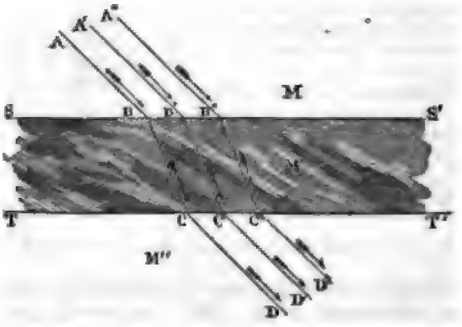

Fig. 310.

908. *Parallel rays.*—If a pencil of parallel rays be incident upon a plane surface s s', *fig.* 310., which separates two refracting media

[redacted]

M and M′, the rays of the pencil, provided they enter the medium M′ at all, will continue to be parallel.

Whether the rays of the pencil enter the medium M′, will be determined by the relative refracting powers of the two media M and M′, and the magnitude of the angle of incidence of the pencil upon the surface S S′.

If the medium M′ be more refracting than the medium M, then the pencil will enter the medium M′, whatever be the angle of incidence; but if the medium M′ be less refracting than the medium M, then the pencil will enter the medium M′ only when the angle of incidence is less than the limit of transmission. If it be greater than that limit, it will be reflected from the surface S S′, according to the common laws of reflection.

If a pencil of parallel rays be incident successively upon parallel plane surfaces separating different media, its rays will, if transmitted at all through them, preserve their parallelism; for, from what has been already proved, the pencil, if parallel in the medium M, will be parallel in the medium M′; and being parallel in the medium M′, it will for the same reason be parallel in the medium M″; and the same will be true for every successive medium through which the pencil passes, provided the surface separating the media be parallel.

But whether the pencil be transmitted at all through the successive media, will depend, as before, upon the relative refracting powers of the media and the angles of incidence. If, for example, at any surface, such as T T′, the medium M″ have less refracting power than the medium M′, the pencil will only enter it provided the angle at which the rays strike the surface T T′ be less than the limit of transmission, otherwise the rays will be reflected.

If a refracting medium M′, bounded by parallel planes, have the same medium at each side of it, as, for example, if the medium M′ be a plate of glass, and the media M and M″ be both the atmosphere, the pencil of rays A B, after passing through the medium M′, will emerge in the direction C D, C′ D′, C″ D″, parallel to the original direction A B, A′ B′, A″ B″, &c.

This has been already proved for a single ray, and will therefore be equally true for any number of parallel rays.

999. *Parallel rays incident on a succession of parallel surfaces.* — If a pencil of parallel rays, after passing through a succession of media bounded by parallel surfaces, be incident upon the surface of a less refracting medium, at an angle greater than the limit of transmission, it will be reflected, and after reflection will return through the several media, making angles with the other surfaces equal to those which it produced passing through them, but on the other side of the perpendicular.

For example, let A B, *fig.* 311., be a ray of the incident pencil, and let it be successively refracted by the media M, M′, M″ in the directions

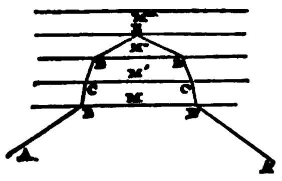

Fig. 311.

B C, C D, and D E; and let it be supposed that, the medium M''' having a less refracting power than the medium M'', the ray D E is incident upon its surface at an angle greater than the angle of transmission.

This ray will consequently be reflected in the direction E D', making an angle with the surface at E equal to that which D E makes with it. The rays E D' and E D, being equally inclined to the surface separating the media M'' and M', will be refracted by the medium M' in the direction D' C', inclined at the same angle as D C to the surface D D', but on the other side of the perpendicular; and in the same way, in passing through the medium M, it will take a direction C' B' inclined to C C' at the same angle as the ray C B is inclined to it. In fine, it will issue from the medium M in the direction B' A', inclined to the surface B B', at the same angle as the incident ray A B is inclined to such surface.

If an eye were placed, therefore, at A', it would see the object from which the ray A B proceeds in the direction A' B', the phenomenon being in all respects similar to that of common reflection.

1000. *Mirage, Fata Morgana, &c. explained.*—These principles serve to explain several atmospheric phenomena, such as Mirage, the Fata Morgana, &c.

In climates subject to sudden and extreme vicissitudes of temperature, the strata of air are often affected in an irregular manner as to their density, and consequently as to their refracting power. If it happen that rays proceeding from a distant object directed upwards after passing through a denser be incident upon the surface of a rarer stratum of air, and that the angle of incidence in this case exceeds the limit of transmission, the ray will be reflected downwards; and if it be received by the eye of an observer, an inverted image of the object will be seen at an elevation much greater than the object itself.

To explain this, let S, *fig.* 312., be an object, which if viewed from E would be seen in the direction E S.

Let M and M' be two atmospheric strata, of which M' is much more rare than M, and let the ray S M be incident upon the surface separating these strata at an angle greater than the angle of transmission. Such ray will in this case be reflected in the direction M E, making with the surface an angle equal to that which S M makes with it. The eye, therefore, will see an image of S, exactly as it would if the surface separating M and M' were a mirror, and consequently the image S' of the object S will be inverted. If no opaque obstacle lie in the line E S. the object S and the inverted image will be seen at the same *time; but if* any object be interposed between the eye and S, such

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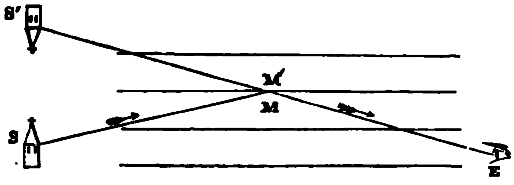

Fig. 312.

as a building, or elevated ground, or the curvature of the earth, then the object S will be invisible, while its inverted image S' will be seen.

It sometimes happens that the reflection takes place from a lower stratum of air towards the eye in an upper stratum, and in such case the inverted image is seen below the object.

1001. *Curious examples of these phenomena.* — Various fantastic optical effects of this kind are recorded as having been observed during the campaign of the French army in Egypt. On this occasion, a corps of savans accompanied the army, in consequence of which, the particulars of the phenomena were accurately observed and explained.

When the surface of the sands was heated by the sun, the land seemed terminated at a certain point by a general inundation. Villages standing at elevated points seemed like islands in the middle of a lake, and under each village appeared an inverted image of it. As the spectator approached the boundary of the apparent inundation, the waters seemed to retire, and the same illusion appeared round the next village.

1002. *Case in which parallel rays are incident successively on surfaces not parallel.* — If a pencil of parallel rays be transmitted successively through several transparent media bounded by plane surfaces which are not parallel, its rays will preserve their parallelism throughout its entire course, whether they strike the successive surfaces at an angle within the limit of transmission or not.

If they strike them at angles within the limit of transmission, they will pass successively through the media, and the preservation of their mutual parallelism may be established by the same reasoning as was applied to parallel surfaces; for the angles of incidence of the parallel rays upon the surface of the first medium being equal, the angles of refraction will also be equal, and therefore the rays through the first medium will be parallel. They will therefore be incident at equal angles on the surfaces of the two media, and the angle of refraction through the strata within the limits of transmission will be also equal, and therefore the rays in passing through the second medium will be parallel; and the same will be true of every successive medium through which the rays would be transmitted. But if they strike upon the surface of any medium at an angle beyond the limit of transmission, they will be reflected, and being reflected at the same angle at which they are incident, the reflected rays must be parallel. In returning successively through the media they will be subject to the like observation, and will therefore preserve their parallelism whether they be refracted or reflected.

In these observations it is assumed that all the rays composing the parallel pencil are equally refrangible by the same refracting medium, and to such only the above inferences are applicable. It will, however, appear hereafter that certain pencils may be composed of rays which are differently refrangible, a case not contemplated here.

1003. *Refraction by prisms.* — The deflection of a pencil from its original course by its successive transmission through refracting surfaces which are not parallel, is attended with consequences of great importance in the theory of light, and it will therefore be necessary here to explain these effects with some detail.

If two plane surfaces be not parallel, they may be considered as forming two sides of a triangular prism, which is a solid, having five sides, three of which are rectangular, and the two ends triangular. Such a solid is represented in *fig.* 313. A B C and A′ B′ C′ are the triangular ends, which are at right angles to the length of the prism, and therefore parallel to each other. The three rectangular sides are A B B′ A′, B C C′ B′, and A C C′ A′.

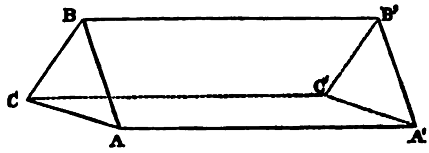

Fig. 313.

1004. *The refracting angle — designations of prisms.* — The refracting angle of the prism is that angle through the sides of which the refracted light passes. Thus, if the light enter at any point of the side A B B′ A′, and emerge from a point of the side B C C′ B′, then the angle of the prism whose edge is B B′ is called the *refracting angle,* and the opposite side A C C′ A′ is called the base of the prism.

Triangular prisms are distinguished according to the properties of the triangles which form their ends. Thus, if the triangle A B C be equilateral, the prism is said to be equilateral; if it be right-angled, the prism is said to be rectangular; if the sides A B and B C of the refracting angle be equal, the prism is said to be isosceles; and so forth.

1005. *Manner of mounting prisms for optical experiments.* — It is usual to mount such prisms for optical purposes on a pillar, as represented in *fig.* 314., having a sliding tube *t* with a tightening screw, by which the elevation may be regulated at pleasure, and a knee-joint at *g*, by which any desired inclination may be given to the prism.

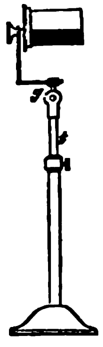

Fig. 314.

By the combination of these arrangements, the apparatus may always be adjusted, so that a pencil may be received in any desired direction with reference to its refracting angle.

If the transparent medium composing the prism be a solid, the prism may be formed by cutting and polishing the solid in the form required; if it be a liquid, the prism may be formed of glass plates hollow, so as to be filled by the liquid.

1006. *Effect produced on parallel rays by a prism.*— Let a pencil of parallel rays be supposed to be incident at O, *fig.* 315., upon one side A B of the refracting angle A B C of a prism. Let it be required to determine under what conditions such a pencil entering the prism and traversing it will be transmitted through the other side B C.

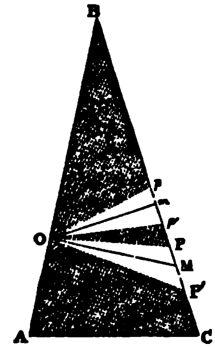

Fig. 315.

We shall here assume that the refracting power of the prism is greater than that of the surrounding medium. This being the case, the pencil incident upon the surface A B will enter the prism, whatever be its angle of incidence. From O draw O M perpendicular to A B, and O *m* perpendicular to B C; draw O P and O P', making with O M the angles P O M and P' O M, each equal to the limit of transmission; and also draw the lines O *p* and O *p'*, making angles with O *m* also equal respectively to the limit of transmission. It is evident, from what has been already explained, that in whatever direction the incident ray would fall at O, it will, when refracted, fall within the angle P O P'.

It follows also from what has been explained, that no ray proceeding from O and incident upon the surface B C can be transmitted through it unless it fall between *p* and *p'*, that is, within the angle *p* O *p'*.

It is evident, then, that if these two angles P O P', and *p* O *p'* lie altogether outside each other, as represented in *fig.* 315., no ray incident at O could pass through the surface B C; and that, consequently, every such ray must be reflected by such surface. In order that any of the rays transmitted through the prism, and therefore falling within the angle P O P′, should be transmitted, it would be necessary that the angle *p* O *p*′, or some part of it, should fall upon or within the angle P O P′.

To determine the conditions which would ensure such a result, we are to consider that the lines O M and O *m*, which are perpendicular respectively to the sides of the refracting angle, must form with each other the same angle, that is, the angle *m* O M must be equal to the refracting angle B.

This angle *m* O M is, as represented in *fig.* 315., equal to the sum of the three angles M O P, *m* O *p*′, and *p*′ O P. Therefore, the angle *p*′ O P will be equal to the angle *m* O M, diminished by twice the limit of transmission, because the two angles *m* O *p*′ and M O P are respectively equal to the limit of transmission.

It follows, therefore, that the angle *p*′ O P, which separates the rays transmitted through the prism from the direction of those rays which it would be possible to transmit through the surface B C, is equal to the difference between the refracting angle B, and twice the limit of transmission. If, therefore, the refracting angle of the prism be greater than twice the limit of transmission, the rays which enter the prism cannot be transmitted through the second surface of the refracting angle, but will be reflected by it. If the angle M O *m* be equal to twice the limit of transmission, then the commencement O P of the rays which pass through the prism will coincide with the commencement O *p*′ of those rays which it would be possible to transmit through the surface B C. This case is represented in *fig.* 316. In this case, none of the rays which pass through the prism can be transmitted through the surface B C, and the line O P is the limit which separates the two cones of rays, one consisting of the rays which traverse the prism, and the other including those directions which would render their transmission through the second surface possible.

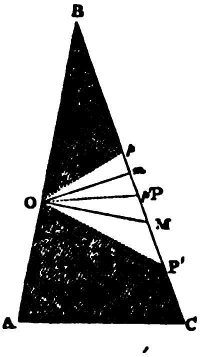

Fig. 316.

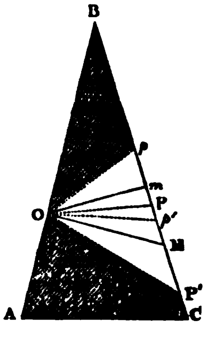

Fig. 317.

[redacted]

If, in fine, the angle *m* O M, as represented in *fig.* 317., be less than twice the limit of transmission, then a portion of the cone *p* O *p'* will lie within the cone P O P' and all the refracted rays which are included between O P and O *p'* will fulfil the condition of transmission, and will consequently pass through the surface B C; but all the others which strike the surface B C between P' and *p'*, will be reflected.

The rays, therefore, incident at the point O, which are capable of being transmitted through the two surfaces B A and B C of the prism, will be those whose angles of refraction are greater than *p'* O M, and less than P O M.

But if L express the limit of transmission, and B the refracting angle of the prism, we shall have

$$p' \text{ O M} = \text{L} - p' \text{ O P} = \text{B} - \text{L}.$$

The condition, therefore, of transmission at the two surfaces is that the refracting angle of the prism shall be less than twice the limit of transmission, and the rays which in this case are capable of transmission are those whose angles of refraction at the first surface are greater than the difference between the refracting angle of the prism and the limit of transmission.

To explain the course of a ray which, passing through the prism, fulfils these conditions of transmission, let A B C, *fig.* 318., be the refracting angle, and P O the incident ray.

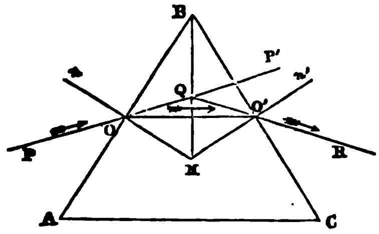

Fig. 318.

The prism being supposed to be more dense than the surrounding medium, or to have a greater refracting power, the ray P O, in passing through it, will be bent towards the perpendicular O N, so that the angle of refraction O' O N will be less than the angle of incidence at O. Thus the refracted ray will be bent out of its course through the angle Q O O', which is the first deviation of the refracted ray from its original direction. The refracted ray O O' being incident on the second surface at O' at the angle O O' N, will pass through this surface, and will emerge in the direction O' R deflected from the perpendicular.

Since O Q is the direction of the original incident ray P O, and Q R the direction of the emergent ray O' R, it follows that the total deviation of the ray from the original direction produced by the two refractions is the angle P' Q R.

1007. *Condition on which the deviation of the refracted ray shall be a minimum.* — If the angle of incidence of the original ray P O be

[redacted]

such that the refracted ray O O′ shall make equal angles with the sides of the prism, that is to say, so that the angles B O O′ and B O′ O shall be equal, then the deviation of the emergent ray O′ R from its original direction will be less than it would be for any other angle of incidence of the original ray P O.

In this case it is easy to see that the angles which the incident and emergent rays P O and O′ R make with the sides of the prism, and with the refracted ray O O′, are equal; for since the angles B O O′ and B O′ O are equal, the angles N O O′ and N O′ O are also equal.

But

$$\frac{\sin.\ \mathrm{P\,O}\,n}{\sin.\ \mathrm{N\,O\,O'}} = \text{index of refraction},$$

$$\frac{\sin.\ \mathrm{R\,O'}\,n'}{\sin.\ \mathrm{N\,O'\,O}} = \text{index of refraction}.$$

But since the angles N O O′ and N O′ O are equal, it follows that the angles P O *n* and R O′ *n*′ are consequently also equal. Therefore the incident and emergent rays make equal angles with the perpendicular to the two surfaces, and therefore with the two surfaces themselves.

It is easy to show experimentally that in this case the deviation of the direction of the emergent from that of the incident ray is a minimum, for the direction of these rays can be determined by observation and the deviation directly measured. By turning the prism on its axis, so as to vary the angle which the first surface makes with the incident ray by increasing or diminishing it, it will be found that the deviation of the direction of the emergent from that of the incident ray will be augmented in whatever way the prism may be turned from that position in which the incident and emergent rays are equally inclined to the sides of the prism.

1008. *How this supplies means of determining the index of refraction.* — Means are thus obtained, by observing the minimum deviation produced upon a ray transmitted through a prism, of determining, by a simple observation, the index of refraction; for the angle of refraction N O O′, being equal to the angle N B O, is one-half the refracting angle of the prism, and the angle of incidence P O *n* is equal to the angle of refraction N O O′, or one-half the angle of the prism, together with the angle O′ O Q, or one-half the deviation O′ Q P′. Thus, if I be the angle of incidence, and R the angle of refraction at the first surface O, and if B be the refracting angle of the prism, and D the angle of deviation, we shall have

$$\mathrm{I} = \tfrac{1}{2}\mathrm{D} + \tfrac{1}{2}\mathrm{B},$$

$$\mathrm{R} = \tfrac{1}{2}\mathrm{B}.$$

Therefore we shall have

$$\frac{\sin.\ \frac{1}{2}\,(D + B)}{\sin.\ \frac{1}{2}\,B} = \text{index of refraction.}$$

By knowing, therefore, the angle of the prism, and by measuring the angle of minimum deviation, the index of refraction of the material composing the prism can be found.

If the ray transmitted through the prism do not fulfil the conditions of transmission at the second surface, it will be reflected, and will therefore return to the first surface, and pass through it into the medium from which it came, or will return to the base, and be transmitted through it, or reflected by it, according as the angle at which it strikes it is within the limit of transmission or not.

In the case represented in *fig.* 319., the incident ray P O striking upon the surface B C at O′, is reflected by it and passes to the base at O″, through which it is transmitted.

1009. ***Rectangular prism used as reflector.*** — A rectangular isosceles prism of glass is often used for an oblique reflector. Such a prism is represented in *fig.* 320. The sides A B and A C being equal, the angles A B C and A C B must be each 45°. If a parallel pencil of rays, of which P O is one, is incident upon B A perpendicularly, it will enter the medium of the prism without refraction, and will proceed to the surface B C, on which it will be incident at O′ at an angle of 45°. Now, the limit of transmission of glass being but 40°, such a ray must suffer total reflection, and will accordingly be reflected from B C at an angle of 45°, that is, in the direction of O′ R, at right angles to the original direction P O′.

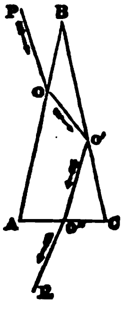

Fig. 319.

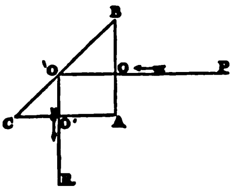

Fig. 320.

An object, therefore, placed at R would be seen by an eye placed at P in the direction P O′, and an object placed at P would be seen by an eye placed at R in the direction R O′.

1010. *Diverging rays refracted at plane surfaces.*—Let I, *figs.* 321., 322., be the focus from which a pencil of diverging rays proceeds, and is incident upon the refracting surface A B C, separating the media M and M′.

Let I B be that ray of the pencil which being perpendicular to the surface is its axis, and will therefore pass into the medium M′ without having its direction changed. Let I D be two other rays equidistant from B falling obliquely on the surface so near the point B as to bring them within the scope of the principle explained in 958. Let D E be the directions of the refracted rays which being continued backwards meet the line B I at R. *Fig.* 321. represents the case in which the medium M′ is more dense than M, and in which, therefore, the refracted rays are deflected towards the perpendicular. *Fig.* 322. represents the case in which the medium M′ is less dense than M, and where, therefore, the refracted rays are deflected from the perpendicular.

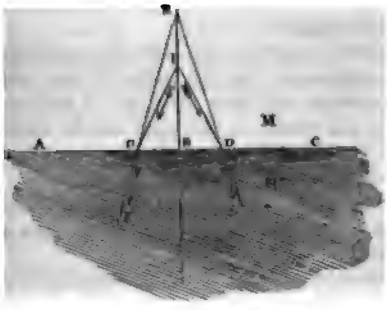

Fig. 321.

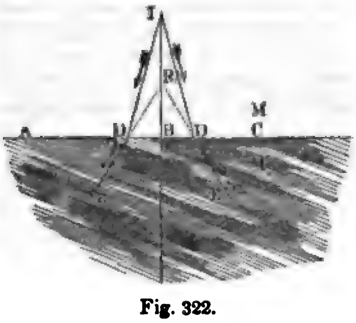

Fig. 322.

In the former case, the point B falls above I, in the latter below it. The point R will then be the focus at which the rays I B and D E, or their continuations, meet. This will therefore be the focus of the refracted rays. The angle D I B which the incident ray makes with the perpendicular I B, is equal to the angle of incidence; and the angle D R B, which the direction of the refracted ray makes with the perpendicular, is the angle of refraction.

Let the distance I B of the focus of incident rays from the surface be expressed by f and R B, that of the focus of refracted rays from the surface by f'.

Since the angles which R D and I D make with R I B are so small as to come within the scope of the principle expressed in 958., we *shall have*

[redacted]

$$\text{I} = \frac{\text{DB}}{f}, \qquad \text{R} = \frac{\text{DB}}{f'};$$

and consequently,

$$\frac{\text{I}}{\text{R}} = \frac{f'}{f}$$

But since the angles I and R are small, their sines, by the principle explained in 958., may be taken to be equal to the angles themselves; and, consequently, we shall have, by the common law of refraction, $\frac{\text{I}}{\text{R}}$ equal to the index of refraction n. Thus we shall have

$$\frac{f'}{f} = n, \quad f' = n \times f \quad . \quad . \quad (\text{C}).$$

In this case, n is the index of refraction of the rays proceeding from the medium M to the medium M′, and is consequently greater than 1 when M′ is more dense than M, and less than 1 when M′ is less dense than M.

The formula (C) is equivalent to a statement that the distance of the foci of refraction and incidence from the refracting surface is in the proportion of the index of refraction to 1; that is to say,

$$f' : f :: n : 1.$$

1011. *Convergent rays incident on plane surfaces.* — The cases represented in *figs.* 321. and 322. are those of diverging rays. Let us now consider the case of converging rays. Let the rays E D be incident upon the surface A B C, *figs.* 323., 324., converging to the point I.

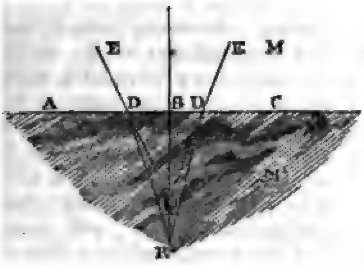

Fig. 323.

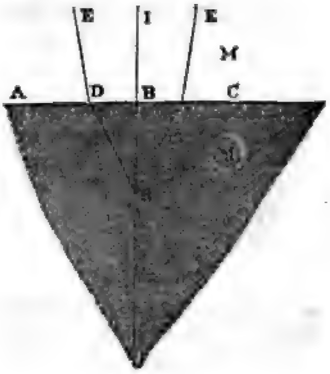

Fig. 324.

[redacted]

If the medium M′ be more dense than M, the rays being deflected towards the perpendicular would meet the axis B I at the point R, more distant than I from B; and if M′ be less dense than M, being deflected from the perpendicular they will meet the axis at the point R, less distant from the surface than I. In this case, the same reasoning will be applicable as in the former, and the same formula (C) for the determination of the relative distances of I and R from B will result.

If I, *figs.* 321., 322., be any point in an object seen by an eye placed within the medium M′, the point I will appear at R, because the rays D E proceeding from it enter the eye as if they came from R. The point will therefore seem to be more distant from the surface A C than it really is in the case represented in *fig.* 321., and less distant in that represented in *fig.* 322.

1012. ***Why water or glass appears shallower than it is.*** – This explains a familiar effect, that when objects sunk in water are viewed by an eye placed above the surface, they appear to be less deep than they are, in the proportion of 3 to 4, this being the index of refraction for water. If thick plates of glass with parallel surfaces be placed in contact with any visible object, as a letter written upon white paper, such object will appear, when seen through the glass, to be at a depth below the surface only of two-thirds the thickness of the glass, the index of refraction for glass being $\frac{3}{2}$.

If a straight wand be immersed in water in a direction perpendicular to the surface, the immersed part will appear to be only three-fourths of its real length, for every point of it will appear to be nearer to the surface than it really is, in the proportion of 3 to 4. If the wand be immersed in a direction oblique to the surface, it will appear to be broken at the point where it meets the surface, the part immersed forming an angle with the part not immersed.

Let A C, *fig.* 325., represent in this case the surface of the water, and let L B L′ be the real direction of the rod, B L′ being the part immersed. From any point P, draw P M perpendicular to the surface A C, and let M p be equal to three-fourths M P. The point P will therefore appear as if it were at p; and the same will be true for all points of the rod from B to L′. The rod, therefore, which really passes from B to L′, will appear as if it passed from B to l', this line B l' being apparently at a distance from the surface of three-fourths the distance B L′.

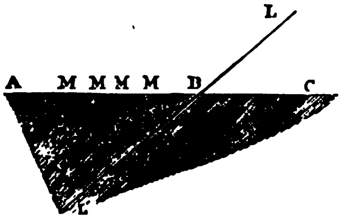

Fig. 325.

1013. *Refracting and refractive power explained.* — Much confusion and consequent obscurity prevails in the works of writers on optics of all countries, arising from the uncertain and varying use of

[redacted]

the terms refracting or refractive power, as applied to the effect of transparent media upon light transmitted through them.

It is evident that if rays of light incident at the same angle on the surfaces of two media be more deflected from their original course in passing through one than in passing through the other, the *refracting power* of the former is properly said to be greater than the *refracting power* of the latter. But it is not enough for the purposes of science merely to determine the *inequality* of refracting power. It is necessary to assign numerically the amount or degree of such inequality, or, in other words, to assign the *numerical ratio* of the refracting powers of the two media.

In some works the *index of refraction* is adopted as the expression of the *refracting power*. Thus the first table in the Appendix to Sir David Brewster's Optics is entitled "Table of *Refracting Powers* of Bodies;" the table being, in fact, a table of the *indices of refraction*.

The correct measure of the refracting power of a medium is, however, not the index of refraction itself, but the number which is found by subtracting 1 from the square of that index. Thus, if n express the index of refraction, $n^2 - 1$ would express the refracting power.

This measure of the refracting power is based upon a principle of physics not easily rendered intelligible without more mathematical knowledge than is expected from readers of a volume so elementary as the present. In the corpuscular theory of light, the number n^2-1 expresses the increment of the square of the velocity of light in passing from the one medium to the other; and in the undulatory theory it depends on the relative degrees of density of the luminous ether in the two media. In each case there are mathematical reasons for assuming it as the measure of the refractive power.

Taking the *refractive power* in this sense, it may be expressed for any medium, either on the supposition that light passes from a vacuum into such medium, or that it passes from one transparent medium to another. If the refractive powers of two media be given, on the supposition that light passes from a vacuum into each of them, the refractive power, where light passes from one medium to the other, can be found by dividing their refractive powers from a vacuum one by the other. Thus the refractive power of glass from vacuum being 1·326, and that of water 0·785, the refractive power of glass, in reference to water, will be

$$\frac{1 \cdot 326}{0 \cdot 785} = 1 \cdot 690.$$

1014. ***Absolute refractive power explained.*** — The term "absolute refracting power" has been adopted to express the ratio of the refracting power of a body to its density. Thus, if D express the density of a medium, and A express its absolute refracting power, we shall have

$$A = \frac{n^2 - 1}{D}.$$

When an elastic fluid or gaseous substance suffers a change of density, its refracting power undergoes a corresponding change, increasing with the density; but in this case the "absolute refracting power" remains sensibly constant, the index of refraction varying in such a manner that $n^2 - 1$ increases or diminishes in the same ratio as the density.

CHAP. IX.

REFRACTION AT SPHERICAL SURFACES.

1015. *The radius of a spherical surface taken as the perpendicular to which all rays are referred.* — It has been already explained that a ray of light incident upon a curved surface suffers the same effect, whether by refraction or reflection, as it would suffer if it were incident upon a plane surface touching the curved surface at the point of incidence; and consequently the perpendicular to which such ray before or after refraction must be referred, will be the normal to the curved surface at the point of incidence. But as the curved surfaces which are chiefly considered in optical researches are spherical, this normal is always the line drawn through the centre of the sphere of which such curved surface forms a part. When a ray of light, therefore, is incident upon any spherical surface separating two media having different refracting powers, its angles of incidence and refraction are those which the incident and refracted rays respectively make with the radius of the surface which passes through the point of incidence.

Thus if A B C, *fig.* 326., be such a surface, of which O is the centre, a ray of light Y P, being incident upon it at P, and refracted in the direction P F, the angle of incidence will be the angle which Y P

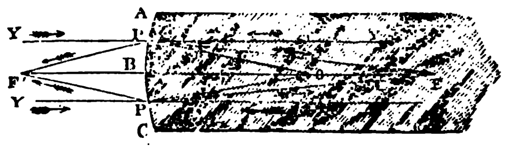

Fig. 326.

makes with the continuation of O P, and the angle of refraction will be O P F. The sine of the angle of incidence will be, according to the common law of refraction, equal to the sine of the angle of refraction multiplied by the index of refraction.

We shall first consider the case of pencils of parallel rays incident on spherical surfaces; and, secondly, that of divergent or convergent rays.

It may be here premised once for all, that in what follows such pencils of rays only will be considered as have angles of incidence or refraction so small as to come within the scope of the principle explained in 958., so that in these cases the angles of incidence and refraction themselves may be substiuted for their sines, and *vice versâ*; and the arcs which subtend these angles, and the perpendiculars drawn from the extremity of either of their sides to the other, may indifferently be taken for each other. The retention of this in the memory of the reader will save the necessity of frequent repetition and recurrence to the same principle.

1016. *Parallel rays.*—Let Y P, *fig.* 326., be two rays of a parallel pencil whose axis is F O B, and which is incident at P upon a spherical surface A B C, whose centre is O.

There are two cases presenting different conditions:

I. When the denser medium is on the concave, and the rarer on the convex side of the refracting surface:

II. When the denser medium is on the convex, and the rarer medium on the concave side of the refracting surface.

1017. *First case. Convex surface of denser medium.*—The rays Y P, *fig.* 326., incident at P, entering a denser medium, will be deflected towards the perpendicular O P, and will consequently meet at a point F beyond O. The angle P O B is equal to the angle of incidence. Let this be called I. The angle O P F is the angle of refraction, which we shall call R.

By the common principles of geometry (Euclid, book 1. prop. 32.), we have

$$R = I - BFP.$$

If the distance B F, of the focus F, from the vertex B be expressed by F, and the radius B O by r, we shall have

$$I = \frac{BP}{r},\ R = \frac{BP}{r} - \frac{BP}{F}.$$

But since I is equal to $n \times R$, we shall have

$$\frac{BP}{r} = n \times \frac{BP}{r} - n \times \frac{BP}{F}.$$

Omitting the common numerator B P, we shall have

$$\frac{1}{r} = \frac{n}{r} - \frac{n}{F};$$

and consequently

$$F = \frac{n\,r}{n-1} \quad . \; . \; (A).$$

1018. *To find the distances of the principal focus from the surface and the centre.*—By this formula, when the index of refraction n, and the radius r of the surface A B C, are known, the distance of the point F from B can always be computed, as it is only necessary to multiply the radius by the index of refraction, and to divide the product by the same index diminished by 1.

To find the distance of the focus F from the centre O, it is only necessary to subtract from the formula expressing its distance from B, the radius r. Thus we have

$$F\,O = \frac{n \times r}{n-1} - r = \frac{r}{n-1} \quad . \; . \; (B).$$

1019. *Case in which the rays pass from the denser into the rarer medium.*—In the case contemplated above, the rays Y P pass from the rarer to the denser medium. If they pass in the contrary direction, that is to say, in the direction Y′ P, then the index n from the denser to the rarer medium will be less than 1, and the expression for F, formula (A), will be negative, showing that in this case the focus lies to the left of the vertex B at F′. The same formula, however, expresses its distance from B, only that the index of refraction n is in this case the reciprocal of the index for the rays passing in the contrary direction. If, then, we express by n' the index of refraction from the denser to the rarer medium, the distance of F′ from B will be expressed by

$$F' = \frac{n' \times r}{n'-1}.$$

It is easy to show that the distance F′ B of the focus of the rays Y′ P from the vertex B is equal to the distance F O of the focus F of the rays Y P from the centre. To show this, it is only necessary to substitute $\frac{1}{n}$ for n', which is its equivalent, and we find

$$F' = \frac{r}{1-n},$$

which is the same as the expression already found for the distance of F from O, but having a different sign, inasmuch as it lies at a different side of the vertex B

1020. *Relative position of the two principal foci.*—The two foci F and F′ of parallel rays incident upon the refracting surface A B C in opposite directions, are called the *principal foci*, one F of the convex surface, and the other F′ of the concave surface.

It follows from what has been just proved that the distance of each of these foci from the vertex B is equal to the distance of the other from the centre O.

It follows, also, from what has been here proved, that parallel rays, whether incident upon the convex surface of a denser, or the concave surface of a rarer medium, will be refracted, converging to a point upon the axis in the other medium, determined by the formulæ above obtained.

1021. *Second case. Concave surface of a denser medium.* — The formulæ (A) and (B) are equally applicable to the case in which the denser medium is on the convex side of the surface A B C. It is only necessary, in this case, to consider that the value of n, for the rays Y P, is less than 1. This condition shows that the value of F, given by the formula (A), is negative, and consequently that the focus will lie to the left of the vertex B, as at F′. Now, since the rays Y P, after passing the surface A B C, have their focus at F′, they must be divergent, and the focus F′ will be imaginary.

In like manner, if the rays pass from the rarer to the denser medium, in the direction Y′ P, the value of F will be positive, because in this case n will be greater than 1, and consequently the focus will lie to the right of the vertex B, as at F, the rays diverging from it being those which, by refraction, pass into the medium to the left of the surface A B C. The focus F, therefore, in this case, is also imaginary.

The same *fig.* 326., therefore, will represent the circumstances attending the case in which the denser medium is at the convex side of the surface, the only difference being that in this latter case F is the focus of the rays Y′ P, and F′ the focus of the rays Y P. The distances of F and F′ from B and O respectively will be the same as in the former case.

1022. *Case of parallel rays passing from air to glass, or* vice versâ. — To illustrate the application of the preceding formulæ, let us suppose, for example, that the denser medium is glass, and the rarer air, and that consequently the value of n, for rays passing from the rarer to the denser, is $\frac{3}{2}$, and its value for rays passing from the denser to the rarer is $\frac{2}{3}$.

We have, consequently, in the case represented in *fig.* 326.,

$$\text{F B} = \frac{n\,r}{n-1} = 3\,r;$$

that is to say, the distance of the principal focus of the parallel rays

F P from B is three times the radius O B, and consequently its distance F O from O is twice its radius.

In like manner, to find the distance F′ B, we have

$$x' = \frac{2}{3},$$

and consequently,

$$r' = -2r;$$

that is to say, the distance F′ B is equal to twice the radius, and is negative, since it lies to the left of B.

In like manner, it will follow that when the surface of the denser medium is concave, B F′ and F O are each equal to twice the radius O B.

1023. *Rays diverging from the principal focus of the convex surface of a denser, or the concave surface of a rarer medium, or converging to the principal focus of the convex surface of a rarer, or the concave surface of a denser medium, are refracted parallel.* — Since the directions of the incident and refracted rays are in all cases reciprocal and interchangeable, it follows that if, in the first case, where the denser medium is on the concave side of the surface, rays are supposed to diverge from either of the foci F or F′, *fig.* 326., they will be refracted parallel to the axis F B in the other medium; and in the second case, if rays be incident upon the refracting surface in directions converging to F or F′, they will be refracted parallel to the axis in the other medium.

It may be asked what utility there can be in considering the case of incident rays converging, inasmuch as rays which proceed from all objects, whether shining by their own light, or rendered visible by light received from a luminary, must be divergent, each point of such objects being a radiant point, which is the focus of a pencil of rays radiating or diverging from it in all directions.

It is true that the rays which proceed immediately from any objects are divergent, and therefore, in the first instance, all pencils of rays which are incident upon reflecting or refracting surfaces are necessarily divergent pencils; but in optical researches and experiments, pencils of rays frequently pass successively from one reflecting or refracting surface to another, and in these cases pencils which were originally divergent, often are rendered convergent, and in this form become pencils incident upon other reflecting or refracting surfaces. In such cases the pencils have imaginary foci behind the surface upon which they are incident, such foci being the points to which they would actually converge if their direction were not changed by the reflecting or refracting surfaces which intercept them.

1024. *Convergent and divergent surfaces defined.* — It appears from the preceding investigation that a spherical refracting surface, having a denser medium on its concave side, always renders parallel *rays convergent*, in whatever direction they are incident upon it; and

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that, on the contrary, a spherical surface, having a denser medium at its convex side, always renders parallel rays divergent in whatever direction they are incident upon it. As these two surfaces possess these distinguishing optical properties, it will be convenient to express the former as a convergent refracting surface, and the latter as a divergent refracting surface.

1025. *Effect of a spherical refracting surface on diverging and converging rays.* — Having explained the conditions which determine the position of the foci of parallel rays incident on spherical reflecting surfaces, we shall now proceed to investigate those by which the focus to which diverging or converging pencils of incident rays are refracted is determined.

Let A B C, *figs.* 327., 328., be a spherical refracting surface, of which the centre is O, and the vertex B. Let I be the focus of the pencil of incident rays, whether diverging or converging; and let R be the conjugate focus of refracted rays, so that the incident pencil may after refraction be converted into another pencil, diverging from or converging to the point R. The angle O P I will be the angle of incidence, and the angle O P R the angle of refraction.

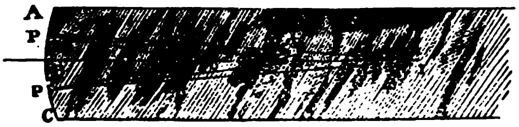

Fig. 327.

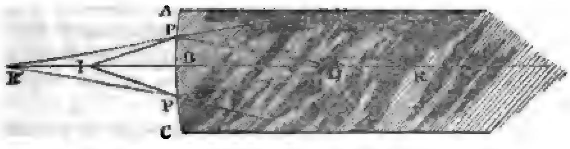

Fig. 328.

Let the radius B O be expressed as before by r, and let I B and R B be expressed respectively by f and f'.

We shall have, by the principles of geometry,[*] *fig.* 327.,

$$OPI = BOP - BIP = \frac{BP}{r} - \frac{BP}{f},$$

$$OPR = BOP - BRP = \frac{BP}{r} - \frac{BP}{f'}.$$

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[*] Euclid, Book 1. Prop. 32.

But since the angle of incidence, being small, is equal to the angle of refraction multiplied by the index of refraction, we shall have

$$\frac{\text{B P}}{r}-\frac{\text{B P}}{f}=n\times\left(\frac{\text{B P}}{r}-\frac{\text{B P}}{f'}\right).$$

Omitting the common numerator B P, we shall have

$$\frac{1}{r}-\frac{1}{f}=n\times\left(\frac{1}{r}-\frac{1}{f'}\right).$$

From this we infer,

$$\frac{1}{f}-\frac{n}{f'}=\frac{1-n}{r}\quad . \quad . \quad (\text{c}).$$

1026. *How to find the focus of refraction when the focus of incidence is given.*—By this formula, when the distance of the focus of incident rays from the vertex, the radius of the surface, and the index of refraction, that is f, n, and r, are known, the position of the focus of refracted rays, that is, its distance f' from the vertex, can always be determined. It is only necessary to observe, that when the value of f' obtained from the formula (c) is positive, it is to be measured to the right of the vertex B, and consequently lies on the concave side of the surface; and that when negative it should be measured to the left of B, and consequently lies on the convex side of the surface.

When the focus of incident rays I lies to the right of B, and therefore on the concave side of the surface, the distance f is positive; but if I lie to the left of B, or on the convex side of the surface, then f in the formula (c) must be taken negatively. The index n is understood in all cases to be the index of refraction of the medium from which the ray proceeds to the medium into which it passes; and is consequently greater than unity when the latter is denser, and less when it is rarer than the former.

With this qualification, the formula (c) will determine the relative position of conjugate foci in every possible case, whether of convergent or divergent rays, and at whichever side of the surface the denser medium may lie.

As an example of the application of this formula, let us take the most common case of a pencil of rays passing from air into glass.

If the pencil be divergent and the refracting surface be convex, as represented in *fig.* 328., the distance of I B, the focus of incident rays, from the vertex, will be negative, and the value of n will be $\frac{3}{2}$. Hence the formula (c) will become

$$-\frac{1}{f}-\frac{3}{2f'}=\frac{-1}{2r}.$$

[redacted]

From whence we infer,

$$f' = \frac{3\,f \times r}{f - 2\,r} \quad . \; . \quad (\text{D}).$$

If I B, or f, therefore, be greater than twice the radius, f' will be positive, and will therefore lie within the surface A B C at a distance from B determined by the formula (D). In this case the rays diverging from I, *fig.* 328., will be made to converge after refraction to R.

But if the distance I B or f be less than twice the radius, then the preceding value of f' will be negative, and must consequently be taken to the left of B, as at R′, *fig.* 328. Consequently, in this case, rays after refraction will diverge, as if they had proceeded from R′.

In fine, if I B be equal to $2\,r$, then the value of f' will be infinite, which indicates that in such case the refracted rays are parallel, their points of intersection being at an infinite distance.

By like reasoning, the position of the focus of refracted rays which corresponds to every other variety of position of the focus of incident rays may be determined.

Principal and secondary pencils. — In the preceding observations, the focus of incident rays is supposed to be placed upon the axis of the spherical surface. Such pencil is, as in the case of reflectors, called the *principal pencil*, and the axis the *principal axis*.

When the focus of a pencil of rays is not on the axis of the refracting surface, or if it be a parallel pencil when its rays are not parallel to such axis, it is called a *secondary pencil*; and its axis, which is the ray passing through the centre of the refracting surface, is called a *secondary axis*.

The focus of refracted rays of a secondary pencil lies upon its axis, and is determined in the same manner as in the case of a principal pencil. The rays, however, from such a pencil will only be refracted to the same point provided the distance of its extreme rays from the axis, measured on the spherical surface, does not exceed a few degrees. If the rays be refracted beyond this limit, they will not be collected into a single point, but will, as in the case of reflectors, be dispersed over a certain space, and produce an aberration of sphericity.

CHAP. X.

PROPERTIES OF LENSES.

1027. *Lens defined.* — When a transparent medium is included between two curved surfaces, or a curved surface and a plane surface, it is called a *lens*.

Lenses are of various species, according to the characters of the curved surfaces which bound them; but those which are almost exclusively used in optical instruments and in optical experiments, are bounded by spherical surfaces, and to these, therefore, we shall here limit our observations.

Spherical surfaces, combined with each other and with plane surfaces, produce the following six species of lens, which are denominated converging and diverging lenses, because, as will be explained hereafter, the first class render a pencil of parallel rays incident upon them convergent, and the second class render such a pencil divergent.

1028. *Three forms of converging lenses, — meniscus, double convex, and plano-convex.* — Converging lenses are of the three following species:—

I. *The meniscus.* The form of this lens may be conceived to be produced as follows: —

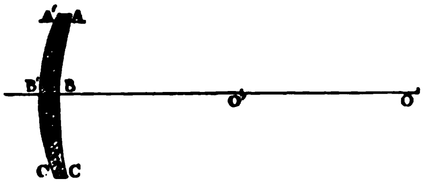

Fig. 329.

Let A B C and A′ B′ C′, *fig.* 329., be two circular arcs, whose middle points are B and B′, and whose centres are O and O′, the radius O B being greater than the radius O′ B′. Let the two arcs be supposed to revolve round a line O O′ B B′ as an axis, and they will in their revolution produce a solid of the form of the meniscus lens.

It is evident from this that the convexity A′ B′ C′ of such a lens is greater than its concavity A B C, the radius C′ B′ of the convexity being less than the radius O B of the concavity.

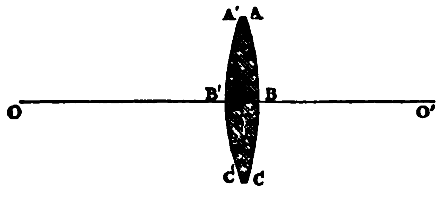

Fig. 330.

II. *Double convex lens.* The form of this lens may in like manner be conceived to be produced as follows:—

Two circular arcs, A B C and A′ B′ C′, *fig.* 330., whose middle points are B and B′, and whose centres are O and O′, being conceived to revolve round a line O B′ B O′ as an axis, will, by their revolution, produce the form of this lens. The convexities of the sides will be equal or unequal according as the radii O B and O′ B′ are equal or unequal.

III. *Plano-convex lens.* The form of this lens may be conceived to be produced as follows: —

Let A′ B′ C′, *fig.* 331., be a circular arc, whose middle point is B′,

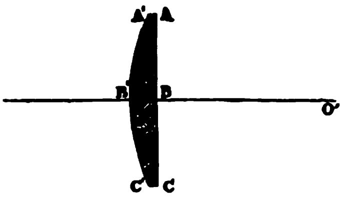

Fig. 331.

and whose centre is O'; and let A B C be a straight line at right angles to B' O', whose middle point is B. If a figure thus formed revolve round the line O' B' as an axis, it will produce the form of a plano-convex lens, the side A B C being plane, and the side A' B' C' being convex.

1029. *Three forms of diverging lenses, — concavo-convex, double concave, and plano-concave.* — Diverging lenses are of the three following species: —

I. *Concavo-convex lens.* To form this lens, as before, proceed as follows:—

Let A B C and A' B' C', *fig.* 332., be two circular arcs, whose middle points are B and B', whose centres are O and O', and whose radii are O B and O B'; the latter being greater than the former. If this be supposed to revolve round the line O' O B B' as an axis, it will produce the form of a concavo-convex lens. Since the radius of the concave side A B C is less than the radius of the convex side A' B' C', the concavity will be greater than the convexity.

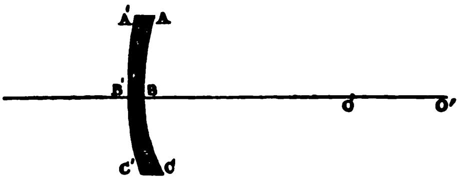

Fig. 332.

II. *Double concave lens.* The form of this lens may be supposed to be produced as follows:—

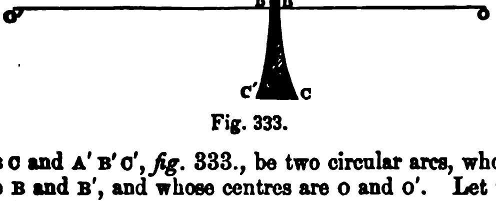

Fig. 333.

Let A B C and A' B' C', *fig.* 333., be two circular arcs, whose middle points are B and B', and whose centres are O and O'. Let this figure be supposed to revolve round the line O O' as an axis, and it will produce the form of a double concave lens. The concavities will be equal or unequal, according as the radii O B and O′ B′ are equal or unequal.

III. *Plano-concave lens.* This lens may be conceived to be produced as follows:—

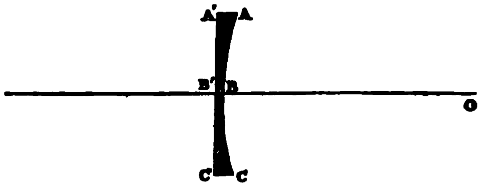

Fig. 334.

Let A B C, *fig.* 334., be a circular arc, whose middle point is B, and whose centre is O. Now let A′ B′ C′ be a straight line perpendicular to O B, whose middle point is B′. Let this figure be supposed to revolve round O B B′ as an axis, and it will produce the form of a plano-concave lens.

1030. *The axis of a lens.* — In all these forms of lens the line O B B′ is called the *axis of the lens.*

1031. *The effect produced by a lens on incident rays.*— To determine the effect produced on a pencil of rays by a lens, we shall first take the case of the meniscus.

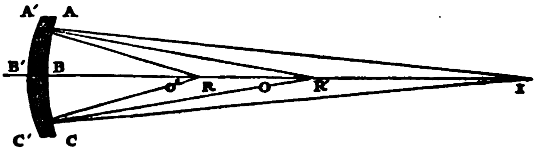

Fig. 335.

Let O, *fig.* 335., be the centre, and O B the radius of the concave surface A B C. Let O′ be the centre, and O′ B′ be the radius of the convex surface A′ B′ C′. Let I be the focus of a pencil of rays incident upon the surface A B C. Let R′ be the focus to which the rays of this pencil would be refracted by the surface A B C, independently of the surface A′ B′ C′.

The pencil whose focus is this point R′ will then be incident upon the second surface A′ B′ C′ of the lens, and the rays from this pencil being again refracted by the second surface will have another focus R, which will be the definitive focus of the rays after refraction by both surfaces of the lens.

In this, and in all other cases of lens, it will be necessary that the

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thickness B B′ of the lens may be disregarded, being inconsiderable compared with the other magnitudes which enter into computation.

Now let the distances of the foci I, R′, and R from the middle point B or B′ of the lens be expressed respectively by f, f'', and f'; and let the radii O B and O′ B′ be expressed by r and r'; we shall then have, by what has been already explained respecting refracting surfaces, the following conditions:

$$\frac{1}{f}-\frac{n}{f''}=\frac{1-n}{r}.$$

$$\frac{1}{f''}-\frac{n'}{f'}=\frac{1-n'}{r'}.$$

In this case n is the index of refraction from air into the medium of the lens, and n' is the index of refraction from the medium of the lens into air.

By what has been already explained, these two indices are reciprocals, and consequently their product is equal to unity, so that we shall have $n \times n' = 1$.

Now if we multiply the latter equation by n, we shall have

$$\frac{n}{f''}-\frac{n \times n'}{f'}=\frac{n-n \times n'}{r'};$$

but since $n \times n' = 1$, this will become

$$\frac{n}{f''}-\frac{1}{f'}=\frac{n-1}{r'};$$

by combining this with the first equation we shall have

$$\frac{1}{f}-\frac{1}{f'}=\frac{1-n}{r}-\frac{1-n}{r'} \quad . \quad . \quad (\text{E}).$$

By these conditions the distance f' can always be determined when f, r, r', and n are known; that is to say, the position of the focus of refracted rays can always be determined when the position of the focus of incident rays, the radii of the lens, and the index of refraction are known.

This formula (E), by a due attention to the signs of the quantities which compose it, may be applied to lenses of every species. If the focus of incident rays lie to the right of the lens, as in *fig.* 335., f must be taken to be positive; if to the left of the lens, f must be taken negatively. If the centre of either surface lie to the right of the lens, the radius will be taken positively; and if to the left of the lens, it will be taken negatively. If one of the surfaces of the lens be a plane surface, it may be considered as having an infinite radius; and accordingly, the term of the equation (E) in the denominator of which such radius enters will become equal to 0, and will therefore disappear from the equation.

When the value of f', which determines the distance of the focus of refracted rays from B, will have been found by the equation (E), it must be taken to the right of the point B if it be positive, and to the left if it be negative.

1082. *To determine the principal focus of a lens.*—If the incident rays whose focus is I be refracted parallel, then the distance f' of the focus of refraction from B will be infinite, and consequently, we shall have $\frac{1}{f'} = 0$. Now, in this case, I will be the *principal focus* for parallel rays incident upon the surface A′ B′ C′. Let this be expressed by F, and we shall have by the equation (E)

$$\frac{1}{F} = \frac{1-n}{r} - \frac{1-n}{r'},$$

from which we infer,

$$F = \frac{r\,r'}{(1-n)\,(r'-r)} \quad . \quad . \quad (F).$$

a formula by which the distance of the focus of parallel rays incident upon A′ B′ C′ can always be calculated.

If the incident rays be parallel, their focus I will be at an infinite distance, and we shall have $\frac{1}{f} = 0$. In this case, the focus R will be the principal focus of the parallel rays incident upon the surface A B C.

Let the distance of this focus from B be expressed by F′, and we shall find as before from equation (E),

$$F' = -\frac{r\,r'}{(1-n)\,(r'-r)} \quad . \quad . \quad (G).$$

Thus it appears that F and F′ differ in nothing save in their sign, the one being positive and the other negative; the inference from which is, that parallel rays, whether incident on the one or the other surface of a lens, will be refracted to points equally distant from the lens, but on opposite sides of it.

1033. *The focal length of a lens.*—The common distance of these principal foci from the lens is called the *focal distance* or *focal length of the lens.*

1034. *The meniscus, double convex, and plano-convex, are convergent lenses.*—If the lens be a meniscus, and composed of a refracting substance more dense than air, it will render a parallel pencil incident upon either of its surfaces convergent, and its principal foci will consequently be real. This follows as a consequence from the formulæ (F) and (G); for in the case of a meniscus, r' is less than r, and, consequently, the value of F given by the formula (F) is positive, and the value of F′ given by the formula (G) is negative; consequently, the focus of parallel rays incident upon A′ B′ C′ lies to the

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right of the lens, and the focus of parallel rays incident on A B C lies to the left of it. Parallel rays are therefore rendered convergent after refraction, and the foci are real in whichever direction they may pass through such a lens.

It is easy to show, that the same will be true for double convex and plano-convex lenses. In the case of double convex lenses, the radius r is negative and r' positive; the consequence of which is, that the value of F is positive, and F' negative. In the case of plano-convex lenses, the radius r is infinite, and the formulæ (F) and (G) become

$$F = + \frac{r'}{n-1}, \quad F = - \frac{r'}{n-1}.$$

Thus it appears, that in all the three forms of convergent lens, parallel rays, whether incident on the one surface or on the other, are refracted, converging to a focus on the other side of the lens; and the foci in all such cases are consequently real.

1035. *Concavo-convex, double concave, and plano-concave, are divergent lenses.* — It is easy to show, by the same formulæ, that parallel rays incident on every species of divergent lens will be refracted diverging from a point on the same side of the lens as that at which they are incident.

In the case of the concavo-convex lens, the radius r' is greater than the radius r; and since n is greater than 1, the value of F (given in the formula F) will be negative, and the value of F' (given in the formula G) positive. Thus it appears that the principal focus of parallel rays incident on the surface A B C, *fig.* 332., will be to the right of B, and the principal focus of the rays incident on the surface A' B' C' to the left of B, the foci in each case being at the same side of the lens with the incident rays; and, consequently, being in such case imaginary.

In the case of the double concave lens, the radius r' is negative; and since n is greater than 1, the value of F will be negative, and that of F' positive.

In the case of the plano-concave lens, the value of r' is infinite, and since n is greater than 1, F will be negative, and F' positive.

Thus it appears that in all the forms of divergent lenses, parallel rays incident upon their surfaces are refracted, diverging from a focus on the same side of the lens as that at which they are incident.

It is from this property that the two classes of convergent and divergent lenses have received their denomination; and it is evident, therefore, that the meniscus and plano-convex lens are optically equivalent to a double convex lens, and that the concavo-convex and plano-concave lens are optically equivalent to a double concave lens.

1036. *Case of a lens with equal radii and convexities in the same direction.* — Among the varieties presented by the preceding formulæ, there is an exceptional case which requires notice. If the radii of the two surfaces of a lens be equal, and their centres be both at the same side of the lens, the lens will hold an intermediate place between a meniscus and a concavo-convex. In the former, the radius of the convex surface is less than that of the concave surface; and in the latter, the radius of the concave surface is less than that of the convex surface. These radii might, however, be in each case as nearly equal as possible, the lenses actually retaining their specific characters. Each species, therefore, would approach indefinitely to an intermediate lens whose surfaces would have equal radii.

It is evident that the condition which would render equal the radii r and r', and give them the same sign, would render both the focal distances F and F' infinite, their denominators being nothing.

To comprehend this it is only necessary to consider that in the case of the meniscus and the concavo-convex lens, the more nearly equal the radii r and r' are, the less will be the denominators of the values of F and F'; and, consequently, the greater will be these values themselves, and if we suppose the difference between the radii to be infinitely diminished, the values of F and F' will be infinitely increased. These conditions lead to the inference that if the radii of the two surfaces be equal, the focus of parallel rays incident upon these two surfaces will be infinitely distant from the lens, that is to say, parallel rays will be refracted parallel.

Thus it appears that a lens formed by spherical surfaces, whose radii are equal, and whose centres lie at the same side of the lens, will have no effect on the direction of rays proceeding through it, and that such lens will be equivalent to transparent plates with parallel surfaces.

An example of such a lens as this is presented in the usual form of a watch-glass.

1037. *Lenses may be solid or liquid.*—Lenses may be composed of any transparent substance, whether solid or liquid.

If they be composed of a solid, such as glass, rock crystal, or diamond, they must be ground to the required form, and have their surfaces polished; if they be composed of liquid, they must then be included between two lenses, such as has been just described, having themselves no refracting power, and having the form required to be given to the liquid lens.

Thus, two watch-glasses, placed with their concavities towards each other, and so inclosed at the sides as to be capable of holding a liquid, would form a double convex liquid lens. If their convexities were presented towards each other, they would form a double concave liquid lens.

1038. *Rules for finding the focal length of lenses of glass.*—The material almost invariably used for the formation of lenses in optical instruments being glass, it will be useful here to give the

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principal formulæ, showing the position of the focus in lenses of this material.

In the case of glass, the index of refraction, the incident rays being supposed to pass from air into that medium, is $\frac{3}{2}$: the formulæ (E) and (F) therefore, in this case, become

$$\frac{1}{f}-\frac{1}{f}=\frac{1}{2r'}-\frac{1}{2r}. \quad . \quad (E').$$

$$F=\frac{2rr'}{r-r'} \quad . \quad . \quad . \quad (F').$$

By the latter formula, the focal length of a glass lens can always be found.

In its application, however, it is necessary to observe that when the convexities of the surface of the lens are turned in opposite directions, as in the cases of double convex and double concave lenses, the denominator will be the *sum* of the radii; and if they are turned in the same direction, as in the case of the meniscus, and the concavo-convex lens, it will be the *difference* of the radii. The following general rule will always serve for the determination of the focus when both surfaces of the lens are spherical

RULE.

Divide twice the product of the radii by their difference for the meniscus and concavo-convex lenses, and by their sum for the double convex and double concave lenses. The quotient will in each case be the focal length sought.

To find the focus of a plano-convex or a plano-concave lens, we are to consider that it has been already proved that the focal length is given by the formula

$$F=\frac{r}{n-1}.$$

and since n is $\frac{3}{2}$, we shall have

$$F=2r;$$

that is to say, the focal length of a plano-convex or plano-concave lens is double the radius of the convexity or concavity.

If a double convex or double concave lens have equal radii, then the formula (F') becomes

$$F=r.$$

The focal length, therefore, of such a lens is equal to the radius of either surface.

For the same class of lens the formula (r′) becomes

$$\frac{1}{f}-\frac{1}{f'}=\frac{1}{r};$$

where r expresses the common magnitude of the radii of the two surfaces. From this we infer,

$$f'=\frac{rf}{r-f};$$

which supplies the following rule for finding the focus of refracted rays, when the focus of incident rays is given.

RULE.

Multiply the common radius of the two surfaces by the distance of the focus of incident rays from the lens, and divide the product by the difference between the radius and the distance of the focus of incident rays from the lens.

If the distance of the focus of incident rays from the lens in this case be less than the radius, the value of f' will be positive, and the focus of refracted rays will lie at the same side of the lens with the focus of incident rays; but if the value of f be greater than r, then the value of f' will be negative, and the focus of refracted rays will lie at the other side of the lens.

1039. *Case of secondary pencils.*—We have here considered those cases only in which the focus of the incident pencil is placed upon the axis of the lens, or of pencils whose rays are parallel to that axis. The focus of the refracted rays may, however, be determined by the same formula for secondary pencils whose axes, passing through the centre of the lens B, are inclined to its axis, provided only the inclination be not so great as to produce such spherical aberration as may prevent the rays from having an exact, or nearly exact, focus.

1040. *Field of a lens. Opening of a lens.*—If X X′, *fig.* 336., be the axis of the lens, and Y Y be the greatest angle at which the axis of the secondary pencil can be inclined to X X′, so that the rays may have a nearly exact focus, the angle included between the two secondary pencils Y Y′ is called the *field of the lens.*

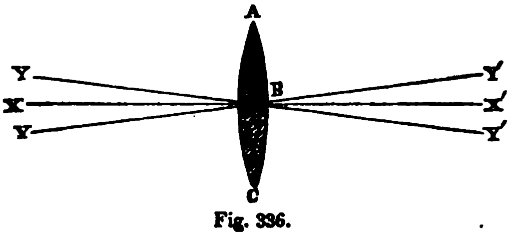

Fig. 336.

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The angle formed by lines drawn from the edge of the lens to its principal focus is called the *opening of the lens*; and this opening cannot in general exceed 10° or 12° without producing an aberration of sphericity, which would prevent the rays of the pencil incident upon it from having an exact focus.

1041. *Images formed by lenses.* — The images of objects formed by lenses are explained upon the same principles as have already been applied to the case of spherical surfaces. If an object, whether it be self-luminous like the sun, or receive light from a luminary like the moon, be placed before a lens, each point upon its surface may be considered as a point from which light radiates in all directions. Such a point will be then the focus of a diverging pencil incident upon the lens, the base of the pencil being the surface of the lens.

If the pencils which thus diverge from all points of the object be rendered, after refraction by the lens, convergent, they will have real foci on the other side of the lens, and the assemblage of such foci will form an *image* of the object. But if these pencils, after passing through the lens, be divergent, their foci will be imaginary, and will be placed at the same side of the lens with the object.

These pencils would in such case be received by an eye on the other side of the lens as if they had originally proceeded from these points, which are the foci of the refracted pencils.

The assemblage of these points would thus form an *imaginary image*.

All these circumstances are analogous to those which have been already explained in the case of reflectors. They will, however, be rendered still more intelligible by explaining their application to glass lenses.

1042. *Every lens, whatever be its form, can be represented by a double convex or double concave lens with equal radii.* — Since all converging lenses, having equal focal lengths, are optically equivalent, a double convex lens with equal radii can always be assigned, which is the optical equivalent of any proposed converging lens, whether it be meniscus, double convex with unequal radii, or plano-convex.

Since, in like manner, all diverging lenses having equal focal lengths are optically equivalent, a double concave lens with equal radii may always be assigned, which is the optical equivalent of any proposed diverging lens, whether it be concavo-convex, double concave with unequal radii, or plano-concave.

1043. *Image formed by double convex lens.* — It will therefore be sufficient to investigate the effects of double convex and double concave lenses with equal radii.

Let A B C, *fig.* 337., therefore, be a double convex lens, with equal radii; and let L M be an object, the centre of which is upon the axis of the lens, and placed beyond the principal focus F. Let the distance of this object from B be expressed by f; let the distance of its image

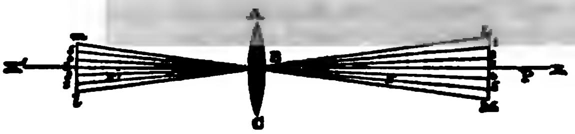

Fig. 237.

be f', and the focal length of the lens, or its radius, be r. By what has been already explained, we shall have

$$\frac{1}{f'} = \frac{1}{f} - \frac{1}{r};$$

and, therefore,

$$f' = \frac{r \times f}{r - f}.$$

Since the distance of the object from the lens is supposed to be greater than B F, we shall have f greater than r; and consequently f' will be negative, which indicates that the image of L M will lie on the other side of the lens.

It appears also, by the preceding formula, that the distance f' of the image from the lens will be greater than r, and the image $l\,m$ will therefore lie beyond the point F'.

If we draw L B l, this line will be the secondary axis of the pencil whose focus is at L, and consequently the focus of refracted rays will be at l; so that an image of the point L will be formed at l. In like manner it may be shown, that an image of the point M will be formed at m; and in like manner the images of all the points of the object, such as, 1, 2, 3, 4, 5, between L and M, will be formed at corresponding points 1, 2, 3, 4, 5 between l and m. It is evident, therefore, that in this case the image will be inverted.

1044. *Conditions which determine the magnitude of the image.*—Since the axis of the extreme secondary pencils L l and M m intersect at the centre of the lens, we shall have the following proportion:—

$$\text{LM} : lm :: \text{LB} : l\text{B};$$

or, which is the same,

$$\text{LM} : lm :: f : f';$$

that is to say, the magnitude of the object is to that of its image, as their distances respectively from the lens. The image, therefore, will be greater, equal to, or less than the object, according as f' is greater, equal to, or less than f.

Now, it appears by the preceding formula, that if f be equal to 2 r, f' will also be equal to 2 r; that is, if the distance of the object from the lens be twice the focal length of the lens, the distance of

[redacted]

the image on the other side of the lens will also be twice the focal length; the image, therefore, in this case will be equal to the object.

An object L M being moved towards F, so as to have less distance from the lens than twice its focal length, the value f' given by the preceding formula becomes greater than f, and increases as the object approaches F. It appears, therefore, that as L M approaches F, its image $l\,m$ recedes from F′ in the direction F′ X′, and that consequently the distance of the image from the lens being greater than that of the object, its magnitude will be greater in the same proportion.

When the object L M comes very near F, the denominator in the preceding formula becoming very small, the magnitude of f' becomes very great, and if we suppose L M to arrive at F, then the denominator becomes O, the value of f' becomes infinite, and consequently the image $l\,m$ would recede to an infinite distance from the lens, and in effect cease to exist.

This circumstance is only what might have been anticipated; for when an object L M is placed at F, a pencil of rays proceeding from any point in it, such as L, will, after passing through the lens, become parallel; and having no point of convergence, it cannot form an image of the pencil L; and the same will be true of any point in the object.

But although no image of the object thus placed in the principal focus of the lens will thus be formed, the lens, in such a position with reference to the object, is attended with optical effects of great importance, which will be explained hereafter.

Let us again suppose an object L M placed at a distance from the lens equal to twice its focal length, and the image $l\,m$ placed at an equal distance at the other side of the lens. If we now suppose the object, instead of approaching F, to recede from it in the direction F X, the distance f being greater than $2\,r$, it follows that the magnitude of f' will be less than $2\,r$; and the farther the object L M is removed from the lens, and consequently the more its distance from the lens exceeds twice the focal length, the nearer will the image $l\,m$ approach to F′.

1045. *Images of very distant objects are formed at the principal focus.* — If we suppose the object L M to recede to a distance actually infinite, then the value of f' would become equal to r, and the image $l\,m$ would coincide with the principal focus F′. This would lead to the inference that the image of a distant object can never be in the principal focus of the lens, because such a supposition would involve the condition that the distance of the object must be actually infinite.

But, practically, it is found, that if the diameter of the lens bear an insignificant proportion to the distance of the object from it, the image of the object will be formed at its principal focus. This is easily explained by reference to the conditions which determine the position of the principal focus.

It will be recollected, that the point F′ is the focus to which parallel rays incident upon the lens A B C would be made to converge. Now, if L M be so distant from the lens that the rays proceeding from any point upon it, such as L, and incident upon the lens, may be considered as parallel, these rays will converge to the principal focus.

This will be the case, provided that the diameter of the lens, which is that of the base of the pencil, is incomparably less than the distance of the object from the lens. Thus, let us suppose the diameter of the base to be 4 inches, and the distance of the object to be 80 feet; the distance will in this case be 240 times the diameter of the lens, and the angle of divergence of the pencil would consequently be less than a quarter of a degree. A pencil which has such a divergence as this would be refracted to a point not sensibly different from the principal focus.

It is evident, that as an object recedes to an increasing distance from the lens, the image approaching the principal focus F' diminishes in magnitude in proportion to the distance from the lens; and when the image is formed at the principal focus, the lines *l* B, *m* B drawn from its extremities to the centre of the lens will form an angle equal to that which would be formed by lines drawn from the extremities of the object to the same point.

1046. *Experimental illustrations.* — All these circumstances admit of easy experimental verification. Let P be a point on the axis at a distance from B equal to 2 B F, so that P F shall be equal to B F. — Let the flame of a candle be held at L M between F and P, the lens A C being inserted in an aperture formed in a screen so as to exclude the light of the candle from the space to the left of the lens. If a white screen be held at right angles to the axis and behind the lens, and be moved to and fro, until a distinct inverted image of the candle shall be seen upon it, its distance from the lens when this takes place will be found to be greater than twice the focal length, and to correspond exactly with that which would be computed by the preceding formula. If the candle be moved towards P, the image will become indistinct upon the screen, but will recover its distinctness by moving the screen towards F'; and if the candle be placed at P, the screen being placed at a distance from B equal to twice B F', a distinct image will be formed on the screen equal in magnitude to the object. If the candle be moved from P towards X, the screen must be moved towards F' to preserve the image distinct; and if the candle be gradually moved in the direction P X, the screen must be continually moved towards F'. If the candle be moved to so great a distance from the lens that the diameter of the lens shall have an insignificant proportion to its distance, then a distinct image will be formed on the screen placed at the principal focus F'. If the candle be placed at the principal focus F, then the screen will show no image of it in whatever position it may be placed behind the lens, but will exhibit merely an illuminated disk formed by parallel rays composing the refracted pencils into which the pencils proceeding from such point of the candle are converted by the lens.

Let us now suppose such object placed at L M, *fig.* 338., between the principal focus F and the lens. In this case, f being less than r, the value of f' obtained by the preceding formula will be positive, and, consequently, the focus of refracted rays will lie at the same side of the lens with the focus of incident rays. If then the pencil of rays diverging from L pass through the lens, it will after refraction diverge from the point l, more distant from the lens than L. In like manner, the pencil diverging from M will after passing through the lens diverge from m; and the same will be true of all the intermediate points of the object, so that the various pencils which diverge from different points of the object and pass through the lens will after refraction diverge from the corresponding points of $l\,m$. The image, therefore, in this case will be imaginary, and an eye placed to the left of the lens A B C would receive the rays of the various pencils as if they diverged, not from a point of the object L M, but from points of the imaginary image $l\,m$.

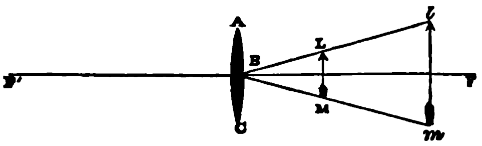

Fig. 338.

The magnitude of the image in this case will be greater than the object in the same proportion as l B is greater than L B.

As the object L M is moved towards F, its distance f from the lens will approach to equality with r, and the denominator of f' in the preceding formula diminishes, and consequently the distance of its image from the lens will be proportionally increased; therefore, as the object L M is moved towards F, its image $l\,m$ will recede indefinitely from the lens, and would become infinite in distance and magnitude when the object arrives at F, which is consistent with what has been already explained of the principal focus.

It appears, therefore, that whenever the object is between the principal focus and the lens, its image will be at a greater distance from the lens on the same side of it, and will be erect, imaginary, and greater than the object.

1047. *Images formed by concave lenses.* — If an object L M, *fig.* 339., be placed before a double concave lens A B C, the focus corresponding to the several points of the object will lie between the object and the lens, at distances determined by the formula

$$f' = \frac{r \times f}{r + f}$$

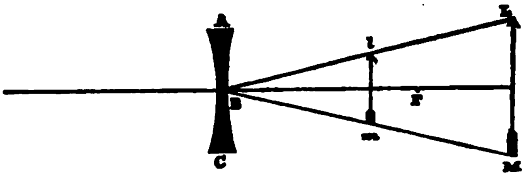

Fig. 339.

It is evident from this formula that f' is less than f, and that consequently the distance of the image $l\,m$ from the lens is less than the distance of the object from it. It appears also that the distances f and f' increase and diminish together, so that when the distance of an object from the lens L M is augmented, the distance of its image $l\,m$ will also be augmented. But the distance of the image from the lens can never be greater than the focal length of the lens, because, as the distance of the object is indefinitely increased, the value of f' obtained from the formula approaches indefinitely to equality with r, though it can only become equal to it when the distance of the object becomes infinite.

1048. *Spherical aberration.* — We have hitherto considered that the pencils of rays proceeding from the lens were brought to an exact focus, and this would be practically the case if the angles of incidence of the extreme rays of the pencils do not exceed a certain limit; but if, from the magnitude of the lens, or the proximity of the object, this be not the case, effects will be produced which have been called *spherical aberration*, which it will be necessary here more clearly to explain.

Let A B C, *fig.* 340., be a plano-convex lens, having its plane side

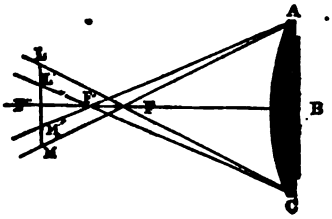

Fig. 340.

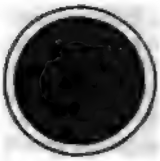

Fig. 341.

presented to the incident rays. Let a circular disk of card or sufficiently thick paper be formed, a little less in diameter than the lens, and let it be attached concentrically with the lens upon the plane side, so as to leave a narrow ring of the glass uncovered round the edge of the lens, as represented in *fig.* 341.

If this lens be now presented to a distant object, such as the sun, none but the extreme rays of each pencil will pass through it, and an image will be formed of the sun by these extreme rays at F, which will therefore be the principal focus of an annulus of parallel rays passing through the edge of the lens. Now let another circular piece of paper or card be cut so as to cover an annular surface surrounding the edge of the lens, and another to cover the central portion of it, so as to leave a ring of the surface uncovered at some distance within the edge, as represented in *fig.* 342. The lens being again presented to the sun, it will be found that an image will be formed at F′, *fig.* 340., somewhat more distant from the lens than F.

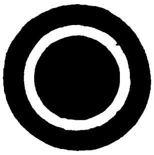

Fig. 342.

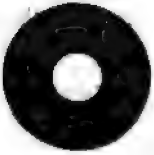

Fig. 343.

If, in fine, a disk of card be cut, equal in magnitude with the lens, having a small circular aperture at its centre, as represented in *fig.* 343., and be in the same manner attached to the lens, so as to allow only the central rays of each pencil to pass, an image of the sun will be formed at F″, *fig.* 340., still further from the lens.

It appears, therefore, that those rays of the pencil which are nearest the centre will have a focus further from the lens than those which are more distant from it, and the more distant the rays of each pencil are from the axis of the lens, the nearer their focus will be to the lens.

If the lens being uncovered be therefore presented to the sun, the rays incident near its edge will be refracted to the focus F, and after passing that focus will diverge in the direction F L and F M. The rays incident nearer to the centre will intersect at F′, and will diverge to L′ M′, while the rays nearer the axis will intersect at F″.

1049. *Longitudinal and lateral aberrations.* — The distance F F″ measured on the axis between the focus of the extreme rays which pass through the edge of the lens, and the focus of the central rays along which the foci of all the intermediate rays are placed, is called the *longitudinal aberration:* the point F″, which is the focus of the central rays, is called the *principal focus* of the lens; and the circle whose diameter is L M, over which the rays are spread, is called the *lateral aberration.*

1050. *Experimental illustration.* — These effects may be rendered apparent by holding a white screen at F″, at right angles to the axis of the lens. An image of the sun will be formed round F″, and beyond the edge of

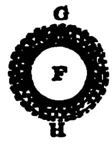

Fig. 344.

this image will be formed a ring or halo of light, growing fainter from the central image outwards, as represented in *fig.* 344.

1051. ***Magnitude of spherical aberration in different forms of lenses.*** — The magnitude of the spherical aberration varies in the different forms of lenses.

1. In a plano-convex lens with its plane side turned to parallel rays, that is, turned to distant objects if it is to form an image behind it, or turned to the eye if it is to be used in magnifying a near object, the spherical aberration will be $4\frac{1}{2}$ times the thickness.

2. In a plano-convex lens with its convex side turned towards parallel rays, the aberration is only $1\frac{17}{100}$ of its thickness. In using a plano-convex lens, therefore, it should always be so placed that parallel rays either enter the convex surface or emerge from it.

3. In a double convex lens with equal convexities, the aberration is $1\frac{67}{100}$ of its thickness.

4. In a double convex lens, having its radii as 2 to 5, the aberration will be the same as in a plano-convex lens in Rule 1., if the side whose radius is 5 is turned towards parallel rays; and the same as the plano-convex lens in Rule 2., if the side whose radius is 2 is turned to parallel rays.

5. The lens which has the least spherical aberration is a double convex one, whose radii are as 1 to 6. When the face whose radius is 1 is turned towards parallel rays, the aberration is only $1\frac{7}{100}$ of its thickness.

These results are equally true of plano-concave and double concave lenses.

If we suppose the lens of least spherical aberration to have its aberration equal to 1, the aberration of the other lenses will be as follows: —

Best form, as in Rule 5	1·000
Double convex or concave, with equal curvatures	1·561
Plano-convex or concave in best position, as in Rule 2.	1·093
Plano-convex or concave in worst position, as in Rule 1.	4·206

CHAP. XI.

ANALYSIS OF LIGHT.

1052. ***Solar light a compound principle.*** — In the preceding chapters, light has been regarded, in relation to transparent media, as a simple and uncompounded principle, each ray composing a pencil being subject to the same effects.

That all light is not thus subject to uniform effects, is rendered *manifest* by the following experiment of Newton: —

Let a pencil of parallel rays of solar light be admitted through a circular opening P, *fig.* 345, about half an inch in diameter, made in

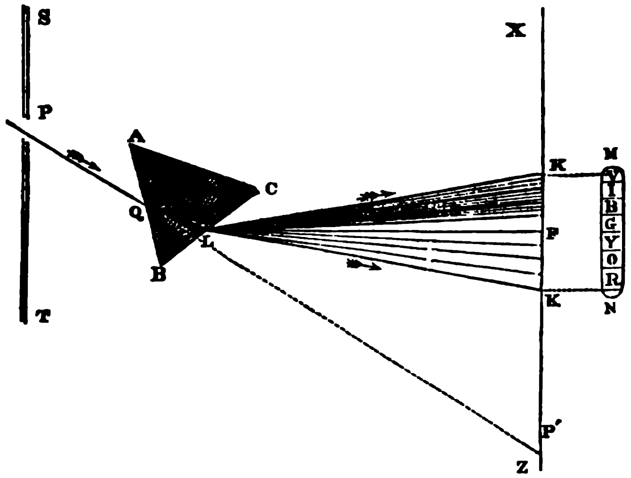

Fig. 345.

a screen or partition S T, all other light being excluded from the space into which the pencil enters. If a white screen X Z be placed parallel to S T, and at a distance from it of about 12 feet, a circular spot of light nearly equal in diameter to the hole will appear upon it at P′, the point where the direction of the pencil meets the screen. Now let a glass prism be placed at A B C, with the edge of its refracting angle B in a horizontal direction, and presented downwards so as to receive the pencil upon its side A B at Q. According to what has been already explained, the pencil would be refracted, in passing through the surface A B, in the direction Q L towards the perpendicular; and it would be again refracted in emerging from the surface C B from the perpendicular in the direction L K. It might therefore be expected that the effect of the prism would be merely to move the spot of light from P′ to some point, such as K, more elevated upon the screen. The phenomenon, however, will be very different. Instead of a spot of light, the screen will present an oblong coloured space, the outline of which is represented at M N as it would appear when viewed in front of the screen.

1053. *The prismatic spectrum.* — The sides of this oblong figure are parallel, straight, and vertical; its ends are semi-circular, and its length consists of a series of seven spaces, vividly coloured, the lowest

space being red, R; the next in ascending orange, O; and the succeeding spaces yellow, Y; green, G; light blue, B; dark blue or indigo, I; and, in fine, violet, V.

These several coloured spaces are neither equal in magnitude nor uniform in colour. The red space R, commencing at the lowest point with a faint red, increases in brilliancy and intensity upwards. The red, losing its intensity, gradually melts into the orange, so that there is no definite line indicating where the red ends and the orange begins. In the same manner, the orange, attaining its greatest intensity near the middle of the space, gradually melts into the yellow; and in the same manner, each of the succeeding colours, having their greatest intensities near the middle of the spaces, melts towards its extremities into the adjacent colours.

The proportion of the whole length occupied by each space will depend upon the sort of glass of which the prism is composed. If it be flint-glass, and the entire length M N be supposed to consist of 360 equal parts, the following will be the length of each succeeding colour, commencing from the red upwards.

Red	56
Orange	27
Yellow	27
Green	46
Blue	48
Indigo	47
Violet	109
	360

It appears, therefore, that the ray of light P Q, after passing through the prism, is not only deflected from its original course P Q P′, but it is resolved into an infinite number of separate rays of light which diverge in a fanlike form, the extreme rays being L K and L K′, the former being directed to the lowest point of the coloured space upon the screen, and the latter to the highest point. The coloured space thus formed upon the screen is called the *prismatic spectrum.*

1054. *Composition of solar light.* — From this experiment the following consequences are inferred:—

1. Solar light is a compound principle, composed of several parts differing from each other in their properties.

2. The several parts composing solar light differ from each other in refrangibility, those rays which are directed to the lowest part of the spectrum being the least refrangible, and those directed to the highest part being the most refrangible; the rays directed to the intermediate parts having intermediate degrees of refrangibility.

3. Rays which are differently refrangible are also differently coloured.

4. The least refrangible rays composing solar light are the red rays, which compose the lowest division R of the spectrum. But these

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red rays are not all equally refrangible, nor are they precisely of the same colour. The most refrangible red rays are those which are deflected to the lowest point of the red space R, and the least refrangible are those which are directed to the point where the red melts into the orange. Between these there are an infinite number of red rays having intermediate degrees of refrangibility. The colour of the red rays varies with their refrangibility, the most intense red being that of rays whose refrangibility is intermediate between those of the extreme rays of the red space.

The same observations will be applicable to rays of all the other colours.

5. Each of these components of solar light having a different refrangibility will have for each transparent substance a different index of refraction. Thus the index of refraction of the red rays will be less than the index of refraction of the orange rays, and these latter will be less than the index of refraction of the yellow rays, and so on, the index of refraction of violet rays being greater than for any other colours.

But the rays of each colour being themselves differently refrangible, according as they fall on different parts of the coloured space, they will, strictly speaking, have different indices of refraction. The index of refraction, therefore, of any particular colour must be understood as expressing the index of refraction of the middle or mean ray of that particular colour. Thus, the index of refraction of the red rays will be the index of refraction of the middle ray of the red space; the index of refraction of the orange rays will be the index of refraction of the middle ray of the orange space; and so on.

It must not, however, be supposed that a pencil of solar light consists of separate and distinct rays of the different colours which form the spectrum, so that it might be possible by any mechanical division of such a pencil to resolve it into such rays. Each individual ray of such a pencil is composed of all the rays of the spectrum, just as the gases oxygen and hydrogen, which are the chemical constituents of water, enter into the composition of each particle of that liquid, no matter how minute it be.

1055. *Experiments which confirm the preceding analysis of light.* — The validity of the preceding analysis of light is confirmed by the following observations and experiments.

If the several rays composing a spectrum be allowed to pass separately through a small hole in a screen, and be received by another prism similar to A B C placed behind the screen, with the same angle of incidence as that with which P Q is incident upon A B, each ray will be refracted by the second prism, and its angle of deflection will be found to be exactly equal to the angle of deflection produced by the first prism A B C upon it. In this refraction of the second prism, the ray will not be dilated as the original ray of solar light was by the first prism, and no second spectrum will be formed; the ray will be merely turned from its direction by the refraction of the prism, but will undergo no other change.

Let a band of white paper, A L, *fig.* 846., be divided into seven spaces, and let those spaces be coloured red, orange, yellow, green, light blue, indigo, and violet, severally, each colour being of uniform tint, and as closely resembling the seven colours of the spectrum as possible. Let this band be placed upon a black ground, and viewed through a prism whose refracting angle is presented upwards, with its edge horizontal and parallel to the band A L. The images of the several coloured spaces seen through the prism will be in positions more or less elevated above A L, according to the greater or less refrangibility of the different colored lights. The image of the red space R will be seen at R′, that of the orange space O at O′, that of the yellow space Y at Y′, and so on. The image O′ will be a little above R′, the image Y′ a little above O′, and so on, as represented in the figure, the image of the violet space V′ being in the highest position. This phenomenon is obviously the result of the relative refrangibilities of the different colours deposited on the spaces of the paper band

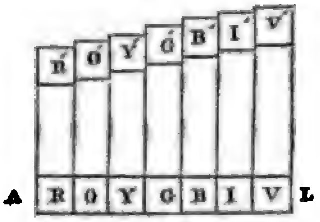


Fig 346.

Instead of artificial colours, let the spectrum itself be thrown on a sheet of white paper in a horizontal position A L, *fig.* 347., which may be done by placing the prism which produces the spectrum with the edge of its refracting angle vertical. Let this spectrum A L be viewed through a prism having the edge of its refracting angle horizontal, and presented upwards. The image of the spectrum seen through the prism will have the position A′ L′, oblique to A L, the violet end being more raised than the red end. The coloured space of the image will not form, as in *fig.* 346., a series of ascending steps, but will be included in one uniform line. This is explained by the fact already stated, that the light composing each of the coloured spaces R, O, Y, &c. of the spectrum is not uniformly refrangible.

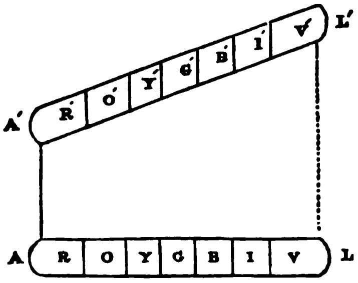


Fig. 347.

The rays which illuminate the red space R increase gradually in refrangibility from the extremity A to the boundary of the orange space; and in like manner, the rays which illuminate the orange space O increase gradually in refrangibility to the boundary of the yellow space; and so on.

Hence it is that the boundary of the image of the spectrum is a line uniformly inclined to A L. The divisions of the coloured spaces in the image correspond, however, with those of the spectrum, each colour in the image being vertically above the corresponding colour in the spectrum.

1056. *Experimental proof by recomposition.* — As the solar light is resolved by the prism into the various coloured lights exhibited in the spectrum, it might be expected that, these coloured lights being mixed together in the proportion in which they are found in the spectrum, white light would be reproduced. This is accordingly found to be the case. If the spectrum formed by the prism A B C, *fig.* 348., instead of being thrown upon a screen, be received upon a concave reflector M N, the rays which diverged from the prism and formed the spectrum will be reflected converging to the focus F, and after intersecting each other at that point, they will again diverge, the ray R F passing in the direction F R', and V F in the direction F V'.

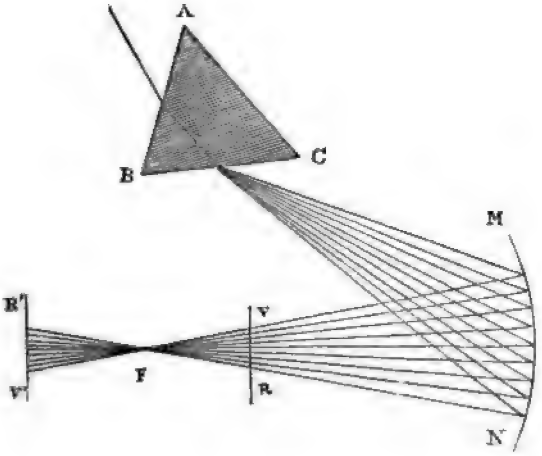


Fig. 348.

Now, if a screen be held between F and the reflector, the spectrum will be seen upon the screen. If the screen be then moved from the reflector towards the focus F, the spectrum upon the screen will gradually diminish in length, the extreme colours R and V approaching each other. When it comes so near to F that the extreme limits of the red and violet touch each other, the central point of the spectrum will become white; and when the screen arrives at the point F, the coloured rays being all mingled together, the spectrum will be reduced to a white colourless spot.

Just before the screen arrives at F, it will present the appearance of a white spot, fringed at the top with the colours forming the upper end of the spectrum, violet, blue, and green, and at the bottom with those forming the lower end of the spectrum, red, orange and yellow. his effect is explained by the fact, that until the screen is brought the focus F, the extreme rays at the other end of the spectrum are ot combined with the other colours.

If the screen be removed beyond F, the same succession of appearances will be produced upon it as were exhibited in its approach to F, but the colours will be shown in a reversed position.

As the screen leaves F, the white spot upon it is fringed as before, ut the upper fringe is composed of red, orange, and yellow, while he lower is composed of violet, blue, and green; and when the screen is removed so far from the focus F as to prevent the superposition of the colours, the spectrum will be produced upon it, with the red at the top, and the violet at the bottom, the position being inverted with respect to that which the screen exhibited at the other side of the focus. These circumstances are all explained by the fact that the rays converging to F intersect each other there.

Similar effects may be produced by receiving the spectrum upon a double convex lens, as represented in *fig.* 349. The rays are made as before to converge to a focus F, where a white spot would be produced upon the screen. Before the screen arrives at F, and after it passes it, the same effects will be produced as with the concave reflector.

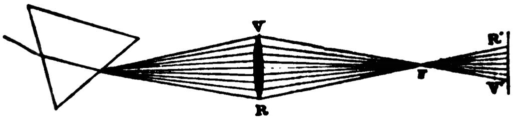


Fig. 349.

The proposition, that the combination of colours exhibited in the prismatic spectrum produces whiteness, may be further verified by the following experiment: —

Let a circular card be framed with a blackened circle, and its centre surrounded by a white circular band, and a black external border, as represented in *fig.* 350.

Let the white circular band be divided into seven spaces proportional in magnitude to the spaces occupied by the seven colours in the prismatic spectrum, these spaces being R, O, Y, G, B, I, and V. Let these spaces be respectively coloured with artificial colours resembling

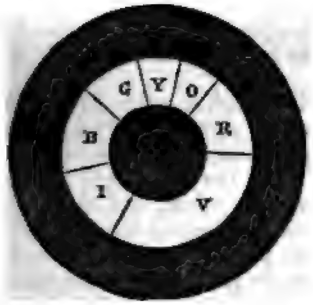


Fig. 350.

as near as practicable in their tints the colours of the spectrum. If the centre of this card be placed upon a spindle, and a very rapid motion of rotation be imparted to it, the ring on which the seven colours are painted will present the appearance of a greyish white. In this case, if all the colours except one were covered with black, the revolving card would present the appearance of a continuous ring of that colour; and, consequently, if all the coloured spaces be uncovered, seven continuous rings of the several colours would be produced; but these rings being superposed and mingled together will produce the same effect on the sight as if all the seven colours were mixed together in the proportion which they occupy on the card. If the colours were as intense and as pure as they are in the spectrum, the revolving card would exhibit a perfectly white ring; but as the colours of natural bodies are never perfectly pure, the colour produced in this case is greyish.

This experiment may be further varied by having uncovered any two, three, or more combinations of the colours depicted on the card. In such case the rotation of the card produces the appearance of a ring of that colour which would result from the mixture of the colours left uncovered: thus, if the red and yellow spaces remain uncovered, the card will produce the appearance of an orange ring; if the yellow and blue remain uncovered, it will produce the appearance of a green ring; and so on.

1057. *Lights of the same colour may have different refrangibilities.* — Although the phenomena attending the prismatic spectrum prove that rays of light which differ in refrangibility also differ in colour, the converse of this proposition must not be inferred; for it is easy to show that two lights which are of precisely the same colour, may suffer very different effects when transmitted through a prism.

Let us suppose two holes made in the screen on which the spectrum is thrown in the middle of the space occupied by the blue and yellow colours, so that rays of these colours may be transmitted through the holes. Let these rays be received upon a double convex lens, and brought to a focus at G,′ *fig.* 351., upon a sheet of white paper, so as to illuminate the spot G′. The colour that it produces then will be a green. Let another spectrum be now thrown by a prism upon the screen, and let a hole be made in the screen at that part of the green space where the tint is precisely similar to the colour produced at G′ on the white paper, and let the light which passes through this hole fall upon the spot G beyond G′.

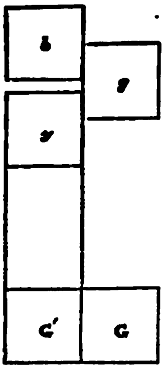


Fig. 351.

The spaces G and G' will then be illuminated by lights of precisely the same colour; but it will be easy to show that these lights are not similarly refrangible.

Let them be viewed through a prism having its refracting angle presented upwards. The image of the illuminated space G will be seen in a more elevated position at *g*; but two images will be produced of the space G', one yellow, and the other blue, at *y* and *b*, the yellow image *y* being a little below *g*, and the blue image *b* a little above it. Thus it is evident that the green light on the space G' is a compound of yellow and blue, and is separable into its constituents by refraction, while the similar green light on the space G is incapable of decomposition by refraction.

1058. *Colours produced by combining different rays of the spectrum.* — An endless variety of tints may be produced by combining in various ways the colours composing the prismatic spectrum; indeed, there is no colour whatever which may not be produced by some combination of these tints. Thus, all the shades of red may be produced by combining some proportion of the yellow and orange with the prismatic red; all the shades of orange may be produced by combining more or less of the red and yellow with each other and with the orange; all the shades of yellow may be produced by varying the proportion of green, yellow, and orange; and so on.

1059. *Complementary colours.* — If two tints T and T' be produced, the former T by combining a certain number of prismatic colours, and the latter T' by combining the remainder together, these two tints T and T' are called *complementary*, because each of these contains just those colours which the other wants to produce complete whiteness; and, consequently, if the two be mixed together, whiteness will be the result. Thus, a colour produced by the combination of the red, orange, yellow, and green of the spectrum in their just proportions, will be complementary to another colour produced by the blue, indigo, and violet in their just proportions, and these two colours, if mixed together, would produce whiteness.

1060. *Colours of natural bodies generally compound.* — Almost all colours, natural or artificial, except those of the prismatic spectrum itself, are more or less compounded, and their combined character belongs to them equally when they have tints identical with the coloured spaces of the spectrum. Thus, a natural object whose colour is indistinguishable from the yellow space of the spectrum, will be found, when subjected to the action of the prism, to refract light in which there is more or less of green or orange; and an object which appears blue will be found to have in its colour more or less of green and violet.

1061. ***Method of observing the spectrum by direct vision.***—Instead of receiving the spectrum on a screen, it may be viewed directly by placing the eye behind the prism A B C, *fig.* 352., at L, so as to receive the light as it emerges. This mode of observing the prismatic

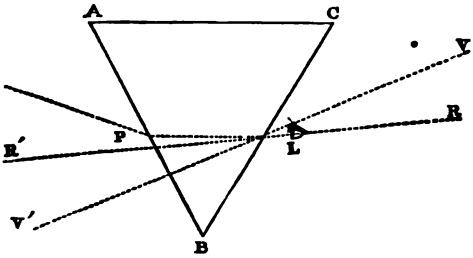


Fig. 352.

effects is in many cases more convenient than by means of the screen, colours being thus rendered observable which would be too feeble to be visible after reflection from the surface of the screen. It is necessary, however, to consider that in this manner of viewing the prismatic phenomena, the colours will be seen in an order the reverse of that which they would hold on the screen; for if the eye be placed at L, it will receive the violet ray which enters in the direction L V as if such ray had proceeded from V', and it will receive the red ray which enters it in the direction R as if it had proceeded from R'; the red will therefore appear at the top, and the violet at the bottom of the spectrum, when the refracting angle B of the prism is turned downwards.

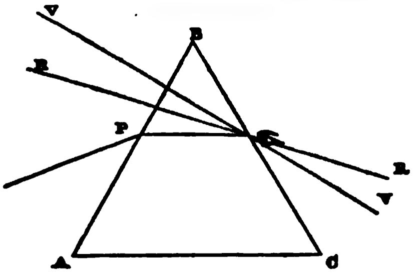
But if the refracting angle B be turned upwards, as represented in *fig.* 353., then the red will appear at the bottom, and the violet at the top of the spectrum, as will be perceived from the figure.

1062. ***Why objects seen through prisms are fringed with colours.***—In general, when objects are viewed through a prism they appear with their proper colours, except at their boundaries, where they are fringed with the prismatic tints in directions parallel to the edge of the reflecting angle of the prism.

Let A A M M, *fig.* 354., be a small rectangular object seen upon a black ground, the sides A M being vertical, and A A and M M horizontal. Let us first suppose that this object has the colour of a pure homogeneous red. If this object be viewed through a prism whose refracting angle is directed upwards with its edge horizontal, it will be seen in a more elevated position, such as *a a m m*, as already explained.

Let us next suppose that the object A A M M has the colour of a pure homogeneous orange. When viewed through the prism it will, as already explained, appear in a position *b b n n*, a little above *a a m m*.

If we next suppose the object A A M M to be coloured with homo-



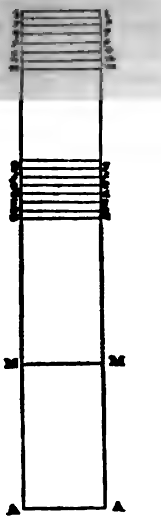


Fig. 353. Fig. 354.

geneous yellow, it will be raised by the prism to *c c o o*, a little above the orange image.

If it be next supposed to have the colour of a prismatic green, it will be seen at *d d p p*, a little above the yellow image; and if it be coloured light blue, its image will be seen at *e e q q*, above the green image; if it be dark blue or indigo, its image will be in the position *f f r r*; if it be violet, its image will be in the position *g g s s*.

Now, if we suppose the object A A M M to be white, that is to say, to have a colour which combines all the prismatic colours together, then all these several images will be seen at once through the prism in the respective positions already described. They will therefore be more or less superposed one upon the other, and the image will exhibit in its different parts those tints which correspond to the mixture of the colours thus superposed.

Hence it appears that the space between *a a* and *b b* from which all colour except the red is excluded, will appear red; in the space between *b b* and *c c*, in which the orange image is superposed upon the red image, a colour will be exhibited corresponding to the mixture of these two colours; in the space between *c c* and *d d*, the three images red, orange, and yellow are superposed, and a colour corresponding to the combination of these will be produced. In fine, the colours which are superposed between every successive division of the upper and lower edges of the combined images are as follows, where the prismatic colours are designated by the capital letters, and their mixture or superposition by the sign +:—

Between *a a* and *b b*	R
" *b b* " *c c*	R + O
" *c c* " *d d*	R + O + Y
" *d d* " *e e*	R + O + Y + G
" *e e* " *f f*	R + O + Y + G + B
" *f f* " *g g*	R + O + Y + G + B + I
" *g g* " *m m*	R + O + Y + G + B + I + V = W.

Thus it appears that the spaces *g g* of the violet image and the top *m m* of the red image are coloured with a white light, because in this space all the seven images are superposed.

In the space between *g g*, the bottom of the violet image, and *f f*, the bottom of the dark blue image, there is a space which is illuminated by all the prismatic colours except the violet, and this space consequently approaches so near a white as to be scarcely distinguishable from it. The space between *f f*, the bottom of the dark blue image, and *e e*, the bottom of the light blue image, is illuminated by all the colours except the dark blue and indigo, and it consequently has a yellowish tint. The succeeding divisions downwards towards *a a* become more and more red until they attain the pure prismatic red of the lowest division. The colours of the upper extremity of the image may in like manner be shown to be as follows:—

Between *s s* and *r r*	V
" *r r* " *q q*	V + I
" *q q* " *p p*	V + I + B
" *p p* " *o o*	V + I + B + G
" *o o* " *n n*	V + I + B + G + Y
" *n n* " *m m*	V + I + B + G + Y + O
" *m m* " *g g*	V + I + B + G + Y + O + R = W.

Thus it appears that the highest fringe at the upper edge is violet, that those which succeed it are formed by the mixture of violet and blue, to which green and yellow are successively added, until the colours become so completely combined that the fringe is scarcely distinguishable from a pure white. It is evident, therefore, that at the lower extremity the reds, and at the upper the blues, prevail.

If the object A A M M viewed through the prism be not white, then the preceding conclusions must be modified according to the analysis of its colour. Thus, if its colour be a green, it may be either a pure homogeneous green, or one formed by the combination of blue and yellow or other prismatic tints. In the former case, the prism will exhibit the object without fringes, but in the latter it will be fringed according to the composition of its colour, determined by the same principles as those which have been applied to the object A A M M.

1063. *Law of refraction applied to compound solar light.* — The analysis of light, which has been here explained and illustrated, will enable us to generalize and extend the law of refraction explained in 979.

Let A M B, *fig.* 355., be a transparent medium having a semi-cylindrical form, C being its centre. Let I C be a ray of solar light incident at C, the angle of incidence being I C O. This ray, on entering the transparent medium, will, according to what has been already explained, be resolved into an infinite number of other rays differently refracted, that which is least refracted being C R, and that which is most refracted being C V. The ray C R is red, and the ray C V is violet; the rays of intermediate colours and intermediate refrangibilities being included between them. The angle R C M is the angle of refraction of the extreme red ray corresponding to the angle of incidence I C O, and the angle V C M is the angle of refraction of the extreme violet ray corresponding to the same angle of incidence.

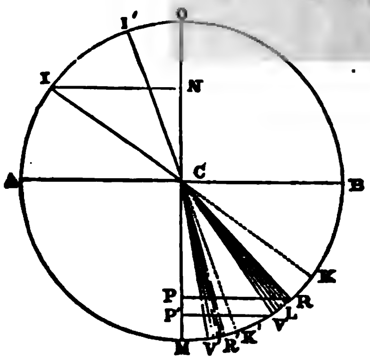


Fig. 355.

The index of refraction of the former will be found by dividing I N by R P; and the index of refraction of the latter will be found by dividing I N by V P'.

It is evident that the indices of refraction for the intermediate rays will be included between these two, being greater than the index of the extreme red, and less than the index of the extreme violet.

If the angle of incidence I C O be diminished, the angles of refraction R C M and V C M will be both diminished, since their sines will still bear the same ratio to the sine of the angle of incidence. Thus, if I' C, *fig.* 355., be the incident ray, and I' C O the angle of incidence, then C R' will be the extreme red, and C V' the extreme violet refracted rays, and the intermediate rays, into which the incident ray is resolved, will lie between these as before.

In this case, the angle R' C V' which measures the divergence of the extreme rays into which the incident ray is resolved, will be less than the angle R C V, which measures their divergence with the greater angle of incidence I C O. Thus it appears that the divergence of the decomposed rays is diminished as the angle of incidence is diminished, and increased as the angle of incidence is increased; but with the same angle of incidence this divergence is always the same in the same transparent medium.

The angle K C R, formed by the direction of any ray, such as C R, with the direction C K, which it would have followed had it not been *refracted*, is called the *refraction* of that ray.

592.

Now it is necessary to distinguish carefully this term from the *angle of refraction* already defined.

Thus it appears that the refraction of the different rays into which the ray C I is resolved is different; that of the extreme red being K C R, and that of the extreme violet being K C V.

1064. ***Dispersion of light.*** — The difference between the refraction of these extreme rays, or the angle of divergence R C V of the rays into which the original solar ray I C has been resolved by refraction, is called the *dispersion* produced upon the solar ray I C by the process of refraction.

It follows from what has been just explained, that this dispersion in the same medium diminishes and increases as the angle of incidence or the angle of refraction, or, in fine, as the refraction itself, diminishes or increases.

1065. ***Mean refraction.*** — But the term refraction, to have a definite meaning, in this case, must be applied to some one of the rays into which the solar ray is resolved, since each of these rays has a different refraction, varying from K C R to K C V. The middle ray, therefore, C L, of the rays diverging from C, is adopted for this purpose; and, accordingly, the ray C L is called the mean ray, and the angle K C L the mean refraction.

The refraction produced by any transparent medium upon a given ray at a given angle of incidence, is the measure of the refracting power of the medium on such ray; but as this refraction is always the difference between the angles of incidence and refraction, and as this difference may be taken to be proportional to the difference between their sines, we shall have the refractive power of the medium expressed thus:

$$\frac{\sin.\ \mathrm{I} - \sin.\ \mathrm{R}}{\sin.\ \mathrm{R}} = n - 1;$$

where n expresses the index of refraction.

The measure, therefore, of the refracting powers of different media, is the number found by subtracting 1 from their index of refraction.

It follows, from what has been explained, that in the same medium the dispersion increases and diminishes as the mean refraction increases or diminishes.

1066. ***Dispersive power.*** — When different media are compared together, it is found, that with the same mean refraction there will be different dispersions, — a fact which supplies a characteristic of different media, which has been called their *dispersive power;* one medium being said to have a greater or less dispersive power than another medium, according as the dispersion it produces with the same mean refraction is greater or less than that produced by the other medium.

The dispersion, therefore, produced by any medium being expressed by the difference of the indices of refraction n'' and n' of the extreme rays, and the refracting power being expressed by $n-1$, the absolute dispersive power is the quotient obtained by dividing the dispersion by the refracting power, and will be

$$\text{D} = \frac{n'' - n'}{n - 1}.$$

In the tables of refraction which have been given in page 65, the indices of refraction must be understood to refer to the mean ray of the spectrum, produced by the various media indicated in the tables.

To illustrate the application of this formula, let us take the case of crown-glass and diamond. The index of refraction of the extreme and mean rays of crown glass are as follows:—

$$n'' = 1{\cdot}5466,\ n' = 1{\cdot}5258,\ n = 1{\cdot}5830;$$

consequently we shall have for crown-glass,

$$\text{D}' = \frac{208}{5330} = 0{\cdot}0390.$$

In like manner, the indices for diamond are

$$n'' = 2{\cdot}4670,\ n' = 2{\cdot}4110,\ n = 2{\cdot}4390;$$

therefore, we shall have

$$\text{D}' = \frac{56}{1439} = 0{\cdot}0389.$$

From whence it appears that although the refracting powers of the diamond and crown-glass are as 3 to 1, their dispersive powers are the same.

This identity of their dispersive powers may be proved experimentally by taking two prisms, one of diamond and the other of crown glass, and producing with them two spectra in the manner represented in *fig*, 345., so that the mean ray L F of each shall be equally inclined to the direction P P′ of the incident ray. It will be found that the two spectra thus produced will have equal lengths, and consequently that the dispersions which correspond to equal refractions are equal.

Transparent media differ from each other, not only in the dispersive powers which they have on solar light, but also in the dispersive powers with which they act on the different elements which compose such light. Thus, for instance, it will happen that although two media, such as the diamond and crown glass, may have equal dispersive powers in relation to the compound light of day, they will have very different dispersive powers upon the several coloured lights of which each compound light is made up.

This may be rendered experimentally apparent by producing two spectra of equal lengths, with prisms of different materials.

If these two spectra be placed in juxtaposition, so that their extremities shall coincide, although their coloured spaces will succeed each other invariably in the order already described, yet the boundaries which separate these coloured spaces will not coincide. The red in the one will be more or less extensive than in the other, and the same will be true of the other colours.

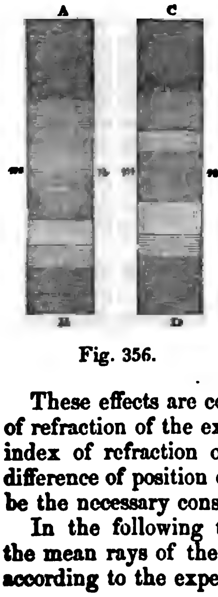


Fig. 356.

Let two spectra, A B and C D, *fig.* 356., be produced in this manner, equal in length, by two hollow prisms, one filled with the oil of cassia, and the other with sulphuric acid. In the spectrum A B, produced by the oil of cassia, the red, orange, and yellow spaces are less than in the spectrum C D, produced by the sulphuric acid, while in the latter the blue, indigo, and violet spaces are less than in the former.

The middle ray *m n* in the spectrum A B passes through the blue space, while it passes through the green space in the spectrum C D.

It appears, therefore, that in this case sulphuric acid has a greater dispersive power upon the less refrangible rays, and a less dispersive power on the more refrangible rays, than the oil of cassia.

These effects are consequences of the fact, that although the indices of refraction of the extreme rays for any two media may be equal, the index of refraction of the intermediate rays may be unequal, and a difference of position of the corresponding colours in the spectrum will be the necessary consequence.

In the following table, the indices of refraction corresponding to the mean rays of the seven coloured spaces of the spectrum are given according to the experiments of Frauenhofer.

1067. *Table of the indices of refraction of the mean rays of each of the prismatic colours for certain media.*

Refracting Substances.	n_1.	n_2.	n_3.	n_4.	n_5.	n_6.	n_7.
Flint glass, No. 13.	1·627749	1·629681	1·635036	1·642024	1·648260	1·660285	1·671062
Crown glass	1·525832	1·526849	1·529587	1·533005	1·536052	1·541657	1·546566
Water	1·330935	1·331712	1·333577	1·335851	1·337818	1·341293	1·314177
Water	1·330077	1·331709	1·333577	1·335849	1·337788	1·341261	1·344162
Potash	1·399629	1·400515	1·402805	1·405632	1·408082	1·412579	1·416368
Oil of turpentine	1·470496	1·471530	1·474434	1·478353	1·481736	1·488198	1·493874
Flint glass, No. 3.	1·602042	1·603800	1·608494	1·614532	1·620042	1·630772	1·640373
Flint glass, No. 30.	1·623570	1·625477	1·630585	1·637356	1·643466	1·655406	1·666072
Crown glass, No. 13.	1·524312	1·525299	1·527982	1·531372	1·534337	1·539908	1·544684
Crown glass, Litt. M.	1·554774	1·555933	1·559075	1·563150	1·566741	1·573535	1·579470
Flint glass, No. 23. and prism of 60°	1·626596	1·628469	1·633667	1·640495	1·646756	1·658848	1·669686
Flint glass, No. 23. and prism of 45°	1·626564	1·628451	1·633666	1·640544	1·646780	1·658849	1·669680

1068. *Dispersion of each component colour, how found.*—The dispersion proper to each successive colour will be found by taking the difference of the two adjacent indices, and the total dispersion produced by each medium by taking the difference between the extreme indices. Thus the total dispersion produced by each medium given in the above table will be as follows:

Flint glass, No. 13.	0·043313
Crown glass, No. 9.	0·020734
Water	0·013242
Water	0·013185
Potash	0·016739
Turpentine	0·023378
Flint glass, No. 3.	0·038331
Flint glass, No. 30.	0·042502
Crown glass, No. 13.	0·020372
Crown glass, Lett. M.	0·024696
Flint glass, No. 23., prism 60°	0·043090
Flint glass, No. 23., prism 45°	0·043116

1069. *Light uniformly refrangible not necessarily simple.*—In all that precedes, it has been assumed that the light composing each part of the prismatic spectrum is simple and homogeneous. This conclusion, deduced by Newton, and adopted generally by all physical investigators since his time, is based on the assumption, that light which, being refracted by transparent media, cannot be resolved into parts differently refrangible, is simple and homogeneous.

Sir David Brewster has, however, published the results of a series of observations, from which it would follow, that a pencil of light which does not consist of parts differently refrangible, may, nevertheless, be resolved into parts which have different colours; in other words, that the light of certain parts of the spectrum, such, for example, as orange and green, although simple so far as respects refraction, is compound so far as respects colour. Thus, the orange light may be resolved into two lights equally refrangible, but differing in colour, one being red and the other yellow; and the green light may in like manner be resolved into two equally refrangible, one being yellow and the other blue.

1070. *Sir D. Brewster's analysis of the spectrum.*—In a word, the observations and experiments of Sir David Brewster have led him to the conclusion that the prismatic spectrum consists in reality of three spectra of nearly equal length, each of uniform colour, superposed one upon another; and that the colours which the actual spectrum exhibits arise from the mixture of the uniform colours of these three spectra superposed. The colours of these three elementary spectra, according to Sir David Brewster, are red, yellow, and blue. He shows that by the combination of these three, not only all the colours exhibited in the prismatic spectrum may be reproduced, but that their combination also produces white light. He contends, therefore, that the white light of the sun consists not of seven, but of three constituent lights, red, yellow, and blue.

This conclusion is established by showing that there is another method by which light may be resolved into its components, besides the method of refraction by prisms. In passing through certain coloured media, it is admitted that a portion of the light incident is intercepted at the surface upon which it is incident and in its passage through the medium; a part only is transmitted.

Now, this property of colours is taken by Sir David Brewster as another method, independently of refraction, of decomposing colours. He assumes that such a medium resolves the light incident upon it into two parts: first, the part which it transmits; and secondly, the part which it intercepts. He concludes that these two parts are complementary, that is to say, that each contains what the other wants to make up white solar light; or, more generally, that the incident light, whatever be its nature, must be assumed to be a compound, consisting of the light transmitted and the light intercepted.

This being assumed, let a coloured medium, such as a plate of blue glass, be held between the eye and the spectrum. Certain colours of the spectrum will be transmitted and others intercepted. If the colours of the spectrum be simple and homogeneous light, such as they are assumed to be in the Newtonian theory of the decomposition of light, then the consequence would be that the appearance of the spectrum seen through the coloured medium would consist of dark and coloured spots; those simple lights intercepted by the glass appearing dark, and those transmitted by the glass having their proper colour. But if each colour of the prism be, as is assumed in the chromatic theory, simple, then the plate of glass can make no change in its colour by transmission.

It must therefore be wholly transmitted, partly transmitted, or wholly intercepted. If it be wholly transmitted, no change will be made, therefore, in its colour or intensity; if it be partly transmitted, its colour will remain the same; but its intensity will be diminished; if it be wholly intercepted, the space it occupied on the spectrum will be black. But these are not the effects, as Sir David Brewster states, which are observed. He finds, on the other hand, that the coloured spaces on the spectrum are not merely diminished in intensity, but actually changed in colour. Now, if any space of the spectrum be changed in colour, it follows from what has been stated, that the light transmitted must be a constituent of the colour of that space, to which the light intercepted being added, would reproduce the colour of the spectrum. By such an experiment as this, Sir David Brewster found that the parts of the spectrum occupied by the orange and green lights produced yellow, from which he inferred that the glass intercepted the red, which combined with the yellow produced orange, and the blue, which combined with the yellow produced green. But

If the glass have the power of thus intercepting the red and blue light, it might be expected that the red and the blue spaces of the spectrum would appear dark. He accordingly found that the light of the middle of the red space was almost entirely absorbed, as was also a considerable part of the blue space.

From experiments like these, which he made in great number, and under various conditions, Sir David Brewster deduced the conclusion to which we have adverted above.

He inferred that at every point of the spectrum, red, yellow, and blue light are combined in various proportions, the colour of each part being determined by the proportional intensities of these three colours in the mixture. In the red space, the proportions of blue and yellow are exactly those necessary to produce white light, but the red is in excess. A portion of it combined with the blue and yellow produces a white light, which is rendered red by the surplusage of red. In the same manner, in the yellow space the proportion of blue and red is that which is proper to white light, but there is a greater than the just proportion of yellow.

A part of this combining with the blue and red produces white light, which is rendered yellow by the surplus. In the same manner the blue space is shown to consist of a surplusage of blue, combined with the proportion of red and yellow, and the remainder in the proportion necessary for whiteness. The other colours of the spectrum, according to Sir David Brewster, are secondary, or the result of combinations of red, yellow and blue.

The manner in which these three primary colours produce the tints of the spectrum may be more clearly understood by reference to *fig.* [illegible], where M N represents the prismatic spectrum with its usual tints. The curve M R N represents the varying intensity of the red spectrum, M Y N that of the yellow, and M B N that of the blue spectrum. The distance of each part of these curves respectively from M N is understood to be proportional to the intensity of the [illegible] at that part, and the relative lengths of the perpendicular in- [illegible] represents the proportion of the intensities [illegible] colours. Thus at the point P, the three colours are [illegible] the lengths of the perpendiculars *p n*, *p' n*, [illegible] representing the proportion of yellow, the second red, [illegible] the red and yellow predominating, the colour at [illegible] orange.

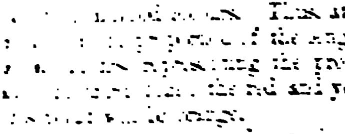


Fig. [illegible]

These observations and experiments, and the conclusions deduced from them by Sir David Brewster, have been now before the scientific world for more than twenty years. The experiments do not appear

[redacted]

to have been repeated, nor the chromatic doctrine inferred from them to have been yet generally assented to or adopted. The chromatic analysis of Newton is the only theory advanced by physical authors.

CHAP. XII.

SPECTRAL LINES—PHOTOMETRIC, THERMAL, AND CHEMICAL PROPERTIES OF THE SPECTRUM.

1071. *Number of spectral lines.*—If the prismatic spectrum produced under certain conditions be examined by the aid of a telescope, it will be found to be crossed throughout its entire length by dark lines of various breadths. The total number of these lines is nearly seven hundred, and they are distributed over the spectrum without any apparent relation to the limits of its coloured spaces.

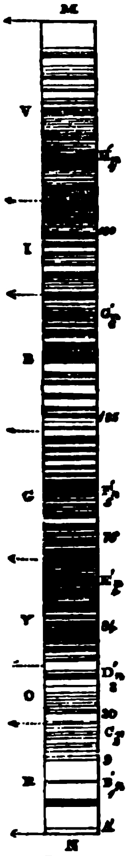


Fig. 358.

In *fig.* 358., M N represents a spectrum, M being its violet, and N its red extremity. The arrows to the left of the diagram represent the boundaries between the coloured spaces, these spaces being indicated by the letters R, O, Y, G, B, I, and V.

The general distribution of the spectral lines is exhibited in the diagram.

It will be observed, that in the distribution of these remarkable phenomena, there is no apparent regularity, either in their arrangement or in their intensity. In some places they are thickly crowded together, while in others they are separated by white spaces, more or less considerable. In some, the lines are extremely fine and scarcely visible; in others they are of distinct breadth

Among these numerous lines, seven were selected by their discoverer, Frauenhofer, as standards of reference or fixed points by which the position of the others could be designated. These seven are those marked on the right by the letters B', C', D', E', F', G', H'.

The first of these, B', is in the middle of the red space; the second, third, and fourth, C', D', and E', are nearer the boundaries which separate the red and orange, the orange and yellow, and the yellow and green; the fifth, F', is near the middle of the green space, and the seventh near the middle of the violet space; while the sixth is near the boundary which separates the blue and indigo.

The numbers which appear in the diagram between each pair of these lines indicate the number of spectral lines which have been ascertained to exist between them.

Thus, between B′ and C′ there are 9, between C′ and D′ 30, and so on; the entire number of lines between the first, B′, and the seventh, H′, being 574. The remainder of the spectral lines between the extreme red and B′, and between the extreme violet and H′, amount to about 100; but they are more difficult of observation, and have not been so precisely ascertained.

A little above the extreme red, there is a well-defined dark line A′; and about half way between that line and the line B′, there is a dark band composed of seven or eight lines.

It was ascertained by Frauenhofer, that these lines are altogether independent either of the magnitude of the refracting angle, or of the matter of the prism; and that their number, order, and intensity are absolutely invariable, no matter what prism be used, provided only the light come through, directly or indirectly, from the sun. Thus it is found that the spectra produced by moonlight and by the light of the planets give exactly the same lines.

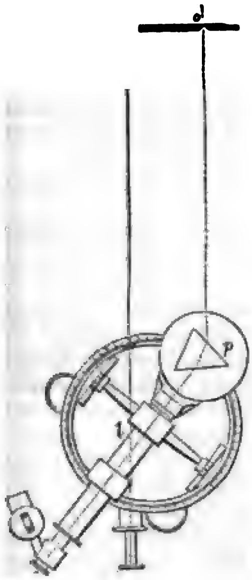


Fig. 359.

1072. *Manner of observing the spectral lines.* — The best method of observing these interesting phenomena is by means of telescopes and a prism, represented in *fig.* 359. Let a narrow slit be made in a window-shutter or a screen, so as to admit a broad thin beam of the sun's light. This slit is represented in section at right angles to its length at *o*. The beam of light is received on a prism of the finest and purest flint glass at *p*. After being refracted by the prism, it is received by a small telescope, which plays upon a graduated arc, on which is a second telescope to indicate the original direction of the ray *o p*. The angle under the two telescopes will indicate the refraction which the ray has suffered by the prism. The prisms used in these observations have been made of the purest and finest flint glass, perfectly free from threads and striæ. The prism ought to be placed at a distance of fifteen or twenty feet from the telescope. By turning, therefore, the telescope or the prism, the successive rays of the spectrum are made to pass through the telescope, so that the spectrum may be viewed successively from one extremity to the other. The telescopes suited to these observations should magnify from eight to ten times.

1073. *Spectral lines of artificial lights and of the moon, planets, and stars.* — By these means the spectra produced not only by solar light, but also by various artificial lights, as well as electric light, have been observed. The electric light gives the spectral lines bright instead of dark, one of the most remarkable for its brilliancy passing through the green space. The flame of a lamp, whether produced by gas, oil, or spirits, also gives the spectral lines bright. Two of these are especially distinguishable in the red and orange spaces.

The moon and planets have the same dark lines as the sun, but less easily distinguishable, especially near the extremities of the spectrum. The spectra produced by the light of the fixed stars are marked with dark lines, but little different in their number, intensity, and disposition from those exhibited in the solar spectrum. It is remarkable that the spectra produced by different fixed stars differ from each other.

1074. *Use of spectral lines as standards of refrangibility.* — The invariable position which Frauenhofer's lines are found to have in the solar spectrum has rendered them eminently useful for establishing standards of refrangibility of the component parts of solar light. From what has been stated respecting the gradual variation of the tints composing the solar spectrum, it may be easily understood that much uncertainty will attend any methods of defining a particular ray to which a certain index of refraction is imputed. Thus the middle of the red or the middle of the green space is necessarily an indefinite term, so long as the limits of these spaces admit of no exact definition.

The seven lines B′, C′, D′, &c., which have been already noticed, have been accordingly adopted as points invariable in their position, of which the indices of refraction once determined may always serve as standards of reference. The indices accordingly which have been given in table, p. 125., are those which belong to these points, n_1 being the index of refraction at B′, n_2 that of the rays at C′, n_3 at D′, and so on.

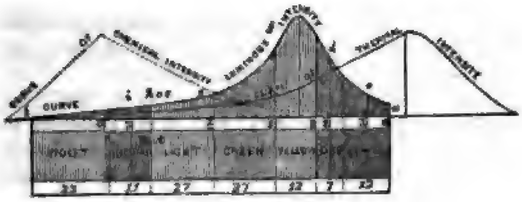


Fig. 360.

1075. ***Relative intensity of light in different parts of the spectrum.*** — Frauenhofer also ascertained by photometric observations the relative intensity of the light in different parts of the spectrum.

The result of these observations is denoted by the curve marked "Luminous intensity," in *fig.* 360.; the perpendicular distance of each point of this curve from the edge of the spectrum being proportional to the brilliancy of the light produced by a flint glass prism. It appears from this that the most intense illumination corresponds to a point about the middle of the yellow space.

In the following table are given the numerical intensities of the other points, the light of the point of greatest intensity being expressed by 1000.

At the red extremity	000
At B′	32
At C′	94
At D′	640
At E′	480
At F′	170
At G′	31
At H′	5·6
At violet extremity	000

1076. *Relative calorific intensity of the spectral rays.* — The heating power of the light composing the different parts of the spectrum was examined first by the late Sir William Herschell, and later by M. Berard, Sir H. Davy, MM. Seebeck, Wunsch, and, in fine, by M. Melloni, who has supplied a vast body of interesting experiments on this subject. The general result of these observations, the details of which would be inadmissible here, are as follows: —

The heating power, being nothing at the violet extremity, augments gradually as the thermometer is moved to the red extremity.

At this point, or near it, the heating power is a maximum; but the presence of thermal rays beyond the red extremity is manifested by the thermometer, which, though it declines on being moved beyond this extremity, continues to show a temperature greater than that of the surrounding air, to a considerable distance from the spectrum.

We are therefore compelled to admit the existence of invisible rays in the sun's light, which have the power of producing heat, and which have a less degree of refrangibility than red light.

The curve marked "Thermal intensity," in *fig.* 360., indicates the variation of the heating power of the rays of the spectrum in the same manner as the former curve represented the luminous intensity. The point of maximum thermal intensity is according to some at the red extremity, and according to others a little below it, but it is found that this depends in some degree upon the material composing the prism.

[redacted]

1077. *Relative chemical intensity of the spectral rays.* — The action of light in changing the colour of certain substances has long been known; but one of the most remarkable of this class of objects has lately acquired increased interest from its application in the art called Daguerreotype.

If the chemical substance called muriate of silver be exposed to solar light, it will be blackened. Now, in order to ascertain whether this effect is due collectively to all the rays composing solar light, or is caused by the action of some rather than other rays, it is only necessary to expose it successively to all the rays composing the prismatic spectrum.

If this be done, it will be found that the least refrangible rays near the red extremity do not produce this effect in any sensible degree, while the more refrangible rays at the violet extremity produce it in a very great degree; in a word, by ascertaining and indicating the intensity of this chemical action in the same manner as the intensities of the illuminating and heating power as already expressed, we shall be enabled to determine the curve of chemical intensity indicated in *fig.* 360., from which it appears that this action is at its maximum near the boundaries between the violet and the indigo.

CHAP. XIII.

CHROMATIC ABERRATION. — ACHROMATISM.

1078. *Chromatic aberration of lenses.* — It appears from what has been established in Chap. X., that the power of a lens whether it be convergent or divergent, and therefore also its focal length, depends not only on the curvature of its surface, but on the index of refraction of the substance composing it.

But, from what has been explained in the last chapter, it appears that the index of refraction for the same transparent medium is different for the different component elements of light. Thus, the index of refraction for flint glass, which corresponds to violet light, is greater than the index of refraction for red light, the former being more refrangible than the latter. The focal length, therefore, of a lens for red light, will be different from the focal length of the same lens for the violet light. This circumstance produces important consequences, which we shall now proceed to explain.

Let A B C, *fig.* 361., be a converging lens, which we will here suppose to be double convex. Its focal length F will, according to what has been explained in Chapter X., be

$$F = \frac{r \times r'}{(n-1)(r+r')},$$

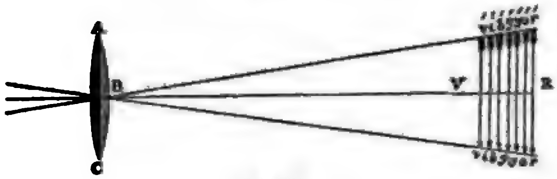


Fig. 361.

where r and r' express the length of the radii of the lens, and n the index of refraction. Now, since the index of refraction which corresponds to the extreme violet rays is greater than the index of refraction which corresponds to the extreme red rays, the value of F will be less for the former than for the latter; and, consequently, the focus of the extreme violet rays will be nearer the lens than the focus of the extreme red rays; and, in like manner, it follows, that the focus of the rays of intermediate refrangibilities will lie between these two points.

If V and R, therefore, be the foci of the extreme violet and extreme red rays respectively, the foci of all the rays of intermediate refrangibilities will be distributed between V and R.

Let us suppose any object which transmits the extreme violet light to be placed before the lens at such a distance that the pencil of rays proceeding from each point upon it to the lens may be considered as consisting of parallel rays; an inverted image of such object will be formed at $v\,v'$, at a distance B V from the lens determined by the preceding formula, n having in it the value which corresponds to the index of the extreme violet rays.

If, now, a similar and equal object be similarly placed before the lens, but emitting the extreme red light instead of the extreme violet light, an inverted image of this object will be formed at $r\,r'$, at a distance B R from the lens, determined in like manner by the above formula, in which the value assigned to n shall be the index of refraction corresponding to the extreme red light.

If, in like manner, the object placed before the lens be supposed to be successively illuminated by all the varying tints of the spectrum, a succession of inverted images corresponding in colour to these tints will be formed at $o\,o'$, $y\,y'$, $g\,g'$, $b\,b'$, and $i\,i'$, between R and V.

Now, if the object placed before the lens, instead of being successively illuminated by these various homogeneous lights, be illuminated with the white light of the sun, or if such object be the sun itself, then the various component parts of the light which it transmits will be brought by the lens to different foci corresponding to their various degrees of refrangibility, and the lens will accordingly produce, not one white image, but an infinite number of coloured images included between the extreme positions V and R. Each ray will form an image, having a position and colour corresponding to its degree of refrangibility, and the space included between V and R will be a truncated cone filled with images, which increase in magnitude from V to R, and which, beginning with a violet colour at V, pass through all the tints of the spectrum; the last image at R having a red colour corresponding to the red of the extreme light of the spectrum.

A white screen held at R would exhibit a well-defined red image of the object, if it did not also receive upon it the pencils of rays forming all the other images between R and V, such pencils diverging from the various points of such images. Thus, a pencil which is brought to an exact focus upon the image *o o'*, would form upon a a screen placed at *r r'*, not a point, but a small spot of orange light. In like manner, a pencil whose focus lies upon the image *y y'* would form upon a screen placed at R a small spot of yellow light, greater in magnitude than the spot of orange light, because of the greater distance of its focus from the screen. In like manner, the points upon the image *g g'*, *b b'*, *i i'*, and *v v'*, would produce upon the screen at *r* luminous spots of green, blue, indigo, and violet light, increasing in magnitude in proportion to their respective distances from the screen.

The image, therefore, formed upon the screen, arising from this combination of pencils of variously coloured lights, will exhibit a confused representation of the object; the colours diffused over the internal parts of its area being those which combined together form white light, the general area of the image will not be coloured; but the coloured pencils thus mingled together, being none of them brought to their foci on the screen, except those of the extreme red light, a confusion will ensue. At the edges there will be coloured fringes, because at the edges the pencils diverging from the edges of the series of images do not overlay each other as they do at the central pencils; and, consequently, the colours necessary for the production of white light are not mingled in these pencils.

The consequence of all this is, that there will be formed upon the screen an image of the object, everywhere indistinct, and fringed with prismatic colours at its edges.

The degree of indistinctness and the breadth of the fringes will depend upon the length of the space V R; that is to say, upon the *dispersion* produced by the lens, and also upon the difference between the magnitudes of the extreme images *r r'* and *v v'*, which latter depends upon the opening of the lens *r* B *r'*, and on the dispersion V R conjointly.

The consequence of this is, the indistinctness of the image and the coloured fringes arising from this cause increase as the focal length of the lens diminishes, as its opening increases, and as the dispersive power of the material of which it is composed increases.

These effects are called the *chromatic aberration* of lenses.

1079. *Aberration of a diverging lens.*—We have assumed in the preceding examples that the lens is a converging lens; and, consequently, that the image of a distant object is real, and may be exhibited on a screen.

If, however, the lens be a diverging lens, the effects of aberration will be the same, but the image being imaginary cannot be exhibited in the same manner. A diverging lens A B C is represented in *fig.* 362.

Let the object, as before, be placed at such a distance from it that the pencils proceeding from it may be considered as parallel. After passing through the lens they will diverge, as if they had proceeded from an object placed at a distance before the lens, equal to its focal length. Thus, if the object emit red light, the rays after passing through the lens will diverge as if they had proceeded from *r r′* at the distance B R, equal to the principal focal length corresponding to

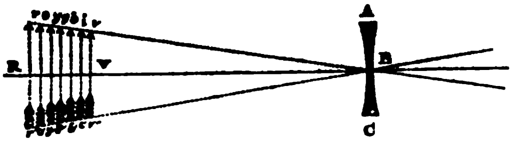


Fig. 362.

the index of refraction of red rays, and in like manner, if the object transmit violet rays, the light, after passing through the lens will diverge as if it had proceeded from points in an object placed at *v v′*, and for the intermediate colours it would diverge as if it had proceeded from intermediate points between R and V.

Thus, if, as before, the object be supposed to emit white solar light, the rays after passing the lens would diverge from points between R and V, varying according to their refrangibilities in the manner already expressed.

1080. *Images formed by single lenses must always be coloured.*—It appears, therefore, from what has been here explained, that no single lens can produce a distinct image of an object free from coloured fringes, since to accomplish this it would be necessary that each lens should possess the same power of convergence over all the component rays proceeding from all points of such object.

But since the converging power of the lens depends upon the index of refraction of the light, and since the index of refraction varies with the colour and refrangibility of the light, it follows that unless the object transmit light of a single refrangibility, that is to say, homogeneous light, the lens cannot cause the pencils which proceed from it to converge to the same focus, and, consequently cannot produce a distinct image. This object, however, which cannot be accomplished by a single lens, may be attained by a combination of lenses

[redacted]

composed of transparent substances, which differ from each other in their dispersive powers.

1081. *Conditions under which combined lenses may be rendered achromatic.*—To put the question first under its most simple form, let it be required to find what form must be given to two lenses composed of media having different refracting powers, so as to render the focal length of the compound lens for light of any one refrangibility, equal to its focal length for light of any other refrangibility.

Let F' and F'' be the focal lengths of the two lenses for light, of which the indices of refraction are n' and n'' for the media composing the lenses respectively.

Let f' and f'' be their focal lengths for light of which the indices of refraction are m' and m''.

Let F be the focal length of the compound lens.

The converging power of the compound lens on each kind of light will be equal to the sum of the converging powers of the two lenses separately on the same kind of light. The converging power of the compound lens, therefore, on the light whose indices of refraction are n' and n'', will be

$$\frac{1}{F'} + \frac{1}{F''};$$

and in like manner its converging powers on the light whose indices of refraction are m' and m'', is

$$\frac{1}{f'} + \frac{1}{f''}.$$

But since, by the supposition, these two converging powers must be rendered equal, we shall have

$$\frac{1}{F'} + \frac{1}{F''} = \frac{1}{f'} + \frac{1}{f''}.$$

The question is, then, to assign such magnitudes to the radii of the surfaces of the lenses as will make them fulfil this condition.

Let R_1 and R_2 be the radii of the surfaces of the first, and r_1 and r_2 those of the surfaces of the second lens. We shall then have, by the formulæ given in 1031. and 1032.,

$$\frac{1}{F'} = \frac{(n'-1)(R_1 - R_2)}{R_1 \times R_2}, \quad \frac{1}{F''} = \frac{(n''-1)(r_1 - r_2)}{r_1 \times r_2};$$

$$\frac{1}{f'} = \frac{(m'-1)(R_1 - R_2)}{R_1 \times R_2}, \quad \frac{1}{f''} = \frac{(m''-1)(r_1 - r_2)}{r_1 \times r_2}.$$

But since

$$\frac{1}{F'} + \frac{1}{F''} = \frac{1}{f'} + \frac{1}{f''},$$

[redacted]

we shall have

$$\frac{1}{\mathrm{F}'} - \frac{1}{f'} = \frac{1}{f''} - \frac{1}{\mathrm{F}''};$$

therefore

$$\frac{(n' - m')\,(\mathrm{R}_1 - \mathrm{R}_2)}{\mathrm{R}_1 \times \mathrm{R}_2} = -\frac{(n'' - m'')\,(r_1 - r_2)}{r_1 \times r_2};$$

and consequently

$$\frac{n' - m'}{n'' - m''} = -\frac{(r_1 - r_2) \times \mathrm{R}_1 \times \mathrm{R}_2}{(\mathrm{R}_1 - \mathrm{R}_2) \times r_1 \times r_2}.$$

The numbers expressed by $n' - m'$ and $n'' - m''$ are the differences between the indices of the two lights having different refrangibilities, which are supposed to be transmitted through the lenses. These are the dispersive powers of the media composing the lenses for each of the two lights. If, then, the radii of the two lenses be so selected as to render the fraction expressed by the second member of the preceding equation equal to the ratio of the dispersive powers of the material of the lenses for the two sorts of light, they will be brought to the same focus by the compound lens.

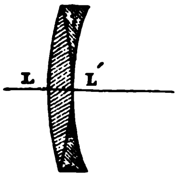


Fig. 363.

To simplify this, let us divest the preceding formula of its generality, and suppose that the first is a double convex lens L, *fig.* 363., with equal radii, and that the second is a double concave lens L', the surface of which, in contact with the first, has the same curvature with it, and consequently the same radius. Observing that when the convexities are turned in contrary directions, the radii have contrary signs, the preceding formulæ will now be reduced to

$$\frac{n' - m'}{n'' - m''} = \frac{r_2 - \mathrm{P}}{2\, r_2}.$$

Now it is always possible so to select the radii as to fulfil this condition; and therefore a compound lens, composed of two lenses of different refracting media, can always be constructed which will bring to the same focus two lights of different refrangibilities.

Let us suppose that the double convex lens is composed of crown glass, for which

$$n' = 1{\cdot}546566,\ m' = 1{\cdot}525832,$$

and the double concave of flint glass, for which

$$n'' = 1{\cdot}671062\ \ m'' = 1{\cdot}627749\cdot$$ 608

[redacted]

we shall therefore have

$$\frac{n'-m'}{n''-m''}=\frac{20734}{43313}=\frac{r_2-\mathrm{R}_1}{2\,r_2};$$

from which we find that

$$r_2 = 23{\cdot}47 \times \mathrm{R}_1.$$

The radius of the second surface of the double concave lens must in this case, therefore, be 23½ times the radius of the double convex.

It is easy to show that if the two lenses were composed of glass having equal dispersions, the result would not supply a solution of the problem; for in that case we should have the radius of the second surface of the double concave lens equal to the radius of the double convex, and consequently the refraction of the two lenses would neutralize each other, and parallel rays would emerge parallel. It is therefore essential to the solution of the problem that the two lenses should be composed of glass or other transparent media having different dispersive powers.

If the dispersive powers of the two lenses for every part of the light composing the spectrum were in the same ratio, which would be the case if the colours filled proportional spaces in the two spectra, the lenses, then, which would bring two coloured rays to the same focus would bring all the colours to that focus, and they would be absolutely achromatic. But it has been already explained that different transparent media not only produce spectra of different lengths, but divide them into coloured spaces in different proportions. It follows, therefore, that although the radii of the lenses be in the necessary proportion to their dispersive powers over lights of two particular colours, they will not be in the proportion necessary to bring the lights of other colours to the same focus. In this case, nevertheless, by bringing together the extreme images *r r'* and *v v'*, the longitudinal chromatic aberration R V is considerably diminished; so much so, that in most cases the indistinctness of the image and the coloured fringes are not perceptible with a triple lens, so adapted as to achromatize rays of three refrangibilities, such as the extreme and mean rays of the spectrum: there is thus an annihilation of chromatic aberration for all practical purposes, so that achromatism may be conceived to be realized.

CHAP. XIV.

THE EYE.

1082. *Sense of sight an extensive source of knowledge.*—Among the organs of sense there is none from which we derive so great a share of knowledge of external nature as the eye. Although, strictly speaking, this organ is cognisant only of light and colours, yet from the effects of them we are enabled by habit and reflection to infer with great promptitude and precision the forms, magnitudes, motions, distances, and positions, not only of the objects around us, but of the great bodies of the universe. Indeed, it is to the information derived from the eye alone that we are indebted for all the knowledge we possess of the material universe beyond the immediate precincts of the world we inhabit.

1083. *Knowledge of the structure of the eye necessary to comprehend optical instruments.* — The eye, therefore, is a subject of interesting inquiry, were it only for the importance of the information it conveys to us; but it is also necessary to understand its structure and functions before we can comprehend the use and application of those optical instruments which have been adapted with such marvellous success to enlarge the range of vision.

It is necessary first to investigate the powers of the organ of sight, and to determine the conditions which limit these powers, before we can appreciate the instruments by which these limits are extended.

1084. *Structure of the eye.* — The eyes, as they exist in the human species, have the form, as is well known, of two spheres, each about an inch in diameter, which are surrounded and protected by strong bony sockets placed on each side of the upper part of the nose. The external coating of these spheres is lubricated by a fluid secreted

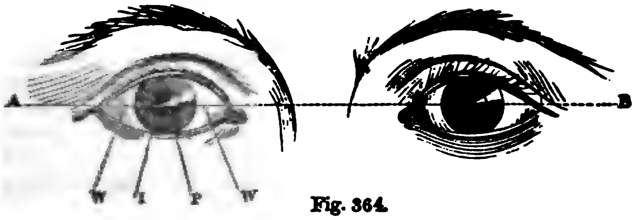


Fig. 364.

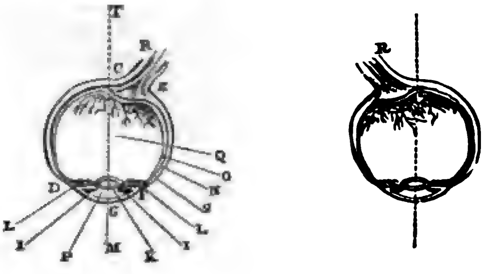


Fig. 365.

in adjacent glands, and spread upon them from time to time by the action of the eyelids in winking.

The eye-balls are moved by muscles connected with them within the socket which move them upon the principle known in mechanics as the ball and socket joint.

A front view of the eyes and surrounding parts is represented in *fig*. 364.; and a section of them made by a horizontal plane through the line A B, which passes through the centre of the point of the eye-balls, is represented in *fig*. 365.

1085. *The sclerotica and cornea.* — The external coating C D F E consists of a strong and tough membrane, called the *sclerotica*, or sclerotic coat. A part of this membrane is visible when the eye-lids are open at W, *fig*. 364., and is called the *white of the eye.* In this part of the eye-ball there is a circular opening formed in this sclerotic coat, which is covered by a thin and perfectly transparent shell D G F, called the *cornea.* This cornea is more convex than the general surface of the eye-ball, and may be compared to a watch-glass. It is connected round its edge with the sclerotica, which differs from it, however, both in colour and opacity, the sclerotica being white and opaque, while the cornea is perfectly colourless and transparent. The thickness of this cornea is everywhere the same.

The cornea covers that part of the point of the eye which is coloured, and is terminated round the coloured part at the commencement of the white of the eye.

1086. *The aqueous humour — the iris — the pupil.* — Within the cornea is a small chamber filled with a transparent liquid, called the *aqueous humour.* This chamber is partially divided by a thin annular partition I, called the iris, in the centre of which there is a circular aperture P, called the *pupil.* The *iris* is a membranous substance varying in colour in different individuals. It is this which gives the peculiar colour to the eye. It is the pupil which presents the appearance of a black spot in the centre of the coloured part of the eye. A front view of the iris and pupil is given at I and P, in *fig*. 364., and a section of them is indicated by the same letters in *fig*. 365.

1087. *The crystalline humour — ciliary processes.* — The chamber containing the aqueous humour is terminated at its posterior part by a substance in the form of a double convex lens, which contains another transparent liquid, called the *crystalline humour.* This lens K is somewhat greater in diameter than the pupil, and it is supported by a ring of muscles, called the *ciliary processes*, represented at L, in such a position that its axis passes through the centre of the pupil.

Thus the crystalline and the ciliary processes, with the cornea, include the membrane containing the aqueous humour.

1088. *The choroid.* — Within the sclerotica is a second coat N, called the *choroid*. This is a vascular membrane which lines the internal surface of the sclerotic coat, and which terminates in front in the ciliary processes, by which the crystalline lens is set in it in the same manner as the cornea is set in the sclerotic coat.

Some anatomists maintain that the iris is only a continuation of the choroid, and that the cornea is a continuation of the sclerotic coat, which there becomes transparent. The inner surface of this choroid coat is covered with a slimy pigment of an intensely black colour, by which the reflection of the light entering the eye is prevented.

1089. *The retina — the vitreous humour.* — A third coating, represented at O, called the *retina*, from the resemblance of its structure to network, lines this black coating.

The internal membrane Q of the eye-ball contains another transparent liquor, called the *vitreous humour*, which is included in a membranous capsule, called the hyaloid.

Thus between the cornea and the posterior surface of the eye there are three successive humours; the aqueous, contained by the cornea; the crystalline, contained by the crystalline lens; and the vitreous, which fills the inner and larger chamber of the eye-ball.

1090. *The optic axis — the optic nerve.* — A straight line M T passing through the centre of the cornea, coinciding with the axis of the crystalline lens, and passing through the centre of the eye-ball, is called the *optical axis*, or the *axis of the eye*.

At a point of the posterior surface of the eye-ball between the optical axis M T and the nose, the sclerotic coat is formed into a tube, which leads backwards and upwards towards the brain. This tube contains within it the *optic nerve*, which at the point C E, where it enters the eye-ball, spreads out over the inner surface of the choroid and forms the retina, and immediately includes the *hyaloid capsule* containing the vitreous humour.

The retina must therefore be regarded as nothing more than the continuation and diffusion of the optic nerve.

The retina, which in dissection admits of being easily separated from the choroid, is absolutely transparent, so that the light or colours which enter the inner chamber of the eye are not intersected by it, but penetrate it as they would any other thin and perfectly transparent substance, and are only arrested by the black coating spread upon the choroid.

1091. *Numerical data connected with the human eye.* — The following are the average numerical data connected with the eye:—

[redacted]

	100ths of Inch.
Radius of sclerotic coating	39 to 48
Radius of cornea	28 — 32
External diameter of iris	43 — 47
Diameter of pupil	12 — 28
Thickness of cornea	4
Distance of pupil from centre of cornea	8
Distance of pupil from centre of crystalline	4
Radius of anterior surface of crystalline	28 — 39
Radius of posterior surface of crystalline	20 — 24
Diameter of crystalline	39
Thickness of do	30
Length of optic axis	87 — 95
Index of refraction from air into aqueous humour	1·3366
Index of refraction from air into vitreous humour	1·3394
Index of refraction from air into crystalline humour:—	
At the surface	1·3767
At the centre	1·3990
At the mean	1·3839
Index of refraction from aqueous humour to crystalline humour:—	
At the surface	1·0466
At the mean	1·0353
Index of refraction from vitreous humour to crystalline humour:—	
At the surface	1·0445
At the mean	1·0332

According to Sir D. Brewster, who has supplied the preceding indices of refraction, the focal length of the crystalline is 1·73 inches.

1092. *Limits of the play of the eye.* — The limits of the play of the eye-ball are as follows: — The optic axis can turn in the horizontal plane through an angle of 60° towards the nose, and 90° outwards, giving an entire horizontal play of 150°. In the vertical direction it is capable of turning through an angle of 50° upwards and 70° downwards, giving a total vertical play of 120°.

1093. *The eye not perfectly achromatic.* — Sir David Brewster is of opinion that the eye is not perfectly achromatic, but that the chromatic aberration is so small as to produce no indistinctness of vision. He says, if we shut up all the pupil, except a part of its edge, or look past the finger held near the eye, until the finger almost hides a narrow line of white light, we shall see a distinct prismatic spectrum of this line containing all the usual colours, — an effect which could not take place if the eye were perfectly achromatic.

1094. *But must be very nearly so.* — Nevertheless, it is certain that if the achromatism of the eye be not perfect, it is very nearly so. In the analogy observable between the forms and relative densities of the transparent humours which compose this organ, the achromatic combination of lenses is too striking to be casual; and we are irresistibly impressed with the conviction that the combination is made to be nearly achromatic. The two meniscuses formed by the aqueous and vitreous humours, having the double convex crystalline placed between them of greater density than either, and the two former differing from each other in density, appear to fulfil the conditions of achromatism in a striking manner; and it is doubtless to this combination that is due the apparent freedom from colour in the image depicted on the retina.

1095. *Spherical aberration of the eye corrected.* — Sir David Brewster is also of opinion that the spherical aberration of the eye is corrected by the varying density of the crystalline lens, which, having a greater refractive power near its centre, refracts the central rays in each pencil to the same point as its external rays.

The optic nerves R, which proceed from the two eyes, *decussate*, that is, cross each other like the letter X, before they reach the brain.

1096. *Effect of an illuminated object placed before the eye.* — The structure of the eye being thus understood, it will be easy to explain the effect produced within it by luminous or illuminated objects placed before it.

Let us suppose a pencil of light proceeding from any luminous object, such as the sun, incident upon that part of the eye-ball which is left uncovered by the open eye-lids.

That part of the pencil which falls upon the white of the eye W, *fig.* 364., is irregularly reflected, and renders visible that part of the eye-ball. Those rays of the pencil which fall upon the cornea pass through it. The exterior rays fall upon the iris, by which they are irregularly reflected, and render it visible. The internal rays pass through the pupil, are incident upon the crystalline, which, being transparent, is also penetrated by them, from which they pass through the vitreous humour, and finally reach the posterior surface of the inner part of the eye, where they penetrate the transparent retina, and are received by the black surface of the choroid, upon which they produce an illuminated spot.

The aqueous humour being more dense than the external air, and the surface of the cornea, which includes it, being convex, rays passing from the air into it will be rendered more convergent or less divergent. In like manner, the anterior surface of the crystalline lens being convex, and that humour being more dense than the aqueous, a further convergent effect will be produced.

Again, the posterior surface of the crystalline being convex towards the vitreous humour, and this latter humour being less dense than the crystalline, another convergent effect will take place. These rays passing successively through these three humours, are rendered at each surface more and more convergent.

1097. *Image formed within the eye.* — If an object be placed before the eye, pencils of rays will proceed from it, and penetrate the successive humours; and if these pencils be brought to a focus at the posterior surface of the eye, an inverted image of the object will be formed there, exactly as it would be formed by lenses composed of

[redacted]

any transparent medium whose refracting powers would correspond with each of the humours of the eye.

1098. *Experimental proof of its existence.* — That this phenomenon is actually produced in the interior of the eye may be rendered experimentally manifest by taking the eye-ball of an ox recently killed, and dissecting the posterior part, so as to lay bare the choroid. If the eye thus prepared be fixed in an aperture in a screen, and a candle be placed before it at a distance of eighteen or twenty inches, an inverted image of the candle will be seen through the retina, as if it were produced upon ground glass or oiled paper.

1099. *Immediate cause of vision.* — It appears, then, that the immediate cause of vision, and the immediate object of perception in the sensorium when we see, is the image thus depicted on the retina by means of the refracting powers of the humours of the eye.

1100. *Conditions of perfect vision.* — In order, therefore, to perfect vision, the following conditions must be fulfilled: —

1°. The image on the retina must be perfectly distinct.

2°. It must have sufficient magnitude.

3°. It must be sufficiently illuminated.

4°. It must continue on the retina for a sufficient length of time.

Let us examine the circumstances which affect these conditions.

1101. 1°. DISTINCTNESS OF THE IMAGE.

The image formed on the retina will be distinct or not, according as the pencils of rays proceeding from each point of the object placed before the eye, are brought to an exact focus on the retina or not. If they be not brought to an exact focus on the retina, their focus will be a point, therefore, beyond the retina, or within it.

In either case, the rays proceeding from any part of the object, instead of forming a corresponding point on the retina, will form a spot of more or less magnitude, according to the distance of the focus of the pencil from the retina, and the assemblage of such luminous spots will form a confused picture of the object. This deviation of the foci of the pencils from the retina is caused by the refracting powers of the eye being either too feeble or too strong. If the refracting power be too feeble, the rays are intercepted by the retina before they are brought to a focus; if the refracting power be too strong, they are brought to a focus before they arrive at the retina.

1102. *Effects of distant and near objects.* — The objects of vision may be distributed into two classes, in relation to the refracting powers of the eye: 1st, those which are at so great a distance from the eye, that the pencils proceeding from them may be regarded as consisting of parallel rays; 2dly, those which are so near that their rays have sensible divergence.

It has been stated that the diameter of the pupil varies from $\frac{1}{8}$ to $\frac{1}{4}$ an inch in magnitude, the variation depending upon a power of dilatation and contraction with which the iris is endued. Taking the diameter of the pupil at its greatest magnitude of a quarter of an inch, pencils proceeding from an object placed at the distance of three feet from the eye would have an extreme divergence amounting to less than half a degree; and if the pupil be in its most contracted state when its diameter is only the one-eighth of an inch, then the divergence of the pencils proceeding from such an object would amount to about fifteen minutes of a degree. It may therefore be concluded, that pencils proceeding from all objects more distant from the eye than two or three feet, may be regarded as consisting of parallel rays.

The pencils of rays, therefore, proceeding from all such objects will be made to converge to the principal focus of the eye.

1103. *Position of the optical centre of the eye.* — Sir David Brewster concludes from observations made by him that the optical centre of the eye, that is to say, the point at which the axes of secondary pencils intersect the optic axis, is situate in the geometric centre of the eye-ball, and consequently must be a little within the crystalline. If, therefore, round this centre we imagine a spherical surface described, whose radius is equal to the focal distance of the combination of the humours of the eye, the image of all objects more distant from the eye than two or three feet will be found on such a surface. Now, since the retina is spread over the surface of the choroid, and since the form of the eye is spherical, and its diameter but an inch, it follows that the retina is a spherical surface, whose centre coincides with the optical centre of the eye, and which is at a distance from that centre of about half an inch. If the distance of the retina from this centre be exactly equal to the focal distance of the humours, then the foci of all pencils of parallel rays entering the eye will be formed upon it, and consequently it will receive distinct images of all objects whose distance from the eye exceeds two or three feet. But if the focal distance of the humours be less or greater than that, then, as already stated, the image on the retina will be indistinct.

1104. *Optical remedies for defects in the refracting powers of the eye.* — The remedy for such a defect in vision is supplied by the properties of convergent and divergent lenses, already explained.

If the eye possess too little convergent power, a convergent lens is placed before it, which, receiving the parallel pencils, renders them convergent when they enter the pupil, and this enables the eye to bring them to a focus on the retina, provided the power of the lens be equal to the deficient convergence of the eye.

If, on the other hand, the convergent power of the eye be too great, so that the parallel rays are brought to a focus before arriving at the retina, a divergent lens is placed before the eye, by means of which parallel pencils are rendered divergent before they enter the pupil; and the power of the lens is so adapted to the convergent power of the eye, that the rays shall be brought to a focus on the retina.

The two opposite defects of vision here indicated are generally called, the one *weak-sightedness* or *far-sightedness*, and the other *near-sightedness*.

If the objects of vision be placed so near the eye that the rays composing the pencils which proceed from them have sensible divergence, then the foci of these rays within the eye will be at a distance from the optical centre greater than the principal focus. If, therefore, in this case, the principal focus fall upon the retina, the focus of rays proceeding from such near objects would fall beyond it, and consequently the image on the retina would be indistinct.

1105. *Power of the eye to adapt itself to objects differently distant.* — It follows, therefore, that eyes which see distant objects at the greater class of distances would see indistinctly all objects at less distances, unless there were in the eye some means of self-adjustment, by which its convergent power may be augmented. Such means of self-adjustment are provided, which operate within certain limits, and by which we are enabled so to accommodate the eye to the divergence of the pencils proceeding from near objects, that the same eyes which are capable of seeing distinctly objects sensibly so distant as to render the rays of the pencils sensibly parallel, are also capable of seeing with equal distinctness objects at distances varying from ten to twelve inches and upwards.

1106. *Experimental proof of this power.* — By what means the convergent power of the humours is thus varied is not certainly known, but that such means of self-adjustment exist may be proved by the following experiment.

Let a small black spot be made upon a thin transparent plate of glass, and let it be placed at a distance of about twelve inches from the eye. If the eye be directed to it, the spot will be seen as well as distant objects visible through the glass. Let the attention be earnestly directed to the black spot, so that a distinct perception of its form may be produced. The objects visible at a distance will then be found to become indistinct.

But if the attention be directed more to the distant objects, so as to obtain a distinct perception of them, the perception of the black spot on the glass will then become indistinct. It is evident, therefore, that when the eye accommodates itself so as to form upon the retina a distinct image of an object at twelve inches' distance, the image produced by objects at great distances will become indistinct; and that, on the other hand, when the eye so accommodates itself as to render the image produced on the retina by distant objects distinct, the image produced by an object at twelve inches distance will become indistinct.

1107. *Hypotheses which explain this power.* — It is evident, therefore, that the power of the eye to refract the pencils of light incident upon it, is to a certain extent under the control of the will;

but by what means this change in the refracting power of the organ is made is not so apparent. Various hypotheses have been advanced to explain it. According to some, the form of the eye-ball, by a muscular action, is changed in such a manner as to increase the length of the optic axis, and thus to remove the posterior surface of the retina to a greater distance from the crystalline, when it is necessary to obtain a distinct view of near objects; and, on the contrary, to elongate the transverse diameter of the eye, and shorten the optic axis so as to bring the retina closer to the crystalline, when it is desired to obtain a distinct view of distant objects.

According to others, this change of form is only effected in the cornea, which being rendered more or less convex by a muscular action, gives a greater or less convergent power to the aqueous humour.

According to others, the eye accommodates itself to different distances by the action of the crystalline, which is moved by the ciliary processes either towards or from the cornea, thus transferring the focus of rays proceeding from it within a certain limit of distance to and from the retina; or, by a similar action of the ciliary process, the crystalline lens may be supposed to be rendered more or less convex, and thus to increase or diminish its convergent power.

None of these hypotheses have, however, found general acceptation. It is denied as a matter of fact, that the eye-ball is elongated, or that the curvature of the cornea is changed; and it is doubted, to say the least of it, that the crystalline is capable either of displacement or change of convexity.

1108. *Explanation proposed by M. Pouillet.* — M. Pouillet maintains (and affirms that his opinion is founded on the dissection of a great number of crystalline lenses) that this humour is composed of layers or strata one within another, differing in curvature and density, so that its section would exhibit a series of concentrical ellipses having varying eccentricities. It would follow from this, that the internal strata being more curved and more dense than the external strata, the rays which pass from the latter will converge to a more distant point than those which pass from the former. The crystalline, there-ore, according to M. Pouillet, has not one but many different foci.

When a pencil of rays falls upon it, those rays which are near the of the pencil, and therefore near the centre of the crystalline, are ught to a shorter focus than those which are near the borders. cording to the hypothesis advanced by M. Pouillet, the eye sees ar objects, therefore, by means of the central rays, and distant ob- ans of those rays which fall near the borders of the crys-

hich proceed from near objects being more divergent proceed from distant objects, are refracted by the crystalline, so as to be brought to a focus on the s of the same pencil which would fall upon the

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borders of the crystalline would be brought to a focus beyond the retina.

When pencils, however, proceed from objects so distant that the rays composing them may be regarded as parallel, the central rays of such pencils would be brought to a focus before arriving at the retina, while the rays falling near the borders of the crystalline would be brought to a focus upon the retina. Now, according to these conditions, it would follow, that a certain confusion of vision would ensue in both cases; for near objects, the image produced by the central rays would be rendered confused by the rays passing near the borders of the crystalline, which meet the retina before they are brought to a focus; and in the case of distant objects, the image formed by the rays passing near the borders of the crystalline would be rendered confused by those which pass near the centre of the crystalline, and which are brought to a focus before they arrive at the retina.

M. Pouillet meets these difficulties by the following considerations. He supposes that when the eye views near objects the pupil contracts itself, so as to intercept to a greater or less extent the external rays of the pencils, and to admit only to the crystalline those which fall immediately under its axis. In this way the confusion which would be produced by the external rays of the pencils is prevented. But the same expedient would not prevent the confusion produced in the image of distant objects by the central rays of the pencils brought to a focus before arriving at the retina. This difficulty M. Pouillet answers, by stating that the comparative number of the central rays is so small that their action upon the retina is inconsiderable compared with that of the external rays, and that consequently their effect is not sensible.

M. Pouillet appeals to observation to establish the fact that the pupil always contracts when the eye views near objects.

1109. *Limits of the power of adaptation to varying distance.*—Whatever be the provisions made in the organization of the eye, by which it is enabled to adapt itself to the reception of divergent pencils proceeding from near objects, the power with which it is thus endued has a certain limit. Thus, eyes which see distinctly distant objects, and which therefore bring parallel rays to a focus on the retina in their ordinary state, are not capable of seeing distinctly objects brought nearer to them than ten or twelve inches. The power of accommodating the vision to different rays is therefore limited to a divergence not exceeding that which is determined by the diameter of the pupil compared with a distance of ten or twelve inches. Now, as the diameter of the pupil is most contracted when the organ is directed to such near objects, we may assume it at its smallest magnitude at one-eighth of an inch, and therefore the divergence of a pencil proceeding from a distance of twelve inches would be $\frac{1}{96}$th, and the angle of divergence would therefore be very nearly half a degree.

It may, therefore, be assumed that eyes adapted to the vision of distant objects are in general incapable of seeing distinctly objects from which pencils have greater divergence than this, or, which is the same, objects applied at less than ten or twelve inches from the eye.

1110. *Case of eyes having feeble convergent power.* — In the case of eyes whose convergent power is too feeble to bring pencils proceeding from distant objects to a focus on the retina, they will be in a still greater degree inadequate to bring pencils to a focus which diverge from near objects; and consequently such eyes will require to be aided, for near as well as distant objects, by the interposition of convergent lenses. It would, however, be necessary to provide lenses of different convergent powers for distant and near objects, the latter requiring a greater convergent power than the former; and in general the nearer the objects viewed, the greater the convergent power required from the lens.

1111. *Case of eyes having strong convergent power.* — In the case of eyes whose convergent power is so great as to bring pencils proceeding from distant objects to a focus short of the retina, and which therefore, for such distant objects, require the intervention of divergent lenses, distinct vision will be attained without the interposition of any lens, provided the object be placed at such a distance that the divergence of the pencils proceeding from it shall be such that the convergent power of the eye bring them to a focus on the retina.

Hence it is that eyes of this sort are called *short-sighted*, because they can see distinctly such objects only as are placed at the distance which gives the pencils proceeding from them such a divergence, that the convergent power of the eye would bring them to a focus on the retina.

1112. *Method of ascertaining the power of the lens required by defective eyes.* — If it be desired to ascertain the focal length of the divergent lens which such an eye would require to see distant objects distinctly, it is only necessary to ascertain at what distance it is enabled to see distinctly the same class of objects without the aid of a lens. A lens having a focal length equal to this distance will enable the eye to see distant objects distinctly, because such a lens would give the parallel rays a divergence equal to the divergence of pencils proceeding from a distance equal to its focal length.

1113. *Power of adaptation to varying distance in short-sighted eyes.* — Persons are said to be more or less near-sighted, according to the distance at which they are enabled to see objects with perfect distinctness, and they accordingly require, to enable them to see distant objects distinctly, diverging lenses of greater or less focal length.

As persons who are enabled to see distant objects distinctly have the power of accommodating the eye so as to see objects at ten or twelve inches' distance, so short-sighted persons have a similar power of accommodation, but within proportionally smaller limits. Thus a short-sighted person will be enabled to see distinctly objects placed at distances from the eye varying from two or three inches upwards, according to the degree of short-sightedness with which he is affected.

1114. *Causes of short sight and long sight.* — The two opposite defects of vision which have been mentioned, arising from too great or too little convergent power in the eye, may arise, either from a defect in the quality of the humours or in the form of the eye. Thus near-sightedness may arise from too great convexity in the cornea or in the crystalline, or it may arise from too great a difference of density between the aqueous humour and the crystalline, or between the crystalline humour and the vitreous, or both of them; or, in fine, it may arise from defects both of the form and of the relative densities of the humours.

1115. *Defective sight arising from imperfect transparency of the humours.* — In a certain class of maladies incidental to the sight, the humours of the eye lose in a greater or less degree their transparency, and the crystalline humour is more especially liable to this. In such cases vision is sometimes recovered by means of the removal of the crystalline humour, in which case the vision is reduced to two humours, the aqueous and the vitreous; but as the eye owes in a greater degree to the crystalline than to the other humours the convergent power, it is necessary in this case to supply the place of the crystalline by a very strong convergent lens placed before the eye.

1116. 2°. MAGNITUDE OF THE IMAGE ON THE RETINA.

In order to obtain a perception of any visible object, it is not enough that the image on the retina be distinct, it must also have a certain magnitude.

Let us suppose that a white circular disk, one foot diameter, is placed before the eye at a distance of 57½ feet.

The axes of the pencils of rays proceeding from such disk to the eye will be included within a cone, whose base is the disk, and whose vertex is in the centre of the eye.

These axes, after intersecting at the centre of the eye, will form another cone, whose base will be the image of the disk formed upon the retina. The common angle of the two cones will in this case be 1°.

Let A B, *fig.* 366., be the diameter of the disk. Let C be the centre of the eye, and let *b a* be the diameter of the image on the

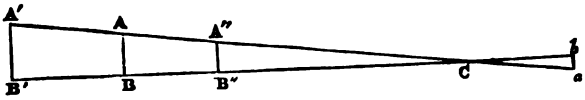

Fig. 366.

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retina. It is clear, from the perfect similarity of the triangles A C B and a C b, that the diameter of the image $b\,a$ will have to the diameter of the object B A the same proportion as the distance a C of the retina from the centre C has to the distance A C of the object from the same centre. Therefore in this case, since one-half the diameter of the eye is but half an inch, and the distance A C is in this case supposed to be 57½ feet, the magnitude of the diameter $b\,a$ of the image on the retina will be found by the following proportion:—

$$a\,b : \text{AB} :: \tfrac{1}{2} : 57\tfrac{1}{2} \times 12 = 690.$$

Therefore we have

$$a\,b = \frac{\frac{1}{2} \times \text{AB}}{690} = \frac{6}{690} = \frac{1}{115}.$$

The total magnitude, therefore, of the diameter of the image on the retina would in this case be the $\frac{1}{115}$th part of an inch; yet such is the exquisite sensibility of the organ, that the object is in this case distinctly visible.

If the disk were removed to twice the distance here supposed, the angle of the cone C would be reduced to half a degree, and the diameter of the image on the retina would be reduced to one-half its former magnitude, that is to say, to the $\frac{1}{230}$th part of an inch. If, on the other hand, the disk were moved towards the eye, and placed at half its original distance, then the angle C of the cone would be 2°, and the diameter of the picture on the retina would be double its first magnitude, that is to say, the $\frac{2}{115}$th of an inch.

In general, it may therefore be inferred that the magnitude of the diameter of the picture on the retina is increased or diminished in exactly the same proportion as the angle of the cone C, formed at the centre of the eye, is increased or diminished.

1117. *The visual angle or apparent magnitude.* — This angle is called the visual angle or apparent magnitude of the object; and when it is said that a certain object subtends at the eye a certain angle, it is meant that lines drawn from the extremities of such object to the centre of the eye form such angle.

The *apparent magnitude* of an object must not be confounded with its apparent superficial magnitude, the term being invariably applied to its *linear magnitude.* The apparent superficial magnitude varies in proportion to the square of the apparent magnitude.

Thus, for example, when the disk A B is removed to double its original distance from the eye, the apparent magnitude, or the angle C, is diminished one-half, and consequently the diameter $a\,b$ of the picture on the retina is also diminished one-half; and since the diameter is diminished in the ratio of 2 to 1, the superficial magnitude of the image, or its area, will be diminished in the proportion of 4 to 1

1118. *Apparent magnitude increases in proportion as the distance diminishes, and vice versâ.* — It is clear from what has been stated

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also, that when the same object is moved from or towards the eye, its apparent magnitude varies inversely as its distance; that is, its apparent magnitude is increased in the same proportion as its distance is diminished, and *vice versâ*.

It is easy to perceive that the objects which are seen under the same visual angle will have the same apparent magnitude. Thus let A′ B′, *fig.* 366., be an object more distant than A B, and of such a magnitude that its highest point A′ shall be in the continuation of the line C A, and its lowest point B′ in the continuation of the line C B. The apparent magnitude of A′ B′ will then be measured by the angle at C. This angle will therefore at the same time represent the apparent magnitude of the object A B and of the object A′ B′. It is evident that an eye placed at C will see every point of the object A B upon the corresponding points of the object A′ B′; so that if the object A B were opaque, and of a form similar to the object A′ B′, every point of the one would be seen upon a corresponding point of the other. In like manner, if an object A″ B″ were placed nearer the eye than A B, so that its highest point may lie upon the line C A, and its lowest point upon the line C B, the object, being similar in form to A B, would appear to be of the same magnitude. Now it is evident that the real magnitudes of the three objects A″ B″, A B, and A′ B′, are in proportion of their respective distances from the eye; A′ B′ is just so much greater than A B, and A B than A″ B″, as C B′ is greater than C B, and as C B greater than C B″.

Thus it appears that if several objects be placed before the eye in the same direction at different distances, and that the real linear magnitudes of these objects are in the proportion of their distances, they will have the same apparent magnitude.

1119. *Case of the sun and moon illustrates this.* — A striking example of this principle is presented by the case of the sun and moon. These objects appear in the heavens equal in size, the full moon being equal in apparent magnitude to the sun. Now it is proved by astronomical observation that the real diameter of the sun is, in round numbers, four hundred times that of the moon; but it is also proved that the distance of the sun from the earth is also, in round numbers, four hundred times greater than that of the moon. The distance, therefore, of these two objects being in the same proportion as their real diameter, their visual or apparent magnitudes are equal.

1120. *Apparent magnitude corresponds with the real magnitude of the picture on the retina.* — It is evident from what has been explained, that objects which have equal apparent magnitudes, and are therefore seen under equal visual angles, will have pictures of equal magnitude on the retina, a fact which proves that the visual angle is the measure of the apparent magnitude.

1121. *The apparent magnitude of an object diminished by removing it from the eye.* — If the same object be moved successively to increasing distances, its apparent magnitude will be diminished in the same proportion, exactly as its distance from the eye is increased. Thus, if L M, *fig.* 367., be such an object, its distance E M being ex-

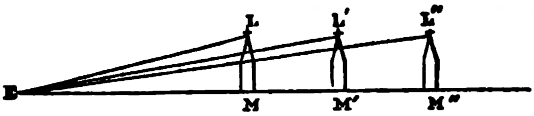

Fig. 367.

pressed by D, and its height L M by H, the visual angle L E M, which measures its apparent magnitude, will be expressed, according to what what has been formerly explained, by $\frac{H}{D}$. If the object be now removed to double its former distance, such as E M′, the visual angle or apparent magnitude L′ E M′ will be expressed by $\frac{H}{2\,D}$, which is just one-half $\frac{H}{D}$, the former visual angle; and, in like manner, if the object be removed to three times its first distance, such as E M″, its visual angle or apparent magnitude will be $\frac{H}{3\,D}$, which is one-third of its original apparent magnitude.

1122. *Apparent superficial magnitude.* — The apparent superficial magnitude of a body is determined by a section of the body made by a plane at right angles to the lines containing the visual angle. Thus, the apparent superficial magnitude of the sun or moon is determined by a section of those bodies passing through the points where lines drawn from the eye touching them would meet them, which, in consequence of the great distance of these bodies, would be a circular section through their centres, and at right angles to a line drawn from the centre to the eye.

1123. *Section of vision.* — This circle, in the case of the sun or moon or other celestial object, is called the *circle of vision;* and a corresponding section of any other object drawn at right angles to the sides of the visual angle would be called the *section of vision.*

For all distant objects, this section is a plane at right angles to the direction in which the object is seen.

1124. *The smallest magnitudes which can be distinctly seen.* — If the circular disk A B, *fig.* 366., which we have supposed to be presented before the eye at a distance of fifty-seven and a half times its own diameter, and which therefore subtends at the centre of the eye a visual angle of 1°, be removed to a distance sixty times greater, or to a distance equal to 3,450 times its own diameter, it will subtend an angle at O proportionally less, which will therefore be, in this case, an angle of one minute; and if it be removed to double the latter dis-

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tance, or 6,900 times its own diameter, it will subtend a visual angle of thirty seconds. Now it is found that if such an object be directly illuminated by the sun, it will be barely visible. This limit, however, depends as well on the colour of the object as on the degree of its illumination. Plateau affirms that a white disk, such as we have here supposed to be presented to the eye, if the light of the sun shone fully upon it, will be visible when seen under a visual angle of twelve seconds, or the one-fifth part of a minute. The disk would subtend thi angle at the eye if placed at a distance equal to 17,250 times its diameter.

He says also that if the disk, under the same circumstances, were red, it would be distinctly seen until its apparent magnitude were reduced to twenty-three seconds; and that if it were blue, the limit would be twenty-six seconds; but that, if instead of being illuminated by the direct solar light, it were illuminated by the light of day reflected from the clouds, these limiting angles would be half as large again.

1125. *Distinctness of vision compared with the magnitude of the pictures on the retina.* — Nothing can be more calculated to excite our wonder and admiration than the distinctness of our perception of visible objects, compared with the magnitude of the picture on the retina, from which immediately we receive such perception.

1126. *Example of the picture of the full moon on the retina.*—If we look at the full moon on a clear night, we perceive with considerable distinctness by the naked eye the lineaments of light and shade which characterize its disk.

Now let us consider only for a moment what are the dimensions of the picture of the moon formed on the retina, from which alone we derive this distinct perception.

The disk of the moon subtends a visual angle of half a degree, and consequently, according to what has been explained, the diameter of its picture on the retina will be $\frac{1}{130}$th part of an inch, and the entire superficial magnitude of the image from which we derive this distinct perception is less than the $\frac{1}{12000}$th part of a square inch; yet within this minute space, we are able to distinguish a multiplicity of still more minute details. We perceive, for example, forms of light and shade, whose linear dimensions do not exceed one-tenth part of the apparent diameter of the moon, and which therefore occupy upon the retina a space whose diameter does not exceed the $\frac{1}{1500000}$th part of a square inch.

1127. *Example of the human figure.* — To take another example, the figure of a man 70 inches high, seen at a distance of 40 feet, produces an image upon the retina the height of which is about one-fourteenth part of an inch. The face of such an image is included in a circle whose diameter is about one-twelfth of the height, and therefore occupies on the retina a circle whose diameter is about the

$\frac{1}{\text{[illegible]}}$th part of an inch; nevertheless, within this circle, the eyes, nose, and lineaments are distinctly seen. The diameter of the eye is about one-twelfth of that of the face, and therefore, though distinctly seen, does not occupy upon the retina a space exceeding the $\frac{1}{\text{[illegible]}}$th of a square inch.

If the retina be the canvas on which this exquisite miniature is delineated, how infinitely delicate must be its structure, to receive and transmit details so minute with such marvellous precision; and if, according to the opinion of some, the perception of these details be obtained by the retina *feeling* the image formed upon the choroid, how exquisitely sensitive must be its touch!

1128. 3°. SUFFICIENCY OF ILLUMINATION.

It is not enough for distinct vision that a well-defined picture of the object shall be formed on the retina. This picture must be sufficiently illuminated to affect the senses, and at the same time not be so intensely illuminated as to overpower the organ.

Thus it is possible to conceive a picture on the retina so extremely faint as to be insufficient to produce sensation, or, on the other hand, so intensely brilliant as to dazzle the eye, to destroy the distinctness of sense, and to produce pain.

When we direct the eye to the sun, near the meridian, in an unclouded sky, we have no distinct perception of his disk, because the splendour is so great as to overpower the sense of vision, just as sounds are sometimes so intense as to be deafening.

That it is the intense splendour alone which prevents a distinct perception of the solar disk in this case is rendered manifest by the fact that if a portion of the solar rays be intercepted by a coloured glass, or by a thin cloud, a distinct image of the sun will be seen.

When we direct the eye to the firmament on a clear night, there are innumerable stars which transmit light to the eye, and which therefore must produce some image on the retina, but of which we are altogether insensible, owing to the faintness of the illumination. That the light, however, does enter the eye and arrive at the retina is proved by the fact that if a telescope be directed to the stars in question, so as to collect a greater quantity of their light upon the retina, they will become visible.

1129. *The eye has power of accommodation to different degrees of illumination.*—The eye possesses a certain limited power of accommodating itself to various degrees of illumination. Circumstances which are familiar to every one render the exercise of this power evident.

If a person, after remaining a certain time in a dark room, pass suddenly into another room strongly illuminated, the eye suffers instantly a degree of inconvenience, and even pain, which causes the eyelids to close; and it is not until after the lapse of a certain time that they can be opened without inconvenience.

The cause of this is easily explained. While the observer remains in the darkened or less illuminated room, the pupil is dilated so as to admit into the eye as great a quantity of light as the structure of the organ allows of. When he passes suddenly into the strongly illuminated room, the flood of light arriving through the widely dilated pupil acts with such violence on the retina as to produce pain, which necessarily calls for the relief and protection of the organ. The iris, then, by an action peculiar to it, contracts the dimensions of the pupil so as to admit proportionally less light, and the eye is opened with impunity.

Effects the reverse of these are observed when a person passes from a strongly illuminated room into one comparatively dark, or into the open air at night. For a certain time he sees nothing, because the contraction of the pupil, which was adapted to the strong light to which it had previously been exposed, admits so little light to the retina that no sensation is produced. The pupil, however, after awhile dilates, and, admitting more light, objects are perceived which were before invisible.*

1130. *Relative brilliancy of equidistant luminaries.* — *Brightness of the picture on the retina.* — If two points from which light radiates be placed at the same distance from the eye, the brightness of their image on the retina will be in proportion to their absolute brilliancy. But if either point be removed to a greater distance, the number of rays passing from it which enter the pupil will be diminished in the same proportion as the square of its distance is increased, and *vice versâ.* It consequently follows that the brightness of each point of the image to an object formed upon the retina will be in direct proportion to the absolute brilliancy of such point, and in the inverse proportion of the square of its distance from the eye.

Thus, if I express the intensity of the light of the point upon the object, and D its distance from the eye, then the brightness of the image of such point upon the retina will be expressed by $\frac{I}{D^2}$.

It is therefore clear that the brightness of the image of each point of an object will be diminished as the square of the distance of the object from the eye is increased.

1131. *Apparent brightness the same at all distances.* — It is sometimes inferred from this, though erroneously, that the apparent splen-

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* "Thus, when the lamp that lighted
The traveller at first, goes out,
He feels awhile benighted,
And wanders on in fear and doubt;
But soon the prospect clearing,
In cloudless starlight on he treads,
And finds no lamp so cheering
As that light which Heaven sheds."—MOORE.

dour of the image of a visible object decreases as the square of the distance increases. This would be the case in the strictest sense, if, while the object were withdrawn from the eye to an increased distance, its image on the retina continued to have the same magnitude; for, in this case, the absolute brightness of each point composing such image would diminish as the square of the distance increases, and the area of the retina over which such points are diffused would remain the same; but it must be considered, that as the object retires from the eye the superficial magnitude of the image on the retina is diminished in the same proportion as the square of the distance of the object from the eye is increased. It therefore follows that while the points composing the image on the retina are diminished in the intensity of their illumination, they are collected into a smaller space, so that what each point of the image on the retina loses in splendour, the entire image gains by concentration.

1182. *If the distance of the sun were increased or diminished, its apparent magnitude would be changed, but its apparent brightness would remain the same.* — If the sun were brought as close to the earth as the moon, its apparent diameter would be 400 times greater, and the area of its apparent disk 160,000 times greater than at present, but the apparent brightness of its surface would not be in any degree increased. In the same manner, if the sun were removed to ten times its present distance, it would appear under a visual angle ten times less than at present, as in fact it would to an observer on the planet Saturn, and its visible area would be a hundred times less than it is, but the splendour of its diminished area would be exactly the same as the present splendour of the sun's disk.

These consequences, which are of considerable physical importance, obviously follow from the principles explained above.

The sun seen from the planet Saturn has an apparent diameter ten times less than it has when seen from the earth.

The appearance from Saturn will then be the same as would be the appearance of a portion of the disk of the sun seen from the earth through a circular aperture in an opaque plate, which would exhibit a portion of the disk whose diameter is one-tenth of the whole.

1133. *An object may be visible even though it have no sensible visual magnitude.* — *The fixed stars examples of this.* — When the light which radiates from a luminous object has a certain intensity, it will continue to affect the retina in a sensible manner, even when the object is removed to such a distance that the visual angle shall cease to have any perceivable magnitude. The fixed stars present innumerable examples of this effect. None of these objects, even the most brilliant of them, subtend any sensible angle to the eye. When viewed through the most perfect telescopes they appear merely as brilliant points. In this case, therefore, the eye is affected by the light alone, and not by the magnitude of the object seen.

l34. *By increase of distance, however, such objects may cease fect the retina sensibly.* — Nevertheless, the distance of such an t may be increased to such an extent that the light, intense as it ill cease to produce a sensible effect upon the retina.

will be explained in the second volume of this series, that seven es of the fixed stars, diminishing gradually in brightness,* pro- an effect on the retina such as to render them visible to a d eye. This diminution of splendour is produced by their in- sed distance. The telescope, however, as has been already stated, gs into view innumerable other stars, whose intrinsic splendour is reat as the brightest among those which we see, but which do not smit to the retina, without the aid of the telescope, enough of ; to produce any sensible effect. Nevertheless it is demonstrable , even without the telescope, they do transmit a certain definite itity of light to the retina; the quantity of light which they thus smit, and which is insufficient to produce a sensible effect, having e quantity obtained by the telescope a ratio depending upon the ortion of the magnitude of the object-glass of the telescope to the nitude of the pupil.

l35. *The intensity of illumination necessary to produce sensa- also depends on the relative splendour of other objects present re the eye.* — The quantity and intensity of the light transmitted n external object to the retina, which is sufficient to produce a eption of such object, depends also upon the light received at the time by the retina from other objects present before the eye. proof of this is, that the same objects which are visible at one are not visible at another, though equally before the eye, and smitting equal quantities of light of the same intensity to the re- . Thus, the stars are present in the heavens by day as well as ight, and transmit the same quantity of light to the retina, yet are not visible in the presence of the sun, because the light pro- ing from that luminary, directly and indirectly reflected and re- ted by the air and innumerable other objects, is so much greater uantity and intensity as to overpower the inferior and much less nse light of the stars. This case is altogether analogous to that he ear, which, when under the impression of loud and intense ds, is incapable of perceiving sounds of less intensity, which ne- heless affect the organ in the same manner as they do when, in absence of louder sounds, they are distinctly heard.

ven when an object is perceived, the intensity of the perception lative, and determined by other perceptions produced at the same . Thus, the moon seen at night is incomparably more splendid the same moon seen by day or in the twilight, although in each

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The term magnitude is used in astronomy, as applied to the fixed stars, xpress their apparent brightness; no fixed star, however splendid, sub- s any sensible angle.

case the moon transmits precisely the same quantity of light, of precisely the same intensity, to the eye; but in the one case the eye is overpowered by the superior splendour of the light of day, which dims the comparatively less intense light proceeding from the moon.

1136. 4°. THE IMAGE MUST CONTINUE A SUFFICIENT TIME UPON THE RETINA TO ENABLE THAT MEMBRANE TO PRODUCE A PERCEPTION OF IT.

It will be proved hereafter that the velocity with which light is propagated through space is at the rate of about 200,000 miles per second. Its transmission, therefore, from all objects at ordinary distances to the eye may be considered as instantaneous. The moment, therefore, any object is placed before the eye an image of it is formed on the retina, and this image continues there until the object is removed. Now it is easy to show experimentally that an object may be placed before the eye for a certain definite interval of time, and that a picture may be painted upon the retina during that interval without producing any perception or any consciousness of the presence of the object.

To illustrate this, let a circular disk A B C D, *fig.* 368, about 20 inches in diameter, be formed in card or tin, and let a circle A′ B′ C′ D′ be described upon it, about 2 inches less in radius than the disk, so as to leave between the circle and the disk a zone about two inches wide. Let the entire zone be blackened, except the space A M M′ A′, forming about the one-twentieth of it. Let the disk thus prepared be attached to the back of a blackened screen, so as to be capable of revolving behind it, and let a hole one inch in diameter be made in the screen at any point, behind which the zone A B C D is placed. If the disk be now made to revolve behind the screen, the hole will appear as a circular white spot so long as the white space A M passes behind it, and will disappear, leaving the same black colour as the screen during the remainder of the revolution of the disk. The hole will therefore be seen as a white circular spot upon the black screen during one-twentieth of each revolution of the disk. If the disk be now put in motion at a slow rate, the white hole will be seen on the screen during one-twentieth of each revolution. If the velocity of rotation imparted to the disk be gradually increased, the white spot will ulti-

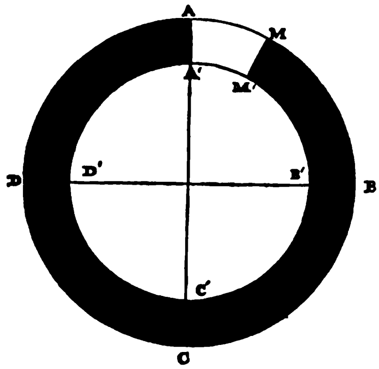

Fig. 368

mately *disappear*, and the screen appear of a uniform black colour, although it be certain that during the twentieth part of each revolution, whatever be the rate of rotation, a picture of the white spot is formed on the retina.

1137. *To determine experimentally the time a picture must continue on the retina to produce sensation.* — The length of time necessary in this case for the action of light upon the retina to produce sensation may be determined by ascertaining the most rapid motion of the disk which is capable of producing a distinct perception of the white spot. This interval will be found to vary with the degree of illumination. If the spot be strongly illuminated, a less interval will be sufficient to produce a perception of it; if it be more feebly illuminated, a longer interval will be required. The experiment may be made by varying the colour of the space A M of the zone, and it will be found that the interval necessary to produce sensation will vary with the colour as well as with the degree of illumination.

1138. *The perception of a visible object is continued for a certain time after the object is removed from before the eye.* — Numerous observations on the most familiar effects of vision, and various experiments expressly contrived for the purpose, show that the retina, when once impressed with the picture of an object placed before the eye, retains this impression, sometimes with its full intensity and sometimes more faintly, just as the ear retains for a time the sensation of a sound after the cause which has put the tympanum in vibration has ceased to act. The duration of this impression on the retina, after the removal of the visible object which produced it, varies according to the degree of illumination and the colour of the object. The more intense the illumination, and the brighter the colour, the longer will be the interval during which the retina will retain their effects.

1139. *Experimental illustration of this.* — To illustrate this experimentally, let a circular disk formed of blackened card or tin, of 12 or 14 inches in diameter, be pierced with 8 holes round its circumference, at equal distances, each hole being about half an inch in diameter, as represented in *fig.* 369.

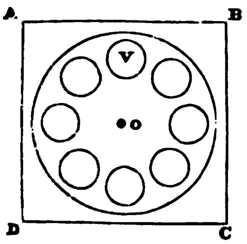

Fig. 369.

Let this disk be attached upon a pivot or pin at its centre O to a board A B C D, which is blackened everywhere, except upon a circular spot at V, corresponding in magnitude to the holes made in the circular plate.

Let this spot be first supposed to be white. Let the circular disk be made to revolve upon the point O, so as to bring the circular holes successively before the white spot at V. The retina will thus be impressed at intervals with the image of this circular white spot. In the intervals between the transits of the holes over it, the entire board will appear black, and the retina will receive no impression. If the disk be made to revolve with a very slow motion, the eye will see the white spot at intervals, but if the velocity of rotation be gradually increased, it will be found that the eye will perceive the white spot permanently represented at V, as if the disk had been placed with one of its holes opposite to it without moving. It is evident, therefore, that in this case the impression produced upon the retina, when any hole is opposite the white spot, remains until the succeeding hole comes opposite to it, and thus a continued perception of the white spot is produced.

If the white spot be illuminated in various degrees, or if it be differently coloured, the velocity of the disk necessary to produce a continuous perception of it will differ. The brighter the colour and the stronger the illumination, the less will be the velocity of rotation of the disk which is necessary to produce a continuous perception of the spot.

These effects show that the stronger the illumination and the brighter the colour, the longer is the interval during which the impression is retained by the retina.

1140. *Why we are not sensible of darkness when we wink.* — This continuance of the impression of external objects on the retina, after the light from the objects ceases to act, is also manifested by the fact, that the continual winking of the eyes for the purpose of lubricating the eye-ball by the eye-lid does not intercept our vision. If we look at any external objects, they never cease for a moment to be visible to us, notwithstanding the frequent intermissions which take place in the action of light upon the retina, in consequence of its being thus intercepted by the eye-lid.

1141. *Experimental illustration suggested by Sir D. Brewster.* — According to Sir David Brewster, the most instructive experiment on this subject, which, however, requires a great deal of practice to be made with success, is to look for a short time at a window at the end of a long room, and then suddenly to turn the eye to a dark wall. In general, a common observer will in this case see a representation of the window on the wall, in which, however, the dark bars of the sash will appear white and the panes of glass dark.

A practised observer, however, who makes the observation with great promptness, will see at the moment his eyes are turned to the wall a correct representation of the window. This representation will almost immediately be succeeded by the reversed picture just mentioned, in which the bars are bright and the panes dark.

1142. *Why a lighted stick revolving produces apparently a luminous ring.* — If a lighted stick be turned round in a circle in a dark room, the appearance to the eye will be a continuous circle of light;

for in this case the impression produced upon the retina by the light, when the stick is at any point of the circle, is retained until the stick returns to that point.

1143. *Flash of lightning.* — In the same manner, a flash of lightning appears to the eye as a continuous line of light, because the light emitted at any point of the line remains upon the retina until the cause of the light passes over the succeeding points.

In the same manner, any objects moving before the eye with such a velocity that the retina shall retain the impression produced at one point in the line of its motion until it passes through the other points, will appear as a continuous line of light or colour.

1144. *Why an object moving with a great speed becomes invisible.* — But to produce this effect, it is not enough that the body change its position so rapidly that the impression produced at one point of its path continue until its arrival at another point; it is necessary, also, that its motion should not be so rapid as to make it pass from any of the positions which it successively assumes before it has time to impress the eye with a perception of it; for it must be remembered, as has been already explained, that the perception of a visible object presented to the eye, though rapid, is not instantaneous.

The object must remain present before the organ of vision a certain definite time, and its position must continue upon the retina during such time, before any perception of it is obtained. Now, if the body move from its position before the lapse of this time, it necessarily follows that no perception of its presence, therefore, will be obtained. If, then, we suppose a body moving so rapidly before the eye that it remains in no position long enough to produce a perception of it, such object will not be seen.

1145. *Example of a cannon-ball.* — Hence it is that the ball discharged from a cannon passing transversely to the line of vision is not seen; but if the eye be placed in the direction in which the ball moves, so that the angular motion of the ball round the eye as a centre will be slow notwithstanding its great velocity, it will be visible, because however rapid its real motion through space, its angular motion with respect to the eye (and consequently of the image of its picture on the retina) will be sufficiently slow to give the necessary time for the production of a perception of it.

1146. *Quickness of vision depends on colour, brightness, and magnitude.*—The time thus necessary to obtain the perception of a visible object varies with the degree of illumination, the colour, and the apparent magnitude of the object. The more intense the illumination, the more vivid the colour, and the greater the apparent magnitude, the less will be the time necessary to produce a perception of the object.

1147. *Conditions which determine apparent motion.*—In applying this principle to the phenomena of vision, it must be carefully remem-

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bered that the question is affected, not by the real but by the apparent motion of the object, that is to say, not by the velocity with which the object really moves through space, but by the angle which the line drawn from the eye to the object describes per second. Now this angle is affected by two conditions, which it is important to attend to: 1°. the direction of the motion of the object compared with the line of vision; and 2°. by the velocity of the motion compared with the distance of the object. If the object were to move exactly in the direction of the line of vision, it would appear to the eye to be absolutely stationary, since the line drawn to it would have no angular motion; and if it were to move in a direction forming an oblique angle to the line of vision, its apparent motion might be indefinitely slow, however great its real velocity might be.

For example, let it be supposed that the eye being at E, *fig.* 370, an object O moves in the direction O O′, so as to move from O to O′ in

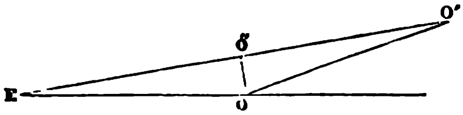

Fig. 370.

one second. Taking E as a centre, and E O as a radius, let a circular arc O O″ be described. The apparent motion of the object will then be the same as if, instead of moving from O to O′ in one second, it moved from O to O″ in one second.

The more nearly, therefore, at right angles to the line of vision the direction of the motion is, the greater will be the apparent motion produced by any real motion of an object.

1148. *How apparent motion is affected by distance.* — A motion which is visible at one distance may be invisible at another, inasmuch as the angular velocity will be increased as the distance is diminished. Thus if an object at a distance of 57½ feet from the eye move at a rate of a foot per second, it will appear to move at the rate of degree per second, inasmuch as a line one foot long at 57½ feet nce subtends an angle of one degree. Now if the eye be removed such an object to a distance of 115 feet, the apparent motion be half a degree, or thirty minutes per second; and if it be wed to thirty times that distance, the apparent motion will be lower. Or if, on the other hand, the eye be brought ject, the apparent motion will be accelerated in exactly tion as the distance of the eye is diminished.

le of a cannon-ball and the moon. — A cannon-ball miles an hour transversely to the line of vision, and 50 yards from the eye, will be invisible, since it will fficient time in any one position to produce perception.

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The moon, however, moving with more than double the velocity of the cannon-ball, being at a distance of 240,000 miles, has an apparent motion, so slow as to be imperceptible.

1150. *What motions are imperceptible.* — The angular motion of the line of vision may be so diminished as to become imperceptible; and the body thus moved will in this case appear stationary. It is found by experience that unless a body moves in such a manner that the line of vision shall describe at least one degree in each minute of time, its motion will not be perceptible.

1151. *Why the diurnal motion of the heavens is not immediately perceptible.* — Thus it is that we are not conscious of the diurnal motion of the firmament. If we look at the moon and stars on a clear night, they appear to the eye to be quiescent; but if we observe them after the lapse of some hours, we find that their positions are changed, those which were near the horizon being nearer the meridian, and those which were in the meridian having descended towards the horizon. Since we are conscious that this change did not take place suddenly, we infer that the entire firmament must have been in continual motion round us, but that this motion is so slow as to be imperceptible.

Since the heavens appear to make a complete revolution in twenty-four hours, each object on the firmament must move at the rate of 15° an hour, or at the rate of one quarter of a degree a minute. But since no motion is perceptible to the eye which has a less apparent velocity than 1° per minute, this motion of the firmament is unperceived. If, however, the earth revolved on its axis in six hours instead of twenty-four hours, then the sun, moon, stars, and other celestial objects, would have a motion at the rate of 60° an hour, or 1° per minute. The sun would appear to move over a space equal to twice its own diameter each minute, and this motion would be distinctly perceived.

The fact that the motion of the hands of a clock is not perceived is explained in the same manner.

1152. *Why objects in extremely rapid motion are not perceivable.* — But if the object which thus moves be not sufficiently illuminated, or be not of a sufficiently bright colour to impress the retina sensibly, it will then, instead of appearing as a continuous line of colour, cease to be visible altogether; for it does not remain in any one position long enough to produce a sensible effect upon the retina. It is for this reason that a ball projected from a cannon or a musket, though passing before the eye, cannot be seen. If two railway trains pass each other with a certain velocity, a person looking out of the window of one of them will be unable to see the other. If the velocity be very moderate, and the light of the day sufficiently strong, the appearance of the passing train will be that of a flash of colour formed by the mixture of the prevailing colours of the vehicles composing it.

An expedient has already been described to show experimentally that the mixture of the seven prismatic colours, in their proper proportions, produce white light, depending on this principle. The colours are laid upon a circular disk surrounding its edge, which they divide into parts proportional to the spaces they occupy in the spectrum. When the disk is made to revolve, each colour produces, like the lighted stick, the impression of a continuous ring, and consequently the eye is sensible of seven rings of the several colours superposed one upon the other, which thus produce the effect of their combination, and appear as white or a whitish grey colour, as already explained.

1153. *The duration of the impression on the retina varies with the brightness of the object.* — The duration of the impression upon the retina, after the object producing it is removed, varies according to the vividness of the light proceeding from the object, being longer according as the light is more intense. It was found that the light proceeding from a piece of coal in combustion moved in a circle at a distance of 165 feet, produced the impression of a continuous circle of light when it revolved at the rate of seven times per second. The inference from this would be that in that particular case the impression upon the retina was continued during the seventh part of a second after the removal of the object.

It is from the cause here indicated that forked lightning presents the appearance of a continuous line of light.

1154. *And also with its colour.* — The duration of the impression on the retina varies also with the colour of the light, that produced by a white object being most visible, and yellow and red being most in degree of durability; the least durable being those tints which belong to the most refrangible lights.

1155. *And with the brightness of the surrounding space.* — The duration of the impression also depends on the state of illumination of the surrounding space; thus the impression produced by a luminous object when in a dark room is more durable than that which would be produced by the same object seen in an illuminated room. This may be ascribed to the greater sensitiveness of the retina when in a state of repose than when its entire surface is excited by surrounding lights. Thus it is found that while the varying duration of the impression of the illuminated object in a dark room was one-third of a second, its duration in a lighted room was only one-sixth of a second.

1156. *Optical toys — thaumatropes, phantascopes, &c.* — Innumerable optical toys and pyrotechnic apparatus owe their effect to this continuance of the impression upon the retina when the object has changed its position.

Amusing toys, called thaumatropes, phenakisticopes, phantascopes, &c., are explained upon this principle. A moving object, which assumes a succession of different positions in performing any action, is

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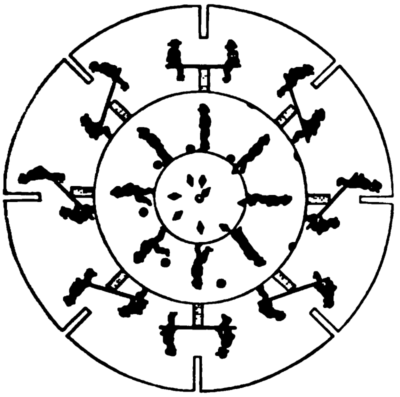

Fig. 371.

represented in the successive divisions of the circumference of a circle, as in *fig* 371., in the successive positions it assumes. These pictures, by causing the disk to revolve, are brought in rapid succession before an aperture, through which the eye is directed, so that the pictures representing the successive attitudes are brought one after another before the eye at intervals; the impression of one remaining until the impression of the next is produced. In this manner the eye never ceases to see the figure, but sees it in such a succession of attitudes as it would assume if it revolved. The effect is, that the figure actually appears to pirouette before the eye. The effects of catherine-wheels and rockets are explained in the same manner.

1157. *The direction in which objects are seen.*—The direction in which any part of an object is seen is that of the line drawn from such point through the optical centre of the eye. This line being carried back to the retina determines the place on the retina where the image of such point is found. If the optical centre of the eye were not at the centre of the eye-ball, the direction of this line would be changed with every movement of the eye-ball in its socket; every such movement would cause the optical centre to revolve round the centre of the eye-ball, and consequently would cause the line drawn from the optical centre to the object to change its direction. The effect of this would be that every movement of the eye-ball would cause an apparent movement of all visible objects. Now, since there is no apparent motion of this kind, and since the apparent position of external objects remains the same, however the eye may be moved in its socket, it follows that its optical centre must be at the centre of the eye-ball.

1158. *Why the motion of the eye-ball does not produce any apparent motion in the object seen.*—Since lines drawn from the various points of a visible object through the centre of the eye remain unchanged, however the eye-ball may move in its socket, and since the corresponding points of the image placed upon these lines must also remain unchanged, it follows that the position of the image formed on the eye remains fixed, even though the eye-ball revolve in the socket. It appears, therefore, that when the eye-ball is moved in the socket

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the picture of an external object remains fixed, while the retina moves under it just as the picture thrown by a magic lantern on a screen would remain fixed, however the screen itself might be moved.

Thus, if we direct the axis of the eye to the centre O, *fig.* 372., of any object, such as A B, the image of the point O will be formed at *o* on the retina, where the optical axis D C meets it. The axis of the pencil of rays which proceed from the point O will pass through the centre of the cornea D, through the axis of the crystalline, and through the centre C of the eye-ball, and the image of O will be formed at *o*.

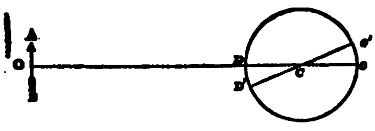

Fig. 372.

Now, if we suppose the eye to be turned a little to the left, so that the optical axis will be inclined to the line O C at the angle D' C O, the image of the point O will still hold the same absolute position *o* as before; but the point of the retina on which it was previously formed will be removed to *o'*. The direction of the point O will be the same as before; but the point of the retina on which its image will be formed will be, not at *o*, at the extremity of the optic axis, but at *o'*, at a distance *o o'* from it, which subtends at the centre C of the eye an angle equal to that through which the optical axis has been turned.

It is evident, therefore, that although the eye in this case be moved round its centre, the point O is still seen in the same direction as before.

But if the optical centre of the eye were different from the centre of the eye-ball, the direction in which the point O would be seen would be changed by a change of position of the eye.

To render this more clear, let C, *fig.* 373., be the centre of the eye-ball, and C' the optical centre of the eye. Let the optical axis C D, as before, be first presented to the point O of the object. The image of this point will, as before, be formed at *o*, the point where the optical axis D C meets the retina. Let us now suppose the axis of the eye to be turned aside through the angle D C D', the optical centre will then be removed from C' to C'', and the image of O will now be formed at the point *o''*, where the line O C'' meets the retina. The direction, therefore, in which O will now be seen, will be that of the line C'' O, whereas the direction in which it was before seen was that of the line C O.

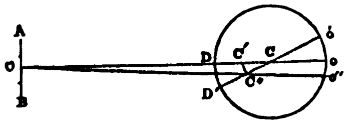

Fig. 373.

The point of the retina at which the image *o* was originally formed is removed to *o′*, while the image is removed to *o″*. Thus there is a displacement not only of the retina behind the image, but also an absolute displacement of the image, and an absolute change in the apparent direction of the object. Since no such change in the apparent direction is consequent upon the movement of the eye in its socket, it follows that the optical centre c′ of the eye must coincide with its geometrical centre c.

1159. *Ocular spectra and accidental colours.* — The continuance of the effect produced by the image of a visible object on the retina after such object has been removed from before the eye, combined with the effect of the image of another object placed before the eye, during such continuance of the effect of that which was removed, produces a class of phenomena called *ocular spectra* and *accidental colours.*

The effect produced by a strongly illuminated image formed on the retina does not appear to be merely the continuance of the same perception after the image is removed, but also a certain diminution or deadening of the sensibility of the membrane to other impressions. If the organ were merely affected by the continuance of the perception of the object for a certain time after its removal, the effect of the immediate perception of another object on the retina would be the perception of the mixture of two colours. Thus, if the eye, after contemplating a bright yellow object, were suddenly directed to a similar object of a light red colour, the effect ought to be the perception of an orange colour; and this perception would continue until the effect of the yellow object on the retina would cease, after which the red object alone would be perceived.

Thus, for example, a disk of white paper being placed upon a black ground, and over it a red wafer which will exactly cover it being laid, if, closing one eye, and gazing intently with the other for a few seconds on the red wafer, the red wafer be suddenly removed so as to expose the white surface under it to the eye, the effect ought to be the combination of the perception of red which continues after the removal of the red wafer, with the perception of white which the uncovered surface produces; and we should consequently expect to see a diluted red disk, similar to that which would be produced by the mixture of red with white.

This, however, is not the case. If the experiment be performed as here described, the eye will, on the removal of the red wafer, perceive, not a reddish, but a greenish-blue disk.

In like manner, if the wafer, instead of being red, were of a bright greenish-blue, when removed the impression on the eye would be that of a reddish disk.

These and like phenomena are explained as follows:—

When the eye is directed with an intensity of gaze for some time

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at the red surface, that part of the retina upon which the image of the red wafer is produced becomes fatigued with the action of the red light, and loses to some extent its sensibility to that light, exactly as the ear is deafened for a moment by an overpowering sound. When the red wafer is removed, the white disk beneath it transmits to the eye the white light, which is composed of all the colours of the spectrum. But the eye, from the previous action of the red light, is comparatively insensible to those tints which form the red end of the spectrum, such as red and orange, but comparatively sensitive to the blues and greens, which occupy the other end. It is therefore that the eye perceives the white disk as if it were a greenish-blue, and continues to perceive it until the retina recovers its sensibility for red light.

1160. ***Experiments of Sir D. Brewster on ocular spectra.***—The experiment above described may be varied by using wafers of various colours; and it will in each case be found that on the removal of the wafer the accidental colour or ocular spectrum produced will be that which is given in the second column of the following table, supplied by the observations of Sir David Brewster:—

Colour of the Wafer.	Accidental Colour, or Colour of the Ocular Spectra.
Red	Bluish-green.
Orange	Blue.
Yellow	Indigo.
Green	Violet, reddish.
Blue	Orange red.
Indigo	Orange yellow.
Violet	Yellow green.
Black	White.
White	Black.

It follows, therefore, from the results in the above table, that the primitive and accidental colours are so related to each other, that if the former be reduced to the same degree of intensity as the latter, me will be the complementary colour of the other, or, which is the me, they will be so related that if mingled together they will pro- ce white light.

The experiment may be varied in the following manner:—

If a small particle of red fire be burned in a dark room, so as to luminate surrounding objects with an intense red light, and it be nguished, the eye will for a time see a green flam n flame will be visible whether the eye be open or

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If, on the other hand, a green fire be burned, it will be succeeded by the perception of a reddish light.

If the eye be directed intently upon the disk of the sun at rising or setting, when it is red, on closing the eyelids a green solar disk will be perceived.

1161. *Why visible objects do not appear inverted.* — A difficulty has been presented in the explanation of the functions of the eye to which, as it appears to me, undue weight has been given. It has been already explained, that the images of external objects which are depicted on the retina are inverted; and it has accordingly been asked why visible objects do not appear upside down. The answer to this appears to be extremely simple. Inversion is a relative term, which it is impossible to explain or even to conceive without reference to something which is not inverted. If we say that any object is inverted, the phrase ceases to have meaning unless some other object or objects are implied which are erect. If all objects whatever hold the same relative position, none can be properly said to be inverted; as the world turns upon its axis once in twenty-four hours, it is certain that the positions which all objects hold at any moment is inverted with respect to that which they held twelve hours before, and to that which they will hold twelve hours later; but the objects as they are contemplated are always erect. In fine, since all the images produced upon the retina hold with relation to each other the same position, none are inverted with respect to others; and as such images alone can be the objects of vision, no one object of vision can be inverted with respect to any other object of vision; and consequently, all being seen in the same position, that position is called the erect position.

1162. *The seat of vision.*—Physiologists are not agreed as to the manner in which the perception of a visible object is obtained from the image formed in the interior of the eye. It is certain, however, that this image is the cause of vision, or that the means whereby it is produced are also instrumental in producing the perception of sight. It may also be considered as established that the perception of a visible object is more or less distinct, according to the greater or less distinctness of the image. But it would be a great error to assume that this image on the retina is itself seen, for that would involve the supposition of a second eye, beyond the first, or within it, by which such image on the retina would be viewed. Now, no means of communicating between the image on the retina and the sensorium exist except the usual conduits of all sensation, the nerves.

It has been already explained that the optic nerve, after entering the eye at a point near the nose, spreads itself over the interior of the globe of the eye behind the vitreous humour, and that this retina or network is perfectly transparent, the coloured image being formed not properly upon it, but upon the black surface of the choroid coat behind it. Now, it has been maintained, that the functions of vision are performed by this nervous membrane in a manner analogous to that by which the sense of touch is affected by external objects. The membrane of the retina, it is supposed, touching the coloured image, and being in the highest degree sensitive to it, just as the hand is sensitive to an object which it touches, receives from the coloured image an action which, being continued to the brain, produces perception there in accordance with the form and colour of the image upon the choroid. According to this view of the functions of vision, the retina *feels*, as it were, the image on the choroid, and transmits to the sensorium the impression of its colour and figure in the same manner as the hand of a blind person would transmit to the sensorium the form of an object which it touches.

1163. *Light and colour acting directly on the retina produce no sensation.* — If this hypothesis be admitted, it would follow that the retina itself would be incapable of exciting the sense of sight by the mere action of light and colours upon it. This is verified by the fact that when the image produced within the eye is formed upon a point of the optic nerve which has not the choroid behind it, no perception is produced.

In order to prove this, let three wafers be applied in a horizontal line upon a vertical screen, each separated from the other by a distance of two feet. Let the screen be placed before the observer at a distance of about 15 feet, the wafers being on a level with the eye; and let the centre wafer be so placed that a line drawn from the right eye to it shall be perpendicular to the screen. Let the left eye be now closed, and let the right eye be directed to the extreme wafer on the left, but so that all three wafers may still be perceived. Let another person now slowly move the screen, so as to bring it nearer to the observer, maintaining, however, the middle wafer in the direction of the eye at C. It will be found that the screen being so moved to a distance of 10 feet from the eye, the middle wafer will appear to be suddenly extinguished, and the extreme wafers on the right and left will be seen.

1164. *The optic nerve is insensible where it does not cover the choroid.* — This remarkable phenomenon is explained by showing that in this particular position of the eye and the screen, the image of the middle wafer falls upon the base of the optic nerve where the choroid coat is not under it.

This will be rendered more intelligible by reference to *fig.* 374., where B is the middle wafer, A the left-hand, and C the right-hand wafer. The image of A is formed at *a*, to the right of the optic nerve; and the image of C is formed at *c*, to the left of that nerve. In both these positions the choroid coat is behind the retina. But the image of B is formed at *b*, directly upon the point where the optic nerve issues from the eye-ball, and where the choroid does not extend behind it.

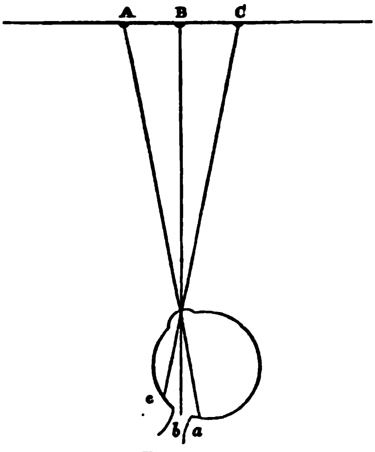

Fig. 374.

1165. *Experiment of Sir D. Brewster to confirm this.* — Sir David Brewster gives the following experiment as a further argument in support of this hypothesis. In the eye of the *Sepia loligo*, or cuttle-fish, an opaque membranous pigment is interposed between the retina and the vitreous humour, so that if the retina were essential to vision, the impression of the image on this black membrane must be conveyed to it by the vibration of this membrane in front of it. Sir David Brewster also mentions that in young persons the choroid coat, instead of being covered with a black pigment, reflects a brilliant crimson, like that of dogs and some other animals; and imagines that if the retina were affected by the rays which pass through it, this crimson light ought to excite a corresponding sensation, which is not the case.

1166. *Why objects are not seen double.* — The question why, having two eyes on which independent impressions are made by external objects, and on the retina of each of which an independent picture of a visible object is formed, we do not see two distinct objects corresponding to each individual external object which impresses the organ, is often asked.

The first reflection which arises on the proposition of this question, is why the same question has not been similarly proposed with reference to the sense of hearing. Why has it not been asked why we do not hear double? why each individual sound produced by a bell or a string is not heard as two distinct sounds, since it must impress independently and separately the two organs of hearing?

It cannot be denied, that, whatever reason there be for demanding a solution of the question, why we do not see double? is equally applicable to the solution of the analogous question, why we do not hear double? Like many disputed questions, this will be stripped of much of its difficulty and obscurity by a strict attention to the meaning of the terms used in the question, and in the discussion consequent upon it. If by seeing double it be meant that the two eyes receive separate and independent impressions from each external object, then it is true that we see double. But if it be meant that the mind receives two distinct and independent impressions of the same external object, then a qualified answer only can be given.

If the two eyes convey to the mind precisely the same impression of the same external object, differing in no respect whatever, then they will produce in the mind precisely the same perception of the object; and as it is impossible to imagine two perceptions to exist in the mind of the same external object which are precisely the same in all respects, it would involve a contradiction in terms to suppose that, in such case, we perceive the object double.

If to perceive the object double mean anything, it means that the mind has two perceptions of the same object, distinct and different from each other in some respect. Now, if this distinctness or difference exist in the mind, a corresponding distinctness and difference must exist in the impression produced of the external object on the organs. It will presently appear, that cases do occur in which the organs are, in fact, differently impressed by the same external object; and it will also appear, that in such cases precisely we *do see double*, meaning by these terms, that we have two perceptions of the same object, as distinct from each other as are our perceptions of two different objects.

To render this point more clear, let us consider in what respects it is possible for the impressions made upon the two eyes by the same object to differ from each other.

A visible object impresses the eye with a sense of a certain apparent form, of a certain apparent magnitude, of certain colours, of a certain intensity of illumination, and of a certain visible direction. Now, if the impression produced by the same object upon the two eyes agree in all these respects, it is impossible to imagine that the mind can receive two distinct perceptions of the object, for it is not possible that the two perceptions could differ from each other in any respect, except in some of those just mentioned. Let us suppose the two eyes to look at the moon, and that such object impresses them with an image of precisely the same apparent form and magnitude, of precisely the same colours and lineaments, of precisely the same intensity of illumination, and, in fine, in precisely the same direction. Now, the impressions conveyed to the mind by each of the eyes corresponding in all these respects, the object must be perceived in virtue of both impressions precisely in the same manner, that is to say, it must be seen in precisely the same direction, of precisely the same magnitude, of precisely the same form, with precisely the same lineaments of light and shade, and with precisely the same brightness or intensity of illumination. It is therefore, in such a case, clearly impossible to have a double perception of the object.

It will be observed, that the same reasoning exactly will be applicable to the sense of hearing. If the same string or the same pipe affect the membrane of each ear-drum in precisely the same manner, so as to produce a perception of a sound of the same pitch, the same loudness, and the same quality, it is impossible to conceive that two different perceptions can be produced by the two ears, for there is no respect in which it is possible for two such perceptions to differ, inasmuch as by the very supposition they agree in all the qualities which belong to sound.

But, if we would conceive by any organic derangement that the same musical string would produce in one ear the note *ut*, and produce in the other ear the note *sol*, then the same effect would be produced as if these two sounds had been simultaneously heard by the two ears properly organized, and we should have a sense of harmony of the *fifth*.

In like manner, if the two eyes, by any defect of organization, produced different pictures on the retina, we should then have two perceptions of the same object having a corresponding difference

It has been already shown, that the apparent visual magnitude of an object, and also that its apparent brilliancy, depend on its distance from the eye.

Now, assuming, as we shall do, unless the contrary be expressed, that the two eyes are similarly constituted, it will follow, that an object whose distance from the two eyes is equal will be seen under the same visual angle, and will therefore have the same apparent magnitude; it will also have the same colour and intensity of illumination, and, in fine, if the distance between the eyes bear an insignificant proportion to the distance of the object from them, the lines drawn from the centre of the eyes to any point on the object will be practically parallel; and since these lines, as has been already explained, determine the direction in which the object is seen, such object will then be seen in the same direction. Now, since the apparent form, the apparent magnitude, the apparent colour, the apparent intensity of illumination, and, in fine, the apparent direction are the same for both eyes, it is clear that the same impression must be produced upon the senses, and the same perceptions conveyed to the mind; consequently it follows, demonstratively, that all objects which are placed at a distance compared with which the distance between the eyes is insignificant, will convey a single perception to the mind, and will consequently not be seen double.

1167. *Exceptional cases in which objects are seen double.* — But we have now to consider a different case, which will present peculiar conditions, and consequences of peculiar interest.

Let us suppose an object placed so near the eyes that its distance shall not bear a considerable proportion to the length of the line which separates the centres of the eyes. In this case, the images produced on the retina of the two eyes may differ in magnitude, and intensity of illumination, and even in form, and, in fine, it is clear that the

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apparent direction of any point on the object as seen by the two eyes will be sensibly different.

In this case, therefore, the two eyes convey to the mind a different impression of the same object; and we may therefore expect that we should see it double, and in fact we do so.

But the observation of this particular phenomenon requires much attention, inasmuch as the perception of which we are conscious is affected not merely by the impression made upon the organ of sense, but by the degree of attention which the mind gives to it. Thus, if the two eyes be differently impressed either by the same or by different objects placed before them, the mind may give its attention so exclusively to either impression, as to lose all consciousness of the other.

Thus, if two stars be at the same time in the field of view of a telescope, as frequently happens, and be viewed together by the eye, we shall be conscious of a perception of both, so long as the attention is not exclusively directed to either; but if we gaze intently on one of them so as to observe its colour, or any other peculiarity attending it, we shall cease to be conscious of the presence of the other. The application of this observation to the question before us will be presently apparent.

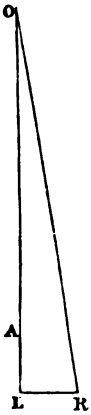

Fig. 375.

Let R L, *fig.* 375., be the line separating the centres of the two eyes, R representing the centre of the right, and L that of the left eye.

Let O be an object, such as the flame of a candle or lamp, seen at the distance of about 40 feet, so that the lines of direction L O and R O converging upon it from the centres of the eyes may be regarded as practically parallel, the distance being about 200 times greater than the distance L R from the eyes. The object O will therefore be seen in the same direction by both eyes, and being at a distance from the two eyes practically equal, will have the same apparent magnitude, form, colour, and intensity of illumination, and, consequently, will be seen single.

Let a small white rod be held at the distance A, of about 8 inches from the left eye L, and in the line L O, so as to intercept the view of the object O from the left eye. The left eye will then see the rod at A, and not the object O; but the right eye will still see the object O, as before. Now, if the attention be earnestly directed to the object O, the object A will not be perceived; but if the attention be directed to the object A, it will be perceived distinctly, but the object O will be seen through it as if it were transparent.

Now, since the object O cannot be seen by the left eye under the circumstances here supposed, the perception we have of it must be derived from the right eye; nevertheless it is seen in the line L A O,

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immediately beyond the intercepting wand, and in the same direction, and in the same manner precisely as it would be seen by the left eye L if the intercepting wand were removed. It follows, therefore, that the perception we obtain of the object O by the right eye is precisely the same as that which we should obtain by the left eye if the right were closed, and the intercepting wand A removed. This may therefore be taken as an experimental proof of what, indeed, may seem sufficiently evident, *à priori*, that an object, such as O, placed at a distance so great that lines drawn to it from the centre of the eyes would be practically parallel, produces precisely the same perception through the vision of both eyes.

But when the distance of an object from the eyes is so small that the line which separates the eyes bears a considerable proportion to it, the directions in which such an object is seen by the two eyes are different, and it is easy to show that in this case such an object would be seen double.

Let L and R, *fig.* 376., as before, be the centres of the two eyes, and let A B be a white screen placed vertically at a distance of 12 or 14 feet, having upon it a horizontal line on a level with the eye, upon which is marked a divided scale, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Let a black wand be held vertically at O, opposite the middle of the line L R. This wand will be seen by the left eye in the direction of the division 8, and by the right eye in the direction of the division 4, on the screen, and two images of the wand will accordingly be perceived; but, according as the attention is directed to the one or to the other, a consciousness of them will be produced. Thus, by an act of the will we may contemplate only the objects as seen with the left eye, in which case the wand will be seen projected on the screen perpendicular to the line A B, at the 8th division; and by a like act of the will, the attention being directed to the impression produced by the right eye, the wand will be seen projected on the screen at the 4th division of the scale. If the attention be withdrawn from either of these, and the wand be viewed indifferently, we shall be conscious of the two images, but not with the same distinctness as that with which we should perceive two wands placed at the 4th and 8th divisions of the scale. It will follow from this, that when we look with both eyes at any object, such as the printed page of a book, at the distance of

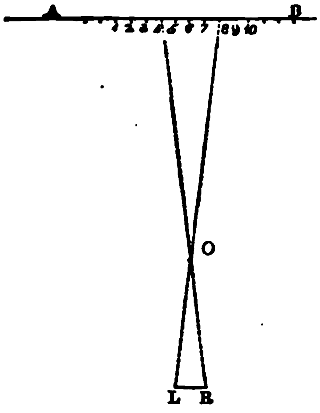

Fig. 376.

8 or 10 inches from the eyes, we have two images of the different parts of the page placed before the eyes, which are seen in different directions, and ought therefore to produce double vision; but this is prevented by habitually directing our attention to one of the two, and neglecting the other.

That the perception of an object will be double if the directions in which it is seen by the two eyes are different, may also be demonstrated in the following manner:—

It has been already shown that the optical centres of the eyes cannot change their position by the mere action of the muscles which move the eye-balls in their sockets, and that the direction in which any distant object is seen by both eyes is the same, and hence it is perceived single; but if a slight pressure be applied to the eye with the finger, the optical centre of the eye may be moved from its position, so that the direction of the same object seen by it and by the other eye will not be the same. A distant object will in this case be seen double, being perceived in one direction by the eye which retains its natural position, and in another by that whose position is deranged by pressure.

1168. *The eye supplies no direct perception of magnitude, figure, or distance.* — It has been already explained that two similar objects whose distances from the eye are to each other in the same proportion as their linear dimensions will have the same apparent magnitude.

In like manner, if an object, such as, for example, a balloon, moves from the eye in a direct line, we have no distinct consciousness of its motion, for the line of direction in which it is seen is still the same. It is true that we may infer its motion through the air by the increase or diminution of its apparent magnitude; for, if we have reason to know that its real magnitude remains unchanged, we ascribe almost intuitively the change of its apparent magnitude to the change of its distance; and we consequently *infer* that it is in motion either towards or from us, according as we perceive its apparent magnitude to be increased or diminished. This information, however, as to the motion of a body in a direct line to or from the centre of the eye, is not a perception obtained directly from vision, but an inference of the reason deduced from certain phenomena. It may therefore be stated generally, that the eye affords no perception of direct distance, and consequently none of direct motion, the term direct being understood here to express a motion in a straight line to or from the optical centre of the eye.

1169. *Manner of estimating the real distance.*—The distance of a visible object is often estimated by comparing it with the apparent magnitude and apparent distance of known objects which intervene between it and the eye.

Thus, the steeple of a church whose real height is unknown cannot *by mere* vision be estimated either as to distance or magnitude, since the apparent height would be the same, provided its magnitude were greater or less in proportion to its supposed distance. But, if between the steeple and the eye there intervene buildings, trees, or other objects, whose average magnitudes may be estimated, a proximate estimate of the magnitude and distance of the steeple may be obtained.

For example, if the height of the most distant building between the eye and the steeple be known, the distance of that building may be estimated by its apparent magnitude, and the distance of the steeple will be inferred to be greater than this.

1170. *Appearance of the sun and moon when rising or setting.*— A remarkably deceptive impression, depending on this principle, is deserving of mention here. When the disk of the sun or moon at rising or setting nearly touches the horizon, it appears of enormous magnitude compared with its apparent size when high in the firmament. Now, if the visual angle which it subtends be actually measured in this case, it will be found to be of the same magnitude. How, then, it may be asked, does it happen that the apparent magnitude of the sun at setting and at noon are by measure the same, when they are by estimation, and by the irresistible evidence of sense, so extremely different? This is explained, not by an error of the sense, for there is none, but by an erroneous application of those means of judging or estimating distance which in ordinary cases supply true and just conclusions.

When the disk of the sun is near the horizon, a number of intervening objects of known magnitude and known relative distances supply the means of spacing and measuring a part at least of the distance between the eye and the sun; but when the sun is in the meridian, no such objects intervene. The mind, therefore, assigns a greater magnitude to the distance, a part of which it has the means of measuring, than to the distance no part of which it can measure; and accordingly an impression is produced, that the sun at setting is at a much greater real distance than the sun in the meridian; and since its apparent magnitude in both cases is the same, its real magnitude must be just so much greater as its estimated distance is greater. The judgment, therefore, and not the eye, assigns this erroneous magnitude to the disk of the sun.

It is true that we are not conscious of this mental operation. But this unconsciousness is explained by the effect of habit, which causes innumerable other operations of the reason to pass unobserved.

1171. *Method of estimating by sight the magnitude of distan objects.* — As the eye forms no immediate perception of distance, neither does it of form or of magnitude, since, as has been already proved, objects of very different real magnitudes have the same apparent magnitude to the eye, of which a striking example is afforded in the case of the sun and moon. Nevertheless, although the eye supplies no immediate perception of the real magnitude of objects, habit and experience enable us to form estimates more or less exact of these magnitudes by the comparison of different effects produced by sight and touch.

Thus, for example, if two objects be seen at the same distance from the eye, the real magnitude of one of which is known, that of the other can be immediately inferred, since, in this case, the apparent magnitudes will be proportional to the real magnitudes. Thus, for example, if we see the figure of a man standing beside a tree, we form an estimate of the height of the latter, that of the former being known or assumed. Ascribing to the individual seen near the tree the average height of the human figure, and comparing the apparent height of the tree with his apparent height, we form an estimate of the height of the tree.

1172. *Singular illusion produced in St. Peter's at Rome.* It is by this kind of inference that buildings constructed upon a scale greatly exceeding common dimensions are estimated, and rendered apparent in pictorial representations of them.

On entering, for example, the aisle of St. Peter's at Rome, or St. Paul's at London, we are not immediately conscious of the vastness of the scale of these structures; but if we happen to see at a distant part of the building a human figure, we immediately become conscious of the scale of the structure, for the known dimensions of this figure supply a modulus which the mind instantly applies to measure the dimensions of the whole. For this reason artists, when they represent these structures, never fail to introduce human figures in or near them.

1173. *Real magnitude may sometimes be inferred from apparent magnitude.* — It has been explained that the apparent magnitude of objects depends conjointly on their real magnitude and their distance. Although, therefore, the eye does not afford any direct perception either of real magnitude or distance, we are by habit enabled to infer one of these from the other.

Thus, if we happen to know the real magnitude of a visible object, we form an estimate of its distance from its apparent magnitude; and, on the other hand, if we happen to know or can ascertain the distance of an object, we immediately form some estimate of its real magnitude.

Thus, for example, the height of a human figure being known, if we observe its apparent visual magnitude to be extremely small, we know that it must be at a distance proportionally great. If we know that at 20 feet the figure of a man will have a certain apparent height, and that we find that his figure seen at a certain distance appears to have only one-fifth of this height, we infer that his distance must be about 100 feet.

In like manner, the real magnitude may be inferred from the apparent magnitude, provided the distance be known or can be ascertained. Thus, for example, in entering Switzerland by its northern frontier, we see in the distance, bounding the horizon, the line of the snowy Alps, and the first impression is that of disappointment, their apparent scale being greatly less than we expected; but when we are informed that their distance is sixty or eighty miles, our estimate is instantly corrected, and we become conscious that the real height of mountains which, seen at so great a distance, is what we observe it, must be proportionally vast.

1174. *The eye not perceiving direct distance can have no perception of any motion but angular motion of which it is the centre.* — When an object moves in any direction which is not in a straight line drawn to or from the centre of the eye, the direction in which it is seen continually changes, and the eye in this case supplies an immediate perception of its motion; but this perception can be easily shown to be one not entirely corresponding to the actual motion of the object, but merely to the continual change of direction which this motion produces in the line drawn from the object to the eye.

Thus, for example, if the eye be at E, *fig.* 377., any object which moves from A to B will cause the line of direction in which it is seen to revolve through the angle A E B, just as though the body which moves were to describe a circular arc, of which E is the centre and E A the radius. But if, instead of moving from A to B, the body were to move from A′ to B′, the impression which its motion would produce upon the sight would be exactly the same. It would still appear to be moving from the direction E A′ A to the direction E B B′.

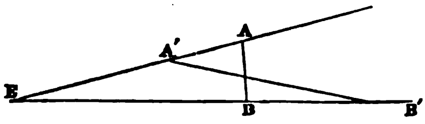

Fig. 377.

In fine, the eye affording no perception of direct distance, supplies no evidence of the extent to which the body may change its distance from the eye during its motion, and the apparent motion will be the same as if the body in motion described a circle of which the eye is the centre.

Hence it is that the only motion of which the eye forms any immediate apprehension is angular motion, that is, a motion which is measured by the angle which a line describes, one extremity of which is at the centre of the eye, and the other at the moving object.

1175. *Real direction of motion may be inferred by comparing apparent motion with apparent magnitude.* — Though the real direction in which a distant object moves cannot be obtained by the direct perception of vision, some estimate of it may be formed by comparing the apparent angular motion of the object with its apparent magnitude.

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Thus, for example, if we observe that the apparent magnitude of an object remains constantly the same while it has a certain apparent angular motion, we infer that its distance must necessarily remain the same, and consequently that it revolves in a circle, in the centre of which the observer is placed; or if we find that it has an angular motion, in virtue of which it changes its direction successively around us, so as to make a complete circuit of 360°, and that in making this circuit its apparent magnitude first diminishes to a certain limit, and then augments until it attains a certain major limit from which it again diminishes, we infer that such a body revolves round us at a varying distance, its distance being greatest when the apparent magnitude is least, and least when its apparent magnitude is greatest. An exact observation of the variation of the apparent magnitude would in such a case supply a corresponding estimate of the variation of the real distance, and would thus form the means of ascertaining the path in which the body moves.

1176. *Examples of the sun and moon.* — An example of this is presented in the cases of the sun and moon, whose apparent magnitudes are subject, during their revolution round the earth, to a slight variation, being a minimum at one point and a maximum at the extreme opposite point, the variation being such as to show that their motions are made in an ellipse in the focus of which the earth is placed.

1177. *How the apparent motion of an object is affected by the motion of the observer.* — As the eye perceives the motion of an object only by the change in the direction of the line joining the object with the eye, and as this change of direction may be produced as well by the motion of the observer as by that of the object, we find accordingly that apparent motions are produced sometimes in this manner. Thus, if a person be placed in the cabin of a boat which is moved upon a river or canal with a motion of which the observer is not conscious, the banks and all objects upon them appear to him to move in a contrary direction. In this case the line drawn from the object to the eye is not moved at the end connected with the object, which it would be if the object itself were in motion, but at the end connected with the eye. The change of its direction, however, is the same as if the end connected with the object had a motion in a contrary direction, the end connected with the eye being at rest; consequently, the apparent motion of the objects seen which are really at rest, is in a direction contrary to the real motion of the observer.

1178. *Example of railway trains.*—In some cases the apparent motion of an object is produced by a combination of a real motion in the object and a real motion in the observer. Thus, if a person transported in a railway carriage meet a train coming in the opposite direction, both extremes of the line joining his eye with the train which passes him are in motion in contrary directions; that extremity which is at his eye is moved

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by the motion of the train which carries him, and the other extremity is moved by the motion of the train which passes him. The change of direction of the line is accordingly produced by the sum of these motions; and as this change of direction is imputed by the sense to the train which passes, this train appears to move with the sum of the velocities of the two trains. Thus, if one train be moved at twenty miles an hour, while the other is moved at twenty-five miles an hour, the apparent motion of the passing train will be the same as would be the motion of a train moved at forty-five miles an hour passing a train at rest.

1179. *Compounded effects of the motion of the observer and of the object observed.* — If the line joining a visible object with the eye be moved at both its extremities in the same direction, which would be the case if the observer and the object were carried in parallel lines, then the change of direction which the line of motion would undergo would arise from the difference of the velocities of the observer and of the object seen.

If the observer in this case moved slower than the object, the extremity of the line of motion connected with the object would be carried forward faster than the extremity connected with the observer, and the object would appear to move in the direction of the observer's motion, with a velocity equal to the difference; but if, on the contrary, the velocity of the observer were greater than that of the object, the extremity of the line connected with the observer would be carried forward faster than that connected with the object, and the change of direction would be the same as if the object were moved in a contrary direction with the difference of the velocities.

It is easy to perceive that a vast variety of complicated relations which may exist between the directions and motions of the observer and of the object observed, will give rise to very complicated phenomena of apparent motion. Thus, relations may be imagined between the motion of the observer and that of the object perceived, by which, though both are in motion, the object will appear stationary; the motion of the one affecting the line of direction in an equal and contrary manner to that with which it is affected by the other; and, in the same manner, either motion may prevail over the other more or less, so as to give the line of direction a motion in accordance with or contrary to the real motion of the object.

1180. *Examples of the planetary motions.* — All these complicated phenomena of vision are presented in the problems which arise on the deduction of the real motion of the bodies composing the solar system from their apparent motions. The observer placed in the middle of this system is transported upon the earth in virtue of its annual motion round the sun with a prodigious velocity, the direction of his motion changing from day to day according to the curvature of the orbit. The bodies which he observes are also affected with various motions at various distances around the sun, the combination of which with the motion of the earth gives rise to complicated phenomena, the analysis of which is made upon the principles here explained.

1181. *Angular or visual distances.* — It is usual to express the relative position in which objects are seen by the relative direction of lines drawn to them from the eye; and the angle contained by any two such lines is called the angular or visual distance between the objects. Thus, the angular distance between the objects A and B, *fig.* 377., is expressed by the magnitude of the angle A E B. If this angle be 30°, the objects are said to be 30° asunder. It is evident from this that all objects which lie in the direction of the same lines will be at the same angular distance asunder, however different their real distance from each other may be. Thus, the angular distance between A and B, *fig.* 377., is the same as the angular distance between A′ and B′.

1182. *Vision affords no direct perception of bulk or form.—How such qualities are inferred.*—Sight does not afford any immediate perception either of the volume or shape of an object. The information which we derive from the sense, of the bulk or figure of distant objects, is obtained by the comparison of different impressions made upon the sense of sight by the same object at different times and in different positions. A body of the spherical form seen at a distance appears to the eye as a flat circular disk, and would never be known to have any other form, unless the impression made upon the eye were combined with other knowledge, derived from other impressions through sight or touch, or both these senses, and thus supplied the understanding with data from which the real figure of the object could be inferred. The sun appears to the eye as a flat, circular disk; but, by comparing observations made upon it at different times, it is ascertained that it revolves round one of its diameters in a certain time, presenting itself under aspects infinitely varying to the observer; and this fact, combined with its invariable appearance as a circular disk, proves it to be a sphere; for no body except a sphere, viewed in every direction, would appear circular.

Although we do not obtain from the sense of sight a perception of the shape of a body, we may obtain a perception of the shape of one of its sections. Thus, if a section of the body be made by a plane passing through it at right angles to the line of vision, the sight supplies a distinct perception of the shape of such section. Thus, if an egg were presented to the eye with its length in the direction of the line of vision, it would appear circular, because a section of it made by a plane at right angles to its length is a circle; but if it were presented to the eye with its length at right angles to the line of vision, it would appear oval, that being the shape of a section made by a plane passing through its length.

If a body, therefore, presents itself successively to the eye in several different positions, we obtain a knowledge by the sense of sight of so many different sections of it, and the combination of these sections may in many cases supply the reason with data by which the exact figure of the body may be known.

1183. *Visible area.*—As the term "apparent magnitude" is used to express the visual angle under which an object is seen, we shall adopt the term *visible area* to express the apparent magnitude of the section of a visible object made by a plane at right angles to the line of vision, that is to say, to the line drawn from the eye to the centre of the object.

1184. *How the shape is inferred from lights and shades.*—Besides receiving through the sight a perception of the figure of the section of the object which forms its visible area, we also obtain a perception of the lights and shades and the various tints of colour which mark and characterize such area. By comparing the perception derived from the sense of touch with those lights and shades, we are enabled by experience and long habit to judge of the figure of the object from these lights and shades and tints of colour. It is true that we are not conscious of this act of the understanding in inferring shape from colour and from light and shade; but the act is nevertheless performed by the mind. The first experience of inference is the comparison of the impressions of sight with the impressions of touch; and one of the earliest acts of the mind is the inference of the one from the other. It is the character of all mental acts, that their frequent performance produces an unconsciousness of them; and hence it is that when we look at a cube or a sphere of a uniform colour, although the impression upon the sense of sight is that of a flat plane variously shaded, and having a certain outline, the mind instantly substitutes the thing signified for the sign, the cause for the effect; and the conclusion of the judgment, that the object before us is a sphere or a cube of uniform colour, and not, as it appears, a flat plane variously shaded, is so instantaneous, that the act of the mind passes unobserved.

The whole art of the painter consists in an intimate practical knowledge of the relation between these two effects of perception of sight and touch. The more accurately he is able to delineate upon a flat surface those varieties of light and shade which visible objects immediately produce upon the sense, the more exact will be his delineation, and the greater the *vraisemblance* of his picture.

What is called relief in painting, is nothing more than the exact representation on a flat surface of the varieties of light and shade produced by a body of determinate figure upon the eye; and it is accordingly found that the flat surface variously shaded, produced by the art of the painter, has upon the eye exactly the same effect as the object itself, which is in reality so different from the coloured canvass which represents it.

1185. *Power of perceiving and distinguishing colours improved by exercise and experience.*—The immediate impressions received from the sense of sight are those of light and colour. The impression of distance, magnitude, form, and motion, are the mixed results of the sense of sight and the experience of touch. Even the power of distinguishing colours is not obtained immediately by vision without some cultivation of this sense. The unpractised eye of the new-born infant obtains a general perception of light; and it is certain that the power of distinguishing colours is only found after the organ has been more or less exercised by the varied impressions produced by different lights upon it. It would not be easy to obtain a summary demonstration of this proposition from the experience of infancy, but sufficient evidence to establish it is supplied by the cases in which sight has been suddenly restored to adults blind from their birth. In these cases, the first impression produced by vision is that the objects seen are in immediate contact with the eye. It is not until the hand is stretched forth to ascertain the absence of the objects seen from the space before the eye that this optical fallacy is dissipated.

The eye which has recently gained the power of vision at first cannot distinguish one colour from another, and it is not until time has been given for experience, that either colour or outline is perceived.

1186. *Of certain defects in vision.*—Besides that imperfection incident to the organs of sight arising from the excess or deficiency of their refractive powers, there is another class which appear to depend upon the quality of the humours through which the light, proceeding from visible objects, passes before attaining the retina. It is evident that if these humours be not absolutely transparent and colourless, the image on the retina, though it may correspond in form and outline with the object, will not correspond in colour; for if the humours be not colourless, some constituents of the light proceeding from the object will be intercepted before reaching the retina, and the picture on the retina will accordingly be deprived of the colours thus intercepted. If, for example, the humours of the eye were so constituted as to intercept all the red and orange rays of white light, white paper, or any other white object, such as the sun, for example, would appear of a bluish-green colour; and if, on the other hand, the humours were so constituted as to intercept the blues and violets of white light, all white objects would appear to have a reddish hue. Such defects in the humours of the eye are fortunately rare, but not unprecedented.

1187. *Curious examples of defective eyes.* — Sir David Brewster, who has curiously examined and collected together cases of this kind, gives the following examples of these defects :—

A singular affection of the retina in reference to colour is shown in the inability of some eyes to distinguish certain colours of the spectrum. The persons who experience this defect have their eyes generally in a sound state, and are capable of performing all the most delicate functions of vision. M. Harris, a shoemaker at Allonby, was unable from his infancy to distinguish the cherries of a cherry-tree from its leaves, in so far as colour was concerned. Two of his brothers were equally defective in this respect, and always mistook orange for grass-green, and light green for yellow. Harris himself could only distinguish black and white. Mr. Scott, who describes his own case in the Philosophical Transactions, mistook pink for a pale blue, and a full red for a full green.

All kinds of yellows and blues, except sky-blue, he could discern with great nicety. His father, his maternal uncle, one of his sisters and her two sons, had all the same defect.

A tailor at Plymouth, whose case is described by Mr. Harvey, regarded the solar spectrum as consisting only of yellow and light blue; and he could distinguish with certainty only yellow, white, and green. He regarded indigo and Prussian blue as black.

M. R. Tucker described the colours of the spectrum as follows:—

Red mistaken for	brown.
Orange "	green.
Yellow sometimes	orange.
Green "	orange.
Blue "	pink.
Indigo "	purple.
Violet "	purple.

A gentleman in the prime of life, whose case I had occasion to examine, saw only two colours in the spectrum, viz. yellow and blue. When the middle of the red space was absorbed by a blue grass, he saw the black space with what he called the yellow on each side of it. This defect in the perception of colour was experienced by the late Mr. Dugald Stewart, who could not perceive any difference in the colour of the scarlet fruit of the Siberian crab, and that of its leaves. Dr. Dalton was unable to distinguish blue from pink by daylight; and in the solar spectrum the red was scarcely visible, the rest of it appearing to consist of two colours. M. Troughton had the same defect, and was capable of fully appreciating only blue and yellow colours; and when he named colours, the names of blue and yellow corresponded to the more and less refrangible rays; all those which belong to the former exciting the sensation of blueness, and those which belong to the latter the sensation of yellowness.

In almost all these cases, the different prismatic colours had the power of exciting the sensation of light, and giving a distinct vision of objects, excepting in the case of Dr. Dalton, who was said to be scarcely able to see the red extremity of the spectrum.

Dr. Dalton endeavoured to explain this peculiarity of vision, by supposing that in his own case the vitreous humour was blue, and therefore absorbed a great portion of the red and other least refrangible rays; but this opinion is, we think, not well founded. Sir J. Herschel attributes this state of vision to a defect in the sensorium, by which it is rendered incapable of appreciating exactly those differences between rays on which their colour depends.

CHAP. XV.

OPTICAL INSTRUMENTS.

[illegible] *Spectacles.* — These are the most simple and most useful class of optical instruments. They consist of two glass lenses mounted in a frame so as to be conveniently supported before the eyes, and to remedy the defects of vision of naturally imperfect eyes.

Whatever be the defects of sight which spectacles may be used to remedy, it is evident that the lenses ought to be so mounted that their axes shall be parallel, and that their centres shall coincide with the centres of the pupils when the optical axes are directed perpendicular to the general plane of the face, that is to say, when the eyes look straight forward.

These conditions, though important, are rarely attended to in the construction of spectacles. If spectacles be mounted in extremely light and flexible frames, the lenses almost invariably lose their parallelism, and their axes not only cease to be parallel, but are frequently in different planes. Spectacles ought therefore to be constructed with mountings sufficiently strong to prevent this derangement of the axes of the lenses, and in their original construction care should be taken that the axes of the lenses be truly parallel.

In the adaptation of spectacles it is necessary that the distance between the centres of the lenses should be precisely equal to the distance between the centres of the pupils. The clearest vision being obtained by looking through the centres of the lenses, the eyes have a constant tendency to look in that direction. Now, if the distance between the centres of the lenses be greater than the distance between the centres of the pupils, the eyes having a tendency to look through the centres of the lenses, their axes will cease to be parallel, and will diverge as in the case of an outsquint. On the other hand, if the distance between the centres of the lenses be less than the distance between the centres of the pupils, there will, for a like reason, be a tendency to produce an insquint.

It has been already shown that the pencils most free from aberration are those whose axes coincide with the axis of the lens, and the more the axes of secondary pencils deviate from this, the greater effects of aberration.

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It follows from this that the most perfect vision with spectacles is produced when the eye looks in the direction of the axis of the lenses, and that more or less imperfection attends oblique vision through them. Persons who use spectacles, therefore, generally turn the head, when those whose sight does not require such aid merely turn the eye.

1189. *Periscopic spectacles.* — To diminish this inconvenience, the late Dr. Wollaston suggested the use of meniscuses or concavo-convex lenses, instead of double concave or double convex lenses with equal radii, which had been invariably used.

For persons requiring convergent lenses he proposed meniscuses with the concave surface next the eye; and for persons requiring divergent glasses, he proposed the concavo-convex lens, with the concave side next the eye. The effect of this is that the secondary pencils have less aberration than in the case of double convex and double concave lenses; and, consequently, that there is a greater freedom of vision by turning the eye without turning the head, from which property they were named *periscopic spectacles.*

1190. *Weak sight and short sight.* — It has been already explained that the optical defects of the eye which are capable of being corrected by lenses placed before it, are either a deficiency or an excess of their refractive power. Eyes which are deficient in refractive power, and which are called *weak-sighted eyes,* are those which are not capable of converging the pencils proceeding from visible objects at the usual distances to a focus on the retina. Eyes, on the other hand, which have too great refractive power, bring the rays proceeding from visible objects to a focus before they come to the retina, and are called *short-sighted eyes,* because objects which are near them are distinctly visible without the interposition of lenses.

1191. *Spectacles for weak-sighted eyes.* — The convergent power of the lenses necessary for weak-sighted eyes will necessarily be determined by the degree of the deficiency which exists in the refractive power of the eye. If the eyes be capable of affording distinct vision of objects so distant that the rays proceeding from them may be regarded as parallel, they will be capable of refracting parallel rays to an exact focus on the retina; but if they are so feeble in their refractive power as to be incapable of converging rays in the slightest degree divergent to a focus, they will be incapable of seeing distinctly any objects whose distances from the eye are less than from two to three feet, because the rays composing the pencils from such objects have a divergence which, though slight, the eye is incapable of surmounting, and the pencils accordingly, after entering the eye, converge to a focus not on the retina, but behind it.

Hence we find that persons having feebly refracting eyes are obliged to remove a printed or written page to a considerable distance from the eye to be able to read it. The pencils are thus rendered parallel, and therefore such as the eye may bring to a focus on the retina, but this increase of distance from the eye is attended with the consequence of rendering the light proceeding from the object more feeble, and often too feeble to produce distinct vision. Hence we find that when weak-sighted persons hold a book or newspaper which they desire to read at a considerable distance from the eye, they are obliged at the same time to place a candle or lamp near the page to produce an illumination of the necessary intensity.

Since such eyes are, according to the supposition, adapted to the refraction of parallel rays, the lenses which they require must be such as to render the pencils proceeding from the objects at which they look parallel, and consequently they must be lenses *whose focal length is equal to the distance of the objects looked at.*

Nothing, therefore, can be more simple than the rule to be followed by such persons in the selection of spectacles. They have only to use for their spectacles lenses whose focal length is equal to the distance of the objects which they desire to see distinctly; and if they have occasion to look at objects at different distances, as, for example, at ten and at twenty inches, they ought to be provided with different pairs of spectacles for the purpose, one having a focal length of ten inches, and the other a focal length of twenty inches. When they look at an object at ten inches from the eye with spectacles of ten inches focal length, the rays will enter the eye exactly as they would if the object were at a distance of several feet from them; and those rays, being parallel, will be refracted to a focus on the retina.

It may be asked, in this case, how it happens that, if it be necessary for such persons to use spectacles having a focal length equal to the distance of the object at which they look, they can, nevertheless, see with the same spectacles distinctly objects at distances greater or less, within certain limits, than the focal distance of the spectacles? The answer is, that this arises from the power with which the eye is endued to adapt itself within certain limits to vision at different distances, as has been already explained.

1192. *How to determine the refracting power of weak-sighted eyes.* — If the weakness of the sight be such that the eye is incapable of bringing even parallel rays to a focus on the retina, it will be necessary to use convergent lenses even for the most distant objects. The power of the lenses which are necessary to render the vision of distant objects clear in that case will supply means of calculating the natural convergent power of the eye; for since the convergent power of the lens, together with the natural convergent power of the eye, bring parallel rays to a focus on the retina, the natural convergent power of the eye will be equal to the difference between the convergent power of the lens and the convergent power of an eye capable of bringing parallel rays to a focus on the retina.

To render this more clear let f be the focal length of a lens which is equivalent to the refracting power of an eye which would bring parallel rays to a focus on the retina. Let f' be the focal length of the lens which is sufficient to enable the defective eye to bring parallel rays to a focus on the retina; and let f'' be the focal length of a lens optically equivalent to the defective eye. We shall then have

$$\frac{1}{f'}+\frac{1}{f''}=\frac{1}{f};$$

consequently we shall have

$$\frac{1}{f''}=\frac{1}{f}-\frac{1}{f'}.$$

From this condition the focal length of the eye can be found, since its reciprocal is equal to the difference between the reciprocals of the focal length of an eye adapted to parallel rays, and the focal length of the lens which produces clear vision in the defective eye.

In the same case, spectacles of different convergent power will be necessary when near objects are viewed; for in this case the pencils, having more divergence, will require a more convergent lens to aid the eye in bringing them to a focus on the retina. Such eyes, therefore, will require spectacles of different powers for distant and near objects; and if the power of the eye in adapting itself to different distances be not great, it may even be advisable to provide different spectacles for near objects which differ in their distance, as already explained in the case of eyes adapted to the refraction of parallel rays.

1193. *Spectacles for near-sighted eyes.* — To determine the focal length of the lens which will enable near-sighted eyes to see distinctly distant objects, it is only necessary to ascertain the distance at which, without an effort, the same eyes can see objects distinctly. This distance determines the degree of divergence of the pencils which the eyes bring to a focus on the retina. If diverging lenses be applied before the eyes whose focal length is equal to this distance, such lenses will give to parallel rays proceeding from distant objects the same degree of divergence as pencils would naturally have proceeding from objects whose distance is equal to their focal length; consequently, according to the supposition, the eye will bring such rays to a focus on the retina. The lenses, therefore, which fulfil this condition, will render the vision of distant objects with such eyes as distinct as would be the vision of objects placed at a distance from the eyes equal to the focal length of the lenses.

If the excess of the refractive power of short-sighted eyes be so great, and the power of adaptation to varying distances so small, that the same divergent lenses which render distant objects distinct will not render objects which are near the eyes, but not near enough for distinct vision without spectacles, distinct, then lenses of less divergent power must be used to produce a distinct vision of such objects.

Thus, for example, suppose the case of eyes so near-sighted as to see distinctly objects only when they are at five inches distance. To enable these eyes to see an object at ten inches distance distinctly, it will be necessary to use divergent lenses; but these lenses must have less diverging power than those which render the vision of distant objects distinct, because the same lenses which would give the necessary divergence to the parallel rays which proceed from distant objects would give too great a divergence to the pencils which proceed from an object at ten inches distance.

1194. *Case in which the eyes of the same person have different refracting powers.* — In the selection and adaptation of spectacles, it is invariably assumed without question, that the two eyes in the same individual have exactly the same refracting power. That this is the case is evident, from the fact that the lenses provided in the same spectacles have invariably the same focal length.

Now, although it is generally true that the two eyes in the same individual have the same refractive power, it is not invariably so; and if it be not, it is evident that lenses of equal focal length cannot be at once adapted to both eyes.

When the difference of the refractive power of the two eyes is not great (which is generally the case when a difference exists at all), this inequality is not perceived. By an instinctive act of the mind of which we are unconscious, the perception obtained by the more perfect of the two eyes in case of inequality is that to which our attention is directed, the impression on the more defective eye not being perceived.

It might be expected, however, that the inequality would become apparent, by looking alternately at the same object with each of the eyes, closing the other; but it is so difficult to compare the powers of vision of the two eyes when they are not very unequal by objects contemplated at different times, even though they should be exhibited in immediate succession, that this method fails.

1195. *Apparatus for comparing the power of vision of the two eyes.* — My attention having been recently directed to this question, I have contrived an apparatus which may not inaptly be called an *Opthalmometer*, by which the least difference in the powers of the two eyes may be rendered immediately apparent.

The principle I have adopted for this purpose resembles that which has been otherwise applied with success in photometers. I have so arranged the apparatus, that two similar objects similarly illuminated shall be at the same time visible in immediate juxtaposition, the one by the right eye being invisible to the left eye, and the other by the left eye being invisible to the right eye.

This apparatus consists of a small box A B C D, *fig.* 378., about five

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inches in width A D, ten inches in length A B, and six inches in height. Within this there slides another box, A′ B′ C′ D′, made nearly to fit it, but to move freely within it, the interior of this box being blackened, or lined with black velvet. In the end B′ C′ is a rectangular aperture M N, the length of which M N is about an inch, and the height about half an inch; the length, however, being capable of being augmented and diminished by slides. Opposite to the end of the box B C is a white screen, on which is traced a horizontal line parallel and opposite to the opening M N, and marked with a divided scale, the 0 of which is opposite to the centre of the aperture M N, and the divisions upon which are numbered in each direction from 0 by 1, 2, 3, 4, 5, 6.

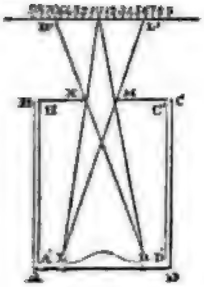

Fig. 378.

Let us suppose the eyes now applied at R and L. Let the sliding interior box B′ C′ be moved until, on closing the left eye, the division 0 of the scale coincides with the edge M of the opening, and at the same time, by closing the right eye, the same division 0 of the scale coincides with the edge N of the opening. It will be always possible to make this adjustment, provided the eyes are placed centrally opposite the opening M N, which may be easily managed by cutting in the edge of the box A D an opening to receive the bridge of the nose. This arrangement being made, it is clear that if we close the left eye we shall see the space upon the scale included by the lines R N and R M continued to the screen R′ L′. Let us suppose this space to include the six divisions of the scale from 0 to 6. If we close the right eye, we shall see with the left eye the six divisions of the scale to the right of 0. Now if we open both eyes and look steadily with them through the aperture M N, giving no more attention to the impression on the one than on the other, we shall see the twelve divisions of the scale, six to the right and six to the left of 0; the six divisions to the left of 0 being seen only with the right eye, and the six divisions to the right of 0 being seen only with the left eye.

In this way we have two similar objects, similarly illuminated and of equal magnitude, in immediate juxtaposition, the one seen by the right and the other by the left eye; and any difference in their distinctness, quality, brilliancy, or colour, will be as clearly and instantly perceivable as the comparative brilliancy of spaces illuminated by two different lights in the photometer. I have already experimented with this apparatus upon my own eyes, the result of which is, that I find that the sight of the right eye is much better than that of the left, the figures to the left of 0 being always more distinct than those to the right of it; but, what is more remarkable, I find that the transparency of the humours of the right eye is more perfect than that of the humours of the left eye, for the space to the right of 0 always appears less bright than the space to the left of it.

1196. *Method of adapting spectacles to eyes with unequal powers of vision.* — To apply this instrument for the purpose of adapting spectacle lenses to eyes of unequal powers of vision, it is necessary first to ascertain the existence of the inequality of power in the manner already explained. It would then be necessary to provide two distinct screens on which similar scales might be drawn, so that they might be placed at different distances from the aperture M N. Let their relative distances be then determined, so that the two eyes would see the scales with equal distinctness. These distances will then represent the focal lengths of the divergent lenses which it would be necessary to provide for the eyes, so as to make them see different objects with equal distinctness.

In the case of weak-sighted eyes, this method will not be applicable. In that case let the two screens be placed at equal distances from the aperture M N, and let lenses be selected for each eye separately, closing the other, so as to give a distinct perception of the scales. The two lenses being then simultaneously applied to the eyes, let the scale be viewed with both eyes open. If the lenses be adapted to correct the defect of vision, the two parts of the scale to the right and to the left of 0, seen at the same time by each eye alone, will appear of uniform brilliancy and distinctness.

If defective eyes were tested by this method, I believe it would be found that inequality of vision would be much more common than is generally supposed, and accordingly the adaptation of spectacles would be considerably improved.

1197. *Remarkable case of vision defective in different degrees in different directions.* — Cases occur not only in which the comparative powers of vision of the two eyes differ, but in which the power of vision, even of the same eye, is different when estimated in different directions.

I have known short-sighted persons who were more short-sighted for objects taken in a vertical than in a horizontal direction. Thus with them the height of an object would be more perceptible than its breadth, and in general vertical dimensions more clearly seen than horizonal. This difference arises from the refractive power of the eye taken in vertical planes being different from the refractive power taken in horizontal planes; and the defect is accordingly removed by the use of lenses whose curvatures, measured in their vertical direction, is different from their curvature, measured in their horizontal direction. The lenses, in fact, instead of having *spherical* surfaces, have *elliptical* surfaces, the eccentricities of which corresponded with the variation of the refractive power of the eye.

1198. *Camera lucida.*—This instrument, the invention of the late Dr Wollaston, has proved of great utility in the arts, presenting a remarkable facility for tracing a drawing of any distant objects, such as a building, a landscape, &c.

A quadrangular prism, *a b c d*, *fig.* 379, having a right angle at *b*, an angle of 135° at *d*, and angles at *a* and *c* equal to 67½°, is supported on a vertical pillar, with one side *a b* of its right angle horizontal, and the other *b c* vertical.

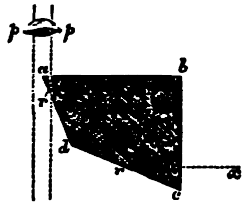

Fig. 379.

If an object be placed at a distance opposite *b c*, the rays proceeding from it will enter the prism in the direction *x r*, and will fall upon the surface *b c* so as to make the angle *x r c* equal to 22½°, and consequently the angle of incidence with the surface *d c* is 67½°. This angle being greater than the limit of transmission from glass into air, the light will be reflected from *r*, making the angle *d r r'* equal to the angle *c r x*; consequently, it will fall upon the surface *d a* at an angle of incidence of 67½°, and will therefore be again reflected from *r'*, making the angle *a r' p* equal to 22½°. The ray will thus fall upon the surface *a b* perpendicularly, and will pass through without further refraction. An eye placed at *p* would therefore see the object from which the original ray *x r* had proceeded in the direction *p r'*, and the same being true of all rays proceeding from the object, an image of the object will be seen by an eye presented downwards over the prism at *a*.

If a sheet of white paper be placed upon the table which supports the prism, an eye placed at *p* will see a picture of the object projected upon the paper; and if the eye be placed so close to the edge *a* of the prism that while it sees the picture projected upon the paper it also sees the paper directly, the observer will be able to trace with a pencil an outline corresponding with the picture, for while the picture is seen through the prism, the point of the pencil is directed upon the paper so seen directly outside the edge of the prism.

The use of this instrument requires some dexterity obtained by practice; but when the necessary skill is acquired, its use is simple and effectual.

1199. *Camera obscura.*—It has been already explained, that if an object be placed before a converging lens, at any distance greater than the focal length, a real image of the object will be formed on the other side of the lens, at the point corresponding to a position of the focus conjugate to the object. If the object, as is generally the case, be so distant that pencils of rays proceeding from it to the lens may be regarded as parallel, the lens will then form a picture of the object at a distance from it equal to its focal length. If a white screen be placed at right angles to the axis of the lens, and at the distance at which the image is formed, the image will be depicted upon it with its proper form and colours; and if arrangements were provided by which a draughtsman could have access to the image thus formed on the screen, he would be enabled to trace its outline.

To obtain the necessary convenience and facilities for accomplishing this, arrangements are necessary by which all other light should be excluded except that which forms the picture, and that the position of the screen or paper on which the picture is formed should be such as may be convenient for the operator, and, in fine, so that the person of the operator may not intercept the rays forming the picture. These objects are attained by different arrangements, one of the most simple of which is represented in *fig.* 380.

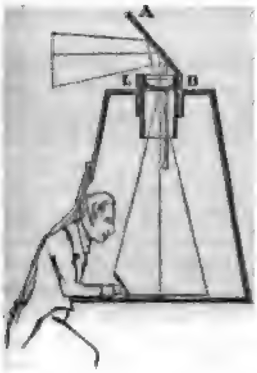

Fig. 380.

The lens L is placed in the centre of the top of a rectangular box, whose height corresponds with the focal length of the lens, whose bottom forms a desk upon which the draughtsman works, and in the side of which is an opening through which he may introduce his head and arms, over which a curtain is suspended, so that while it includes his person it may exclude the light. A plane reflector A B is placed above the lens, and is moveable on hinges. It is capable of being adjusted by a handle, which descends into the box, so that the operator may raise and lower it until the picture is thrown in a proper position. Means are also provided by which the lens L and the mirror A B can be moved round their centre so as to receive any required direction. The lens L is adjusted in a sliding tube, by which the focus can be brought exactly to correspond with the surface of the paper.

The oblique mirror A B and the lens L may be replaced by a prism with curved faces, such as that represented in *fig.* 381. The face of the prism *a c*, at which the rays first enter, is convex, by which the rays are made to converge; they then fall upon the plane side *a b*, by which they are reflected, and pass through the curved side *c b*, by which they are again refracted. The curvatures of the two sides *a c* and *c b* may be related in any required manner, so that their convergent powers may be equivalent to that of a lens of any proposed focal length.

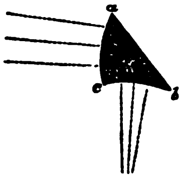

Fig. 381.

Strictly speaking, the picture of distant objects may be formed free from spherical aberration, if the surface of the paper form the surface of a sphere of which the optical centre of the lens is the centre. This is sometimes accomplished by throwing the picture upon a concave surface formed of plaster of Paris, whose centre corresponds with that of the lens.

The phenomena exhibited by this instrument are rendered especially pleasing, inasmuch as it exhibits not only a picture of the external scenery, but shows all the objects in motion upon it as in the real scene. Thus carriages, horses, and pedestrians, appear with their proper motion, the leaves tremble on the trees, and the smoke curls from the chimney.

The opening at which the observer is placed, ought, of course, to be at that side of the box at which the picture appears erect.

1200. ***The magic lantern.*** — The magic lantern is an instrument adapted for exhibiting, on an enlarged scale, pictures painted in transparent colours on glass, by means of magnifying lenses, by which the rays proceeding from the picture, after being transmitted, are brought to a focus at a distance from it upon a screen. The position of the screen and that of the picture are conjugate foci, and their linear dimensions are in the proportion of their distances from the lens. In proportion as the picture approaches the lens, the image formed on the screen recedes from it, and consequently becomes more magnified. These instruments vary in their form, according to the circumstances under which they are placed, and the cost expended on their construction.

They are usually arranged as represented in *fig.* 382. A lamp L is included in a dark lantern; behind it is placed a metallic reflector

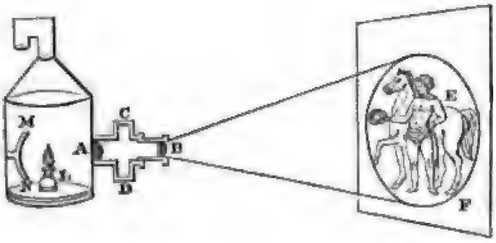

Fig. 382.

M N, and before it a large converging lens A, usually plano-convex. A groove is provided in the nozzle of the lantern, at C D, to receive the pictures, which are called sliders, in consequence of being successively passed in and out of the groove C D.

The colours in which they are painted and prepared are transparent gums, so that the light which passes through them may have corresponding colours. The magnifying lens B, which is also a convergent lens, is set in a tube, which slides in that which forms the fixed nozzle of the lantern, and can be moved to and from the picture with a motion like that of a telescope or opera-glass. The light proceeding directly from the lamp L, and that which is reflected by M N passing through the lens A, which is called the *illuminating lens*, is made to converge upon the picture placed in the groove C D, so as to produce an equal illumination of every part of it. The light which thus proceeds from the picture being received by the convergent lens B, is brought to a focus on a screen E F, the screen being placed at such a distance from B as to correspond with the focus conjugate to that of the picture. We are, therefore, to consider the slider as placed in the focus of incident rays, and the screen in the focus of refracted rays. If the tube B be pushed in so that the lens be brought nearer to the picture, the screen must be moved to a greater distance, since by what has been already established, according as the distance of the focus of incident rays from the lens B approaches to equality with its focal length, the conjugate focus must recede from it. The magnifying power of the lantern is measured by dividing the height of the picture formed on the screen by the height of the picture formed on the slider.

Thus, if the slide be two inches and a half wide, and the picture formed on the screen be twenty-five inches high, then the magnifying power will be ten. The picture may be either viewed by spectators placed before the screen or behind it. In the former case, the screen must be formed of some material which is not penetrable by the light; in the latter case, it must be semi-transparent.

The best surface for exhibiting such pictures to spectators placed in front of the screen, is white paper or pasteboard. To exhibit them to the spectator placed on the other side of the screen, it is usual to prepare the screen with fine wax, so as to stop all its pores, and prevent the direct transmission of light. In this case, the screen being semi-transparent, the picture formed on the side next the lantern is visible on the other side, just as it would be through a plate of ground glass.

Since, however, at least one half the light which falls on the screen is reflected from it in the direction of the lantern, the pictures thus formed are never so vivid as those which are produced upon a properly prepared opaque screen.

Since the light which forms the picture on the screen is in all cases that which proceeds from the picture on the slider, it is evident that the greater the magnifying power used, the less intense will be the brilliancy of the picture. Whether the picture on the screen be great or small, the same quantity of light will be diffused over it, being the light which proceeds from the picture on the slider. This light, therefore, will be less intense in the same proportion as the magnitude of the picture on the screen is increased. If, therefore, the picture on the screen be ten times the height of the picture on the slider, its brilliancy will be a hundred times less than the brilliancy of the picture on the slider.

1201. *Phantasmagoria.* — When the pictures produced by a magic lantern are shown through a transparent screen not visible to the spectator, they may be made to vary in magnitude, gradually increasing and gradually diminishing, by moving the light to or from the screen, and at the same time moving the lens B proportionally from or to the slider. In this case the effect produced on the spectators is that of an object approaching to or receding from them; the change of apparent magnitude which the picture undergoes being imputed to a change of distance in the object. When the picture diminishes, it is supposed to recede from the eye, and when it increases, it is supposed to approach the eye. Various effects of this kind, combined by means of different lanterns used at the same time, are called Phantasmagoria.

1202. *Dissolving views.* — The exhibition called Dissolving Views is produced by placing two lanterns of equal power so as to throw pictures of equal magnitude in the same position on the same screen.

A sliding shutter is placed upon the nozzle of each lantern, and the two shutters are moved simultaneously, in such a manner that when the nozzle of one lantern is open, that of the other is completely closed; and according as the nozzle of the former is gradually closed, that of the latter is gradually opened.

Let us suppose, then, that two slides are placed in the lanterns, one representing a landscape by day, and the other representing precisely the same landscape by night; and let the nozzle of that which contains the landscape by day be open, the other being closed, the picture on the screen will then represent the landscape by day. If the slides be now slowly moved, the nozzle of the lantern which shows the day landscape will begin gradually to close, and that which shows the night landscape will gradually open. The effect will be, that the daylight will gradually decline upon the picture, and the objects represented will assume by slow degrees the appearance of approaching night. This gradual change will go on until the nozzle of the lantern containing the day picture is completely closed and that containing the night picture completely open, when the change from day to night will have been completed.

Various other effects, familiar to those who have witnessed phantasmagoric exhibitions, are produced by combining two or more magic lanterns. Thus, for example, the picture of a castle with a portcullis and drawbridge is exhibited. The portcullis rises, and a knight in armour on horseback issues from it and crosses the drawbridge. The opening of the portcullis is in this case produced by a moveable plate attached to the slider representing the castle, and the figure of the knight is produced by means of a second lantern, so skillfully managed as to throw the image of the knight upon the screen, and to move it so as to cross the drawbridge.

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1208. *Simple microscopes.*—If the eye possessed the faculty of increasing its convergent power without limit, it would be capable of rendering itself microscopic, and of perceiving accurately and distinctly, without artificial aid, the most minute objects.

It has been already shown that the apparent magnitude of an object, measured by the angle it subtends at the eye, varies inversely as the distance at which the object is viewed. It therefore follows, that as we approach an object, its apparent magnitude increases in the same proportion as its distance from the eye is diminished. Now if there were no circumstance to prevent the eye from seeing distinctly an object at any distance, however small, we could see it with any apparent magnitude, however great, by merely bringing it close to the eye. Thus, if an insect placed at six inches from the eye be seen with a certain apparent magnitude, it will be seen with ten times that apparent magnitude at a distance of $\frac{6}{10}$ths of an inch, and with a hundred times that apparent magnitude at the distance of $\frac{6}{100}$ths of an inch; and so on.

But while the eye approaches to any object, the divergence of the pencils of light proceeding from each point of this object and passing through the pupil, increases exactly in the same proportion as the distance of the eye from the object is diminished. At half the distance the pencil would have double the divergence, and at one-tenth of the distance it would have ten times the divergence; and so on. Now in order to obtain distinct vision of an object, it is not enough that the picture on the retina be large. It is necessary also that the pencils proceeding from the various points of the object should be severally brought to a focus on the retina. Now the more divergent these pencils are, the greater will be the refracting power necessary to bring them to a focus on the retina; and although the eye possesses the faculty of augmenting its refractive power, the exercise of this faculty has a narrow limit, and there is accordingly a distance from the eye, within which a visible point being placed a pencil proceeding from it cannot be made to converge upon the retina; and though an object placed within such distance may produce a large image on the retina, such image will be so indistinct and confused, owing to the pencils not coming to a focus, as to afford no clear perception of the object.

But if, by any contrivance, we could enable the eye, as it approaches an object, to increase its converging power in the same proportion as its distance from the object is diminished, we should then enable it to see such object distinctly, however diminished its distance from the eye might be; and we should consequently, by the same means, obtain a picture on the retina at once magnified and distinct.

Now this object is attained by the simple microscope, which is nothing more than a convergent lens applied between the eye and the object, the effect of which is to cause the pencils of rays which diverge

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from the several points of the object to enter the pupil with a degree of divergence so much diminished as to enable the eye to bring them to a focus on the retina. It must be remembered, that a convergent lens placed at a distance equal to its focal length from the focus of a divergent pencil renders the rays of such pencil parallel, thus destroying altogether their divergence.

1204. *How simple microscopes are adapted to different eyes.* — If such a lens be applied at a still less distance than the focal length from the focus of a divergent pencil, its effect will be not altogether to destroy, but merely to diminish the divergence of the rays of the pencil.

Now some eyes, such as those called *far-sighted*, are adapted to the reception of parallel rays, which they bring without effort to a focus on the retina. Others, called *near-sighted*, are adapted to the reception of rays more or less divergent, which they bring to a focus on the retina. A convergent lens placed between the eye and an object near it may, in either case, be so adapted as to bring the rays diverging from the points of such object to a focus on the retina, and therefore to afford clear vision of it.

If the eye, for example, be *far-sighted*, and therefore adapted to the reception of parallel rays, the lens must be held at a distance from the object equal to its focal length, in which case the pencils diverging from the various points of the object will be parallel after passing through the lens, and will therefore be brought to a focus on the retina.

If the eye be *near-sighted*, so as to be adapted to rays more or less divergent, then the lens must be placed at a distance from the object less than its focal length, and the distance must be regulated, which it always may be by trial, so that the rays of the pencil, after passing through it, shall have just that degree of divergence which will enable the eye to bring them to a focus on the retina.

The more short-sighted the eye is in this case, the greater will be the divergence with which the rays will enter it, and consequently the nearer to the object the lens must be brought. The apparent magnitude, therefore, of an object, when seen through a single converging lens, is equal to the apparent magnitude which it would have if the eye could view it at the same distance without the intervention of any lens; and from what has been just explained it follows that the same lens will give a greater apparent magnitude to an object to a near-sighted than to a far-sighted person, and the more near-sighted the eye is, the greater will be the apparent magnitude of the object seen through the lens.

1205. *Magnifying power explained.* — The term *magnifying power*, as applied to a microscope, is one which, in its ordinary application, is generally vague and uncertain. It has in all cases a relative signification, the object viewed being said to be magnified or increased in magnitude, in comparison with the apparent magnitude which it would have if viewed without the interposition of a lens. But since the eye is capable of obtaining distinct vision of the same object at different distances, it is capable of seeing the same object with different apparent magnitudes.

To give a distinct meaning, therefore, to the term magnifying power, it is necessary to state what is the standard magnitude with which the effect of the lens is to be compared.

This standard is, or ought to be, the greatest apparent magnitude under which the object is capable of being distinctly seen by an eye without the interposition of a lens. But here a distinction becomes necessary. The greatest apparent magnitude under which a given object can be seen by one person is not the same as the greatest apparent magnitude under which it can be seen by another. The greatest apparent magnitude under which an object appears to a short-sighted eye, which can obtain a clear perception of it when viewed at five inches' distance, is greater than the greatest apparent magnitude under which it can be seen by a long-sighted eye, which is not capable of obtaining a clear perception of it at a less distance than ten inches. It is evident, therefore, that the magnifying power of a given microscope applied to the eye of the one person will be different from its magnifying power applied to the eye of the other.

In general, however, it may be stated that the apparent magnitude of an object seen through a simple microscope, is so many times greater than the apparent magnitude of the same object seen at any distance at which distinct vision can be obtained, as the latter distance is greater than the distance of the object from the microscope.

Thus, if an object which cannot be distinctly seen at a less distance than eight inches be made distinctly visible at the distance of one inch by the interposition of a convergent lens, the magnifying power of such lens to the eye which thus views the object is eight times, inasmuch as the angle subtended by the object at the distance of an inch is eight times that which it would subtend at the distance of eight inches.

1206. *Apparent brightness is diminished as the square of the magnifying power is increased.* — What has been stated respecting the brilliancy of a picture thrown upon a screen by a magic lantern, is equally applicable to the brilliancy of the image of an object formed on the retina either by means of the naked eye or by the interposition of a convergent lens. The light diffused over such a picture can only be that which is transmitted from the object, and it follows that the larger the picture the less, proportionally, will be its brilliancy; and in this case it must be remembered, that the area of the picture is increased in proportion to the square of the magnifying power Thus, if the magnifying power be four, the height or diameter of the picture on the retina formed by the lens will be four times the height or diameter of the picture formed on the retina without the lens. But if the height or diameter be increased in a four-fold proportion, the area of the picture will be increased in a sixteen-fold proportion, and the light which was before diffused over the smaller area will, by means of the lens, be spread over an area sixteen times greater, and consequently the brilliancy of the image will be sixteen times less.

If it be desired that the magnified image of an object produced by a microscope should have the same brilliancy as the object itself has when viewed without a microscope, it would be necessary to illuminate the object, when viewed through the microscope, with light of an intensity proportional to the square of the magnifying power. Thus, if a magnifying power of four were used, the image cannot have the same intensity of illumination as the object, unless it be illuminated with light of sixteen times the intensity.

1207. *Compound microscope.* — With the simple microscope the object itself is viewed directly by the eye, but at a less distance than would be compatible with distinctness of vision without the interposition of the lens. It is, however, sometimes necessary to submit to microscopic observation objects so minute that practical difficulties would arise in viewing them with simple lenses of sufficiently small focal length to produce the requisite magnifying effect. In such cases, instead of submitting the object itself to immediate observation by means of the simple microscope, an optical image of it is produced by means of convergent lenses or concave reflectors.

The image thus formed may, according to the principles already established (Chap. X.), be rendered larger in any desired proportion than the object, and may, therefore, be viewed with a simple microscope of proportionally less magnifying power. Thus, for example, if an image of a minute object be produced, having linear dimensions ten times greater than those of the object, and if such image be viewed by a simple microscope whose magnifying power is twenty, the object will, by such a combination, be magnified two hundred times; for the image which forms the immediate object of examination with the simple microscope has ten times the linear magnitude of the object, and is itself magnified twenty times by the simple microscope.

A compound microscope consists, then, of such a combination. If the image, which is the immediate object of observation be formed by lenses, the microscope is called a *compound refracting microscope*, and if it be formed by a concave reflector it is called a *compound reflecting microscope*.

The form, dimensions, and power of compound microscopes are infinitely various, according to the purposes to which they are applied, to the exigencies of the observer for whose use they are intended, and to the taste and ability of their constructors.

The principles common to all refracting microscopes, which are more generally used than reflectors, may be more clearly understood by reference to *fig.* 888., where *b* represents a small convergent lens of very short focal length, before which a minute object *o*, which it is

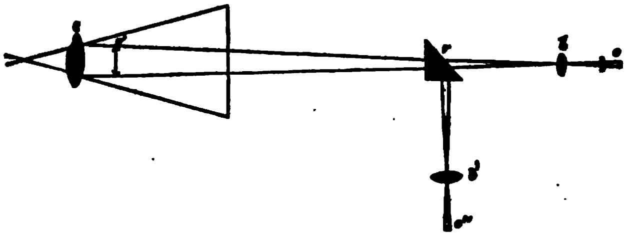

Fig. 383.

desired to magnify, is placed. The distance *b o* will a little exceed the focal length of the lens *b*. An image of *o* will be formed at *o'*, the focus conjugate to *o*. The magnitude of the image at *o'* will be just so much greater than the magnitude of the object, as the distance *o' b* is greater than the distance *o b*. This image *o'* is the immediate object of examination with the simple microscope *e*; and everything which has been explained with respect to the application of simple microscopes to magnify objects will be equally applicable to the microscope *e* as applied to the image *o'*, considered as an object. The lens *b* is called the object-glass, and the lens *e* is called the eye-glass. In some cases, instead of a single lens two or more are used at *b*, so as to form a compound object-glass. The lens *b* is rendered achromatic in the manner already explained, by constructing it of two lenses having different dispersive powers and corresponding curvatures, as already explained.

The magnifying power of the object-glass is generally increased, not by increasing its curvature, but by combining together two or more lenses of equal magnifying power, and of equal opening. In this way the observer is enabled with great facility to vary the power of his microscope at pleasure, according to the magnitude of the object he examines.

It is only necessary to screw upon the end of the microscope next the object one, two, or three lenses, so as to increase the magnifying power.

In like manner the eye-glass *e* may be varied in power by combining different lenses. It is usual in compound microscopes to provide several *eye-pieces*, as they are called, having different powers; the eye-piece being the name given to the sliding tube which contains the eye-glass, and which moves with the motion of a telescope joint, so as to vary at pleasure its distance from the image *o'*. These eye pieces usually consist of two plano-convex lenses instead of a single double convex lens.

It is sometimes convenient to enable the observer looking in a horizontal direction to see an object which is placed vertically below the end of the instrument. This is accomplished by placing the great tube containing the object-glass at right angles to the tube along which the observer looks, a rectangular prism being placed, as represented at r, in the angle formed by the two tubes. This arrangement is represented at $r\ b'$, where b' is the object-glass, and o'' the object, and r the prism by which the pencils proceeding from the object-glass b' are reflected at right angles, so that the image is formed at o'.

It has been already explained that in all cases where a distinct image is required to be formed by means of a convergent lens, the divergence of the pencils proceeding from the object must not exceed such a limit as would render the diameter of the lens sensibly different from the length of a circular arc described with the extreme pencils as radii, and the point of the object from which the rays diverge as a centre. Consequently it follows that in all cases the diameter of the lens must bear a very small proportion to the distance of the object from it.

Now, since in microscopes of every kind where an eye-piece is used the focal length of the lens itself, if it be a simple microscope, and of the object-glass, if it be a compound microscope, is extremely short, and the distance of the object from the lens very small, it follows that, according to the principle just explained, the diameter of such lenses must be also extremely small, since their diameters must bear an inconsiderable proportion to such distance, and the higher the magnifying power, the smaller must be the diameter of the lens. Thus the diameter of such lenses used for the object-glasses of compound microscopes do not exceed in general a small fraction of an inch. The same observation is applicable to the reflectors or specula used to form the image in reflecting microscopes.

1208. ***Compound universal microscope of Charles Chevalier.*** — It would not be compatible with the object of this volume to enter into any detailed description of the various forms of compound microscopes. It may be useful, however, to indicate the arrangement adopted in one as an example. For this purpose I shall here briefly describe the form of microscope constructed by Mr. Charles Chevalier, and called from its general utility the Universal Microscope.

This instrument is represented in *fig.* 384. The object-glass is at Y, and the eye-piece at S. The rectangular prism by which the pencils are turned along the horizontal tube is at V. The object-glasses consist of three achromatic lenses, whose focal lengths vary from three-tenths to four-tenths of an inch. These lenses are constructed with screws so as to be successively screwed upon the end Y of the tube

They may be used separately or together, according to the magnifying power required.

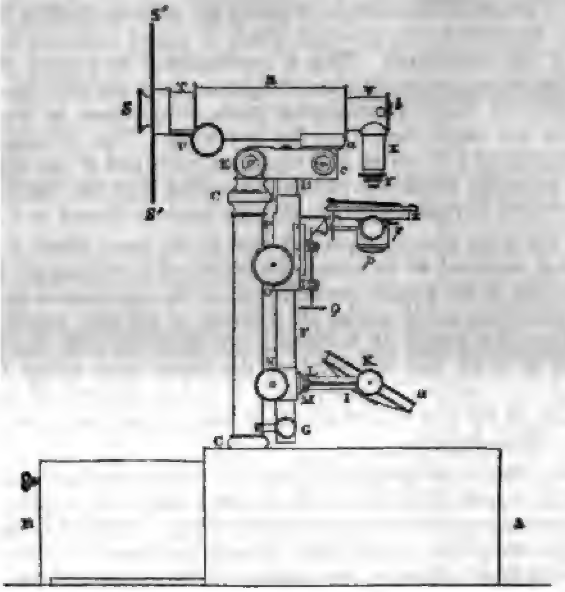

Fig. 384.

The instrument is usually provided with six eye-pieces; the first four are constructed upon the same principle, each being composed of two plano-convex lenses, whose convexities are turned towards the image. The two others are simple converging lenses of short focus. Z is a stage provided to receive the slider on which the object to be examined is placed. This stage can be moved gradually by an adjusting screw O, upwards and downwards on a square vertical rod P G, so as to vary its distance from the object-glass Y. The observer looking through the eye-piece S, places his right hand upon the screw O, and turns it in the one direction or the other, until he brings the object to the focus, the final adjustment being made by a fine micrometric screw Q, which gives a slower motion to the stage than can be imparted to it by the screw O.

If the object be transparent, it is illuminated by a concave mirror H, which is capable of being adjusted at such an angle with reference to the light as to throw the illumination directly upon the object by means of a horizontal axis K, on which the reflector H turns, and the intensity of the illumination may be varied by moving the reflector vertically upon the bar P G by means of a screw N, which works in a rack behind the bar and carries with it the frame L, by which the mirror is sustained. In this manner the reflector may be made to approach to or recede from the slider which supports the object, and the light thrown upon it may be accordingly rendered more or less intense.

If the instrument be used in the day-time it will be convenient to place it upon a table near a window, so that the light from the clouds may be received upon the reflector H. If it be used at night, a lamp or candle placed at a convenient height in front of the instrument will form a sufficient illumination.

In order to intercept all light proceeding from the reflector H, except that which falls upon the object, a circular movable stage is placed beneath the stage Z, which supports the object, and pierced by a number of holes of different magnitudes, which, being brought successively under the object, regulate the quantity of light reflected from H which is transmitted to the object.

The focus is determined by the adjustment with the screw O, and still more accurately by the micrometer screw.

When it is required to view opaque objects, they are usually placed upon a blackened glass laid upon the slide Z, and are illuminated by another lens or reflector attached to the side of the slide Z.

When high magnifying powers are used, so that only a part of a minute object can be seen at one time in the field of view, it is desirable to be enabled to move the object slowly under the microscope, so as to bring all its parts successively under examination. To dc this by moving the slide with the hand is impracticable; for the motion being magnified in the same proportion as the object, a movement of the hand which is imperceptible will throw the object completely out of the field. To enable, therefore, the observer to move the object so as to bring all its parts successively into view, and to keep any part steadily under examination, two micrometer screws under the stage Z are provided: the first moves the object backwards and forwards to or from the observer, and the second moves it laterally right and left. By the combination of these two motions every possible position can be given to the object.

To keep the eye of the observer undisturbed by extraneous light, a large circular screen S′ S′ is placed between the socket of the eye-piece and the end of the instrument. By this means the observer is not under the necessity of closing that eye which is not directed to the eye-piece.

1209. *Adaptation of the camera lucida to the microscope.* — The adaptation of the camera lucida to this instrument has greatly extended its interest and utility. The instrument is attached to the eye-piece S, so that when the observer looks upon it he sees, by the reflection of the prism, a sheet of paper placed vertically under the eye-piece on a table before him, and he sees directly the image of the magnified object projected upon the paper. In this manner he is enabled to make a tracing of the object, as already described in the application of the camera lucida.

1210. ***Method of determining the magnifying power.*** — To determine the magnifying power of the instrument, a slide is provided, of the form represented in *fig.* 385., upon which is engraved a micrometer scale, made to the $\frac{1}{100}$th part of a millimetre, which is equivalent to the $\frac{1}{2500}$th part of an inch. There are usually ten of these divisions engraved upon the slider, the entire length of which is therefore the $\frac{1}{250}$th part of an inch. This slide being placed upon the stage, a camera lucida, constructed upon the principle of Amici, is attached to the eye-piece O, *fig.* 385., the effect of which is that the eye sees a magnified image of the scale projected upon a sheet of paper A spread upon the table under the hand of the observer. A divided scale being applied to this magnified image, the length of the magnified divisions may be at once compared with the divisions on the divided scale, which are seen directly without being magnified. The arrangement by which this is accomplished is represented in *fig.* 385., where P represents the eye, B a metallic speculum applied at an angle of 45° with the line of vision, and having a hole pierced in its centre, through which the eye-piece and the magnified image of the scale, *fig.* 386., are seen. At the same time the eye sees by reflection, in the speculum B, the sheet of paper A, upon which a scale is placed at C′, parallel to, and coincident with, the image of the scale, *fig.* 386., which is also seen projected on the paper. By comparing the divisions of the image of the scale, *fig.* 386., seen magnified, with the image of the divisions of the scale on the paper A, seen by reflection without being magnified, the magnifying power can be immediately ascertained

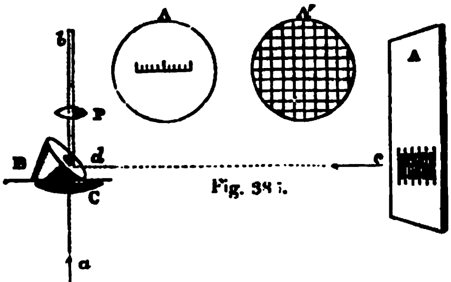

Fig. 385.

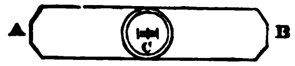

Fig. 386.

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The magnifying power being thus ascertained, the real dimensions of any object viewed through the microscope can be easily measured. For this purpose, let a system of parallel lines be described upon the paper A, at right angles to each other as represented at A′, so that the distance between them shall correspond to any fraction of an inch, such as the 2000th part, as ascertained by the method just explained.

When the magnified image of a minute object is seen projected upon the paper upon which this rectangular scale is drawn, the dimensions of the object can be determined by inspection, by merely observing how many squares of the rectangular scale the image occupies.

The details of the microscope represented in *fig.* 384., present many practical conveniences to the general microscopic observer.

When it is desirable to present the main tube R directly upon the object without reflection by the rectangular prism V, the elbow tube which contains this prism can be detached both from the main tube R, and from the tube X, which bears the object-glass, being attached to them by bayonet joints. The tube X is then inserted in the main tube R of the instrument, so as to form a compound microscope, in which the pencils refracted by the object-glass Y will proceed directly to the eye-piece S. The body of the microscope R turns upon a joint at *c*, so that it can be moved through a right angle, so as to be applied in a vertical position, and to enable the observer to look vertically downwards on the stage Z, which supports the object.

The instrument can also be applied at any required angle with the vertical. To accomplish this, it is only necessary to loosen the screw G, by which the straight bar P G is held in the vertical position. When this is done, the bar P G, carrying the reflector H and the stage Z, can be turned at any angle with the pillar O O by means of the pivot E, upon which the entire instrument, including the body of the microscope and the bar P G, turns with a common motion. The pivot works so stiffly, and the weight of the instrument is so equally balanced upon it, that it will rest at any required angle with the vertical.

By means of the two pivots E and *c*, the bar P G, and the body of the instrument R, can be brought into a horizontal direction, so that the stage Z shall be vertical and opposite to the object-glass of the microscope, the axis of which is horizontal. Proper holders are provided on the stage to keep the sliders in their position upon it, at whatever angle it may be applied with the vertical.

The entire instrument is screwed at O upon its own case O A, in which is a drawer, B, properly constructed to receive it and all its accessories.

1211. *Solar microscope.* — This instrument partakes of the character of a magic lantern and a compound microscope. Light proceeding directly from the sun is received upon a plane mirror, and reflected by it into the tube of the instrument, where it is received upon a large convergent lens called the *illuminating lens*. By this

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lens the rays are made to converge to a focus, near which they are received upon the minute object which it is desired to exhibit. In this manner, the light thrown upon the object is just so much more intense than that which it would receive directly from the sun, as the area of the illuminating lens is greater than the area of the object. The object being thus illuminated, a magnifying lens is applied before it at a distance greater than the focal length; and an image is accordingly formed of the object at the other side of this lens, which image is greater than the object in the same proportion as its distance from the magnifying lens is greater than the distance of the object from it. A white screen being properly suspended at the necessary distance from the magnifying lens, receives the image, upon which it is seen in the same manner as in the case of the magic lantern.

It is evident that such objects only can be exhibited in this instrument as are naturally transparent, or such as may be rendered so. If opaque objects are exhibited, nothing appears upon the screen except a gigantic *silhouette* or profile of their form.

Such an instrument could only be exhibited in the day-time, and when the sun is unclouded. Its application, however, has recently been rendered more convenient and extensive by adapting it to artificial light called the Drummond light. Instead of the solar rays, a small cylinder of lime rendered incandescent by the oxyhydrogen blow-pipe is applied behind the illuminating lens in such a position that its light is brought to a focus upon the object to be exhibited, and the same effect is thereby produced as with sun-light. Several successful attempts have been still more recently made to apply the electric light to this instrument.

1212. *The telescope.* — What the compound microscope is to minute and near objects, the telescope is to distant objects. The principle in both instruments is the same, the details of its application alone being different. In the telescope, however, as the objects to which it is directed are always at a considerable distance from the object-glass, and generally at a distance which may be considered infinite as compared with any possible magnitude of that lens, it is possible to give the object-glass any desired magnitude without producing such spherical aberration as would render the image indistinct. In fine, the objects to which a telescope is directed being at distances incomparably greater than the diameter of the object-glass, their images will always be formed at a distance from such lens equal to its focal length. The pencils which proceed from the extreme limits of the object passing through the centre of the object-glass, and interesecting there, are continued to the corresponding extreme limits of the image which is formed in an inverted position with respect to the object at a distance from the object-glass equal to its focal length.

This image therefore subtends, at the centre of the object-glass, an angle equal to that which the object subtends at the same point. If we imagine that an eye be placed at the centre of the object-glass, the apparent magnitude of the image seen from that point would be equal to the apparent magnitude of the object.

The image which is thus formed at the focus of the object-glass is, as in the case of the compound microscope, viewed by the observer with a simple convergent lens, called as in the compound microscope the *eye-glass*. All the observations which have been made in relation to the eye-piece of the compound microscope are equally applicable to the eye-piece of the telescope, which performs precisely the same functions in relation to the image formed at the focus of the object-glass as the eye-piece of a compound microscope with respect to the image formed in the focus of the object-glass of that instrument.

Telescopes differ from each other in the details of their construction, according as the images of the different objects are produced by object-glasses or by concave reflectors. In this respect telescopes, like microscopes, consist of two classes, reflecting telescopes and refracting telescopes. They are also classed in relation to the objects to the vision of which they are directed; those which are used for astronomical purposes being called *astronomical telescopes*, and those which are used for observing objects at less distance on the surface of the earth being called *terrestrial telescopes*.

In this last class it is important that the object should be seen erect, which it would not be if the image formed by the object-glass were the immediate subject of observation by the eye-glass; such image being, as already explained, always inverted. An expedient, however, is adopted in one class of telescopes, as will be presently explained, by which this inconvenience is removed without the introduction of additional lenses.

Having thus explained the general properties upon which telescopes are constructed, we shall briefly explain the different kinds of telescopes.

1213. *The Gregorian reflecting telescope.* — A longitudinal section of this instrument is represented in *fig.* 387. A B is a large concave speculum formed of an alloy of metals adapted to receive a high polish. A circular aperture is made in the centre, so that the reflecting portion of the speculum is that part only which is outside

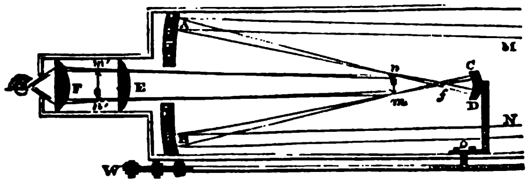

Fig. 387.

the circular aperture. A second concave speculum C D is placed with its concavity in the other direction, at a distance from A B greater than the focal length of the great speculum. The eye-glass F is placed in a smaller tube inserted in the greater one opposite the opening of the great speculum.

The extremity of the great tube being open, and presented towards the object of observation, an inverted image of this object is formed at m n in the principal focus of the great speculum A B. This image forms an object for the small speculum C D, and another image is formed in the conjugate focus m′ n′; this latter image being inverted with respect to m n, and therefore erect with respect to the object.

The pencils proceeding from C D are sometimes brought to a focus by the interposition of a converging lens E, but this is not necessary.

The image m′ n′ is viewed by the eye-glass F, which, as already explained, may be considered as a simple microscope.

The telescope is mounted with proper apparatus, by which it can be directed to the object, and by which its focus can be regulated.

1214. *Cassegrain's reflecting telescope.* — A longitudinal section

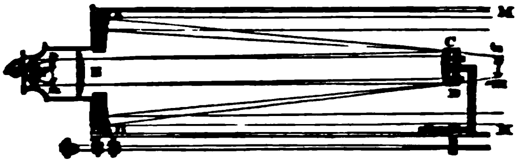

Fig. 388.

of this instrument is given in *fig.* 388. Its details are in all respects similar to the Gregorian reflector, except that the second speculum C D is convex instead of being concave, and receives the pencils proceeding from A B before they come to a focus. It turns them back towards the eye-piece, where an image is formed, as in the former case.

1215. *The Newtonian reflecting telescope.*—A longitudinal section of this instrument is represented in *fig.* 389., where A B is the great

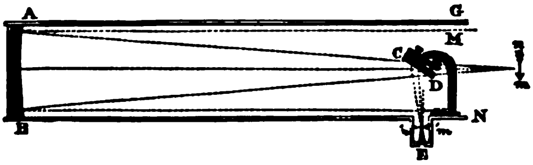

Fig. 389.

speculum which would form an image of the object at m n in its principal focus.

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But the pencils, before they arrive at that point, being received upon a plane reflector C D placed at an angle of 45° with the axis of the telescope, the image is formed at *m′ n′* in a lateral tube inserted in the great tube, where it is viewed by an eye-piece, as before explained. In this case the open end A of the great tube is directed towards the object, and the observer examines the object by looking in at the side of the telescope in a direction at right angles to its length.

In all these cases, the central rays of the pencils directed upon the great speculum are lost. In the Gregorian and Cassegrain, the central portion of the speculum is removed, and in the Newtonian telescope the central rays are intercepted by the plane reflector C D.

1216. *Herschel's telescope.*—The form of reflecting telescope which has attained by far the greatest celebrity of any that have been hitherto constructed, is that which was erected by Sir W. Herschel, and used by him with such signal success, as to render his name memorable in the history of astronomical science. Herschel, after having constructed a great number of reflecting telescopes on the Newtonian principle, varying from seven to twenty feet in length, aided by the patronage of George III., completed in 1789 his celebrated telescope, forty feet in length, by which, on the very day it was completed, he discovered the sixth satellite of Saturn. The great speculum of this telescope measured nearly fifty inches in diameter, its thickness being three inches and a half, and its weight about a ton. The open end of the telescope being directed to the point of the heavens under observation, and the speculum being fixed at its lower end, the observer is suspended in a chair, so as to be able to look over the lowest part of the edge of the opening. The speculum being a little inclined to the axis of the tube, the image is formed near the lowest point of the edge of the opening, where it is viewed by the observer with proper eye-pieces.

The quantity of light obtained by this prodigious speculum enabled Sir W. Herschel to use magnifying powers which greatly exceeded any which before his time had been applied. He was thus enabled, in examining the fixed stars, to apply in some cases a magnifying power of 6450.

1217. *The Galilean telescope.* — *Opera-glass.* — This telescope, which takes its name from Galileo, by whom it was first used, is a refracting telescope, the principle of which is represented in *fig.* 390.

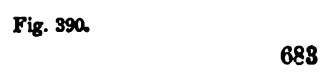

Fig. 390.

A B is the object-glass, in the principal focus of which, E, an inverted image of the object would be formed; but before the pencils arrive at this point, they are received by a divergent lens C D, which, destroying their convergence, causes them to enter the eye parallel, as they would if they proceeded from an object at a considerable distance.

The general direction of the axes of the pencils, however, is not changed, and the eye consequently receives them as if they had proceeded from an object at the same distance from the eye as the image *m n* is from the eye-glass C D. The apparent magnitude, therefore, of the object, as seen with the eye-glass C D, is measured by the angle which the image *m n* subtends at the centre of the lens C D; and the apparent magnitude of the object as seen directly is equal to the angle which the same image subtends at the centre of the object-glass F.

If, therefore, we divide the focal length of the object-glass by the distance of the eye-glass from the image, we shall then obtain the magnifying power.

Let us suppose, for example, that the focal length of the object-glass is fifty inches, that the focal length of the eye-glass is one inch, and that the eye of the observer is adapted to the reception of parallel rays. In this case, the focal length of the object-glass will be fifty times the distance of the eye-glass from the image, and the telescope will magnify accordingly fifty times. But if the eye of the observer be adapted to the reception of diverging rays, then the eye-glass C D must be removed further from the image than its focal length, and, consequently, the magnifying power will be less than it would be for an eye adapted to parallel rays; and if, on the contrary, the eye of the observer be adapted to converging rays, the eye-glass must be moved near to the image, and the magnifying power will be greater.

In all cases, the distance of the eye-glass from the object-glass is equal to the difference between their focal lengths for eyes adapted to parallel rays. It is a little less for short-sighted, and a little more for long-sighted eyes.

This form of telescope has long been disused for all purposes where very distant objects are observed. It is, however, still continued with great convenience where the objects of observation are nearer, as in the case of opera-glasses, which are nothing more than Galilean telescopes.

These instruments have lately been mounted in pairs, so as to enable the spectator to use both his eyes, as with spectacles.

1218. *The astronomical telescope.* — This is the name given to a refracting telescope, consisting of two convergent lenses, one used as an object-lens, to form an image of the object to be observed, and the other as a simple microscope, to examine this image. The principle of this instrument has been already sufficiently explained in the case of the compound microscope, from which it differs in nothing but in

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the proportion of its parts. A B, *fig.* 391., is the object-glass; an inverted image *m n* of the object M N is formed at its focus.

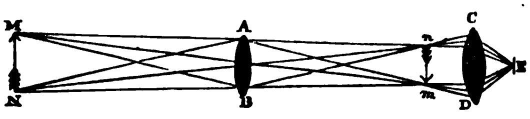

Fig. 391.

This image is viewed by the eye-piece C D, which for eyes adapted to parallel rays is placed at a distance from *m n* equal to its focal length. The image *m n* is seen under an angle equal to that which it subtends at the centre of the eye-glass C D, and its apparent magnitude being equal to the angle which it subtends at the centre of the object-glass A B, it follows that the magnifying power of the instrument is found by dividing the focal length of the object-glass by the focal length of the eye-glass. The image, as seen in this instrument, is always inverted with respect to the object; but as it is used for astronomical purposes, this is unimportant.

1219. *Terrestrial telescope.* — When the telescope described above is applied to terrestrial objects, it exhibits them inverted. This is corrected by interposing between the eye and the image other lenses, by which a second image is formed, inverted with respect to the first, and therefore erect with respect to the object. This arrangement is represented in *fig.* 392., where A B is the object and *m n* the first inverted image. A convergent lens C D is placed before this image, at a distance equal to its focal length; consequently, the pencils proceeding from *m n*, after passing through C D, will emerge with their rays parallel. These pencils are received by another converging lens of equal focal length E F, by which they are again rendered convergent, and are made to form the image *m' n'*, which is inverted with respect to *m n*, and erect with respect to the object. This image *m' n'* is viewed by the eye-glass G H in the usual manner.

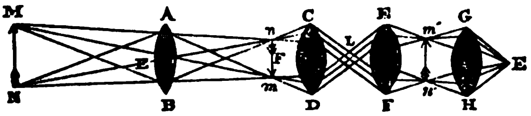

Fig. 392.

The eye-pieces of telescopes, like those of microscopes, do not necessarily consist of a single lens, but are frequently composed of two.

The object-glass, as well as the other lenses composing refracting telescopes, are usually constructed so as to be achromatic, upon the principle already explained in Chap. XIII.

CHAP. XVI.

THEORIES OF LIGHT.

1220. *Ordinary reflection and refraction explicable independently of theories.* — The optical phenomena attending ordinary reflection and refraction, which have formed the subjects of the preceding chapters, have been explained without reference to any hypothesis or theory. They have been deduced directly from experiments, the results of which are so simple and obvious, that the laws which prevail among them have been rendered evident without reference to theoretical considerations.

Other phenomena, however, will now have to be examined, in which the same simplicity does not prevail, and which do not admit of being explained or reduced to general laws without the occasional use of language derived from one or other of the theories respecting the nature of light which have been imagined by scientific inquirers.

1221. *Two theories of light.* — We shall therefore now explain briefly those theories or hypotheses which have been proposed respecting the nature of light, for the purpose of explaining the phenomena of optics.

It has been already stated that the scientific world for ages has been more or less divided by two theories or hypotheses concerning the nature of light, one of which is known as the corpuscular theory, or the theory of emission, and the other as the undulatory theory, or the theory of undulation.

1222. *Corpuscular theory.* — In the corpuscular theory, which was adopted by Newton as the basis of his optical inquiries, light is considered as a material substance, consisting of infinitely minute molecules which issue from luminous bodies and pass through space with prodigious velocities. Thus, in this hypothesis, the sun is re-
as a source from which such molecules or corpuscles proceed
direction, with such a velocity that they pass from that lumi-
he earth, over a distance of ninety-five millions of miles, in
t minutes and thirteen seconds.

mense velocity with which they are endued, amounting to
o hundred thousand miles per second, united with the fact
l by observation, that they do not impress with the slightest

momentum the lightest objects which they strike, render it necessary to suppose that they are so minute as to be altogether destitute of inertia or gravity. The strongest beam of sunlight acting upon the most delicate substance, upon the fibres of silk or the web of the spider, or upon gold-leaf, does not impress upon them the slightest perceptible motion. Now, in order that a particle of matter endued with a velocity so great should have no perceptible momentum, it is necessary to suppose it to be almost infinitely minute.

But this minuteness requires to be admitted to a still greater extent, when it is considered that particle after particle striking upon bodies so light, even after the communication of their forces, impart to them no perceptible motion.

1223. *Difference of colour explained.* — In this system the difference of colour which prevails among the different homogeneous lights, the combination of which constitutes solar light, is ascribed to different velocities.

Thus the sensation of red is produced by luminous molecules animated by one velocity, orange by another, blue by another, and so on.

1224. *Laws of refraction and reflection explained.* — The law which renders the angle of reflection equal to the angle of incidence, is explained by supposing such molecules to have perfect elasticity. The law of refraction is explained by supposing that such molecules are subject to an attraction towards the perpendicular when they enter a denser, and by a repulsion from it when they enter a rarer medium.

1225. *Undulatory theory.* — In the undulatory theory which was adopted by Huygens, and after him by most continental philosophers, light is regarded as in all respects analogous to sound.

The luminous body in this system does not transmit any matter through space any more than a bell transmits matter when it sounds. The luminous body is regarded as a centre of vibration; but in order to explain the transmission of this vibration through space, the existence of a subtle fluid is assumed, which plays, with regard to light, nearly the same part as the atmosphere plays with regard to sound. The sun in this theory, then, is a centre of vibration, and the space which surrounds him being filled with an atmosphere of this subtle fluid, transmits this vibration exactly as the atmosphere transmits the vibration of a sounding body.

1226. *The luminous ether.* — This hypothetical fluid has received the name of *ether*. It is supposed not only to fill all the vacant spaces of the universe which are unoccupied by bodies, but also to fill the interstices which exist between the component parts of bodies. Thus it is not only mingled with the atmosphere which surrounds the earth, but also with the component parts of water, glass, and all transparent substances; and since opaque substances, when rendered sufficiently thin, are penetrable more or less by light, it is necessary to admit that it also fills the pores of such bodies. If this luminous ether did not prevail throughout the whole extent of the atmosphere, the light of the stars could not reach our eyes. If it did not exist in water, glass, precious stones, and all transparent substances, these bodies could not be penetrable by light as they are; in fine, if it did not exist in the humours of the eye, light could not affect this organ, and the undulations could not reach the membrane of the retina.

1227. *Effects ascribed to its varying density.*—But although this luminous ether is thus assumed to be omnipresent, it does not everywhere prevail with the same density. It is probable that its density in the celestial spaces which intervene between planet and planet is the same which it has under the exhausted receiver of an air-pump or above the mercurial column in a barometer.

But its density in transparent media must be different, because to explain the phenomena of light passing through them it is necessary to suppose that the undulations change their magnitude, a supposition which is only compatible with a change in the elasticity of the ether. We shall see further, that in some transparent bodies existing in a crystallized state it is necessary to suppose also that the density of the ether in different directions in the same medium varies.

If this universal ether were for a moment in a state of perfect repose, the universe would be in absolute darkness; but the moment its equilibrium is disturbed, and that an undulation or vibration is imparted to it, that instant light is created, and is propagated indefinitely on all sides, as, in an atmosphere perfectly tranquil, the vibrations of a musical string or the sound of a blow is propagated to a distance in all directions according to determinate laws.

Light itself must not, however, be confounded with the ether which is the medium of its propagation. Light is no more identical with the hypothetical ether than sound is identical with air. The ether, in the one case, and the air in the other, are merely the media by which the systems of undulations which constitute the real sense of light and sound are propagated.

1228. *Analogy of light and sound.*—In considering the analogy between light and sound, however, there is an important distinction which must not escape notice. Sound is propagated, not only by undulations transmitted through the air, but also by undulations transmitted through other fluids as well as solids, as has been already explained. Light, however, according to the undulatory theory, is transmitted only by the undulations of the luminous ether. Light, therefore, does not pass through a transparent body, such as glass, in the same manner as sound is transmitted through the same body. The undulations by which sound is propagated through the air would be imparted to glass itself, which will continue them and transmit them to another portion of air, and thence to the ear; but when the undulations of light are transmitted through glass or any other transparent medium, they must be supposed to be propagated, not by the vibration of the glass itself, but by the vibration of the subtle ether which pervades its pores.

1229. *The undulatory theory affords a more complete explanation of the phenomena.* — These two celebrated theories have, as has been already stated, divided the scientific world for ages; nevertheless, many of the more recent optical discoveries having failed to obtain a satisfactory explanation by means of the corpuscular hypothesis, the other theory has now obtained much more general, if not universal acceptation. We shall therefore, in the succeeding chapters, where it is necessary to use the language of theory, adopt that of the undulatory hypothesis.

All the general principles connected with the theory of undulations, as explained in Book VII., will be applicable to the undulations imparted to the luminous ether in this case.

Thus, the velocity with which such undulations are propagated is the velocity of light, the breadth of the waves determine the colours of the light, and the height of the waves its intensity. Thus the undulations with which red light is propagated are broader than those by which violet light is produced, and the same of the other colours.

CHAP. XVIII.

INTERFERENCE AND INFLECTION.

It has been shown in Book VII., that in all cases where systems of undulation are propagated along the surface of a fluid or through an elastic medium, phenomena are produced by the intersection of systems of waves, by which, at certain points the undulations obliterate each other.

Such effects are called *interference*, one system of waves being said to *interfere* with another when such reciprocal obliterations take place.

An instructive class of interesting optical phenomena are explained upon this principle.

1230. *Fresnel's experiments exhibiting the effects of the interference of light.* — In order to exhibit the phenomena of the interference of light in such a manner as to develop the laws which govern it, and to supply numerical estimates of the data and constants of the undulatory theory, it is necessary to contrive means by which two pencils of light, whether homogeneous or compound, of the same intensity, shall intersect each other at a very oblique angle and at a considerable distance from their foci. Fresnel, to whose experimental researches in this department of physics science is largely indebted, accomplished this object by reflection and refraction in the following manner.

1. *By reflection.*—Let M C, M′ C, *fig.* 393., be two plane reflectors inclined to each other at a very obtuse angle. Let F be a focus of light produced by transmitting the light through a converging lens of short focus, or by reflecting it from a concave speculum. The rays diverging from F are received upon the two plane reflectors M C and M′ C. An image of F will be formed by the reflector M C at P just as far behind the plane of M C as F is before it; and, in like manner, another image of F will be produced by the reflector M′ C at P′ just as far behind the plane of M′ C as F is before it. It follows, therefore, that those rays which proceed from F and are incident upon M C will after reflection diverge as if they had originally proceeded from P, and those rays which are incident upon M′ C will after reflection diverge as if they had originally proceeded from P′. Therefore the pencils after reflection will be optically equivalent to two pencils radiating from P and P′. Thus we shall have a single pencil radiating from the point F converted into two pencils intersecting each other at a very oblique angle, and proceeding from the distant foci P and P′.

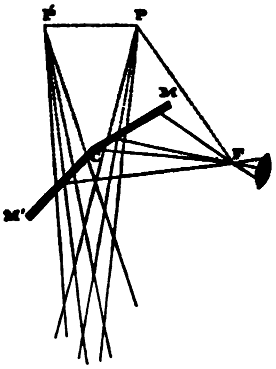

Fig. 393.

2. *By refraction.*—Let A B C, *fig.* 394., be a prism, with a very obtuse angle at B, and let F be a radiant point produced as before by a converging lens or concave reflector. The rays diverging from F, and incident on the surface A B, will be refracted as if they proceeded from F′; and, in like manner, the rays proceeding from F and incident upon B C will be refracted as if they proceeded from F″. Thus we shall have two pencils, as before, the rays of which will intersect each other obliquely at the points I, these pencils consisting of light of the same quality and intensity.

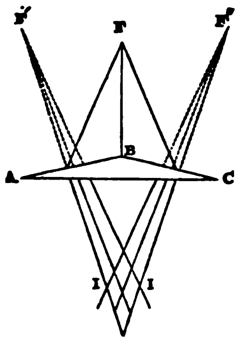

Fig. 394.

1231. *Phenomena of interference exhibited in the case of homogeneous light.*—If

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two pencils of homogeneous light thus obtained be made to diverge from two points F and F′, *fig.* 395., and if the rays of these pencils intersect at very oblique angles below the line A B, which is drawn parallel to the line F F′, which joins the foci of the two pencils, the following effects will ensue: —

If a line C *o* O be drawn from the middle point of F F′ perpendicular to it, any point on this line *o* O will be illuminated; in fact, an illuminated line will be formed from *o* to O, as indicated by the dotted line in the figure. On either side of this illuminated line *o* O will be found a dark curved line 1 1 and 1′ 1′, so that any object held in either of these lines would be deprived of light. Outside these two dark curved lines will be found two other curved lines, 2 2 and 2′ 2′, which will be lines of light, so that any object held at any point of either of them will be illuminated. Beyond this again will be found two other dark curved lines, 3 3 and 3′ 3′, so that any object held in them will be in shadow or darkness; beyond these again will be two curved lines of light, as before, 4 4 and 4′ 4′, so that any object held in either of these will be illuminated. Thus there succeed each other a series of curved lines of light and darkness, the light lines having the colour and qualities of the light of the two pencils. The series of the illuminated curves of light and darkness at each side of the central line *o* O are symmetrically arranged, those on the one side having corresponding forms, positions, and distances to those on the other side.

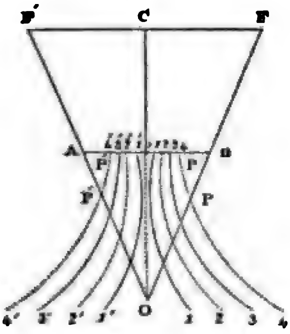

Fig. 395.

The curves formed by these light and dark lines are those known in geometry as the species of conic section called the hyperbola, the points F and F′ being their common foci.

Now, it is a well-known property of this curve that the difference between the distances of every point in it from the two foci is the same. Thus, if lines be drawn from F and F′ to any point in any one of these curves, their difference will be the same as that of lines drawn from F and F′ to any other point in the same curve.

Thus, for example, if P and P be two points upon the curve 4 4, then the differences between the distances of P and P from F and F′ will be equal; and, in like manner, if P′ and P′ be two points on the curve 4′ 4′, the differences between their distances from F and F′ will be equal.

It will presently be seen that this property gives rise to important consequences.

If an opaque screen be interposed between the line A B and either of the foci, F′ for example, all these curves of bright and dark lines vanish, and there is a uniform illumination produced throughout the space below the line A B. This illumination, however, will be found to have only half the intensity of the bright curves which were previously formed.

Now, since by the interposition of the screen no light has been diffused below the line A B which was not there before, but, on the contrary, all the light proceeding from the focus F′, which was there before, is now excluded, it follows that the effect of the rays which, proceeding from the focus F′, intersect those proceeding from the focus F, is to deprive the spaces marked by the dark curves 1 1, 3 3, 1′ 1′, and 3′ 3′ of light, and to increase in a two-fold proportion the light in the spaces marked *o* o, 2 2, 4 4, 2′ 2′, and 4′ 4′.

Thus it appears, that at the intersections of the rays proceeding from F and F′, which take place upon the dark curves, the one light extinguishes the other; and that at the intersections which take place upon the bright curves, the lights add their mutual intensities, and an intensity is produced equal to their sum; for since they are equal to each other, this intensity is double the intensity of either.

Now it will be evident, by reference to what has been established in Book VII. relating to undulations, that this fact is merely a consequence of the interference of the waves of light. The foci F and F′ may be considered as the centres round which two systems of luminous undulations are propagated. These systems, encountering each other, intersect below the line A B. At those points where the waves meet under corresponding phases, that is to say, where the crest of one wave coincides with the crest of another, or the depression of one with the depression of another, they produce waves of double the height or double the depression of either. But at those points where they meet under contrary phases, that is, where the crest of one wave coincides with the depression of the other, or *vice versâ*, then the waves obliterate each other, and no undulation takes place at such point. In the former case, the light at the point of intersection has double the intensity which it would have if the light from one focus alone was received; in the other case, the lights extinguish each other, and there is darkness.

Now it will be easy to show, that the bright curves indicated by the dotted lines in the figure correspond to points where the systems of waves intersect under the first condition above mentioned, and that the dark curves correspond to those points where they intersect under the second condition.

The middle line *o* O, which is a line of light, is at all its points equally distant from F and F′. Thus two lines F O and F′ O drawn from the focus to the same point in it are always equal; consequently the undulations which meet at any point such as O on this line, must necessarily meet under similar phases; for since the waves are of equal lengths, and since the distance F O is equal to the distance F′ O, the same number of waves and parts of a wave must occupy the two distances, and consequently the waves must arrive at O under corresponding phases.

The distance of any point of the first dark curve 1 1 from the focus F′ exceeds its distance from the focus F by half an undulation. If, therefore, the crest of a wave proceeding from F′ arrive at any point on this curve, the depression of a wave proceeding from F must arrive at the same point at the same time; and the same will be true of all points in the dark curve 1 1. The same observation will also be applicable to the curve 1′ 1′, only that in this case the distance of any point from F exceeds its distance from F′ by half an undulation.

Thus it appears that the waves propagated from the centres F and F′ always intersect on the dark curves 1 1 and 1′ 1′ under contrary phases, and consequently obliterate each other's effects and produce darkness.

The distance of any point in the bright curve 2 2 from F′ exceeds the distance of the same point from F by the length of a complete undulation; consequently, if the crest of a wave proceeding from F′ arrive at any point in such line, the crest of the preceding wave proceeding from F must arrive at it at the same time; and the same will be true for every point, so that throughout this bright line 2 2 the intersecting waves increase each other's effect. The same will be true of the line 2′ 2′. Hence the illumination produced along these two bright curves will be equal to the sum of the illuminations proceeding from the two foci.

In the same manner, it appears that the distance of any point on the dark curve 3 3 from F′ exceeds the distance of the same point from F by the length of an undulation and a half, and the same consequences as in the case of the first curve follow, so that the waves intersecting on the dark curves 3 3 and 3′ 3′, meet under opposite phases and obliterate each other.

It is evident, therefore, that the several hyperbolic curves formed by the successive light and dark lines on either side of the central bright line *o* O derive their character from the multiple of only half a wave's length, which expresses the difference between the distance of their successive points from the two centres of undulation F and F, which are the common foci of all the curves; and this multiple is in such case the length of the transverse axis of the hyperbola, of which the point C is the centre.

The spaces intervening between the bright and dark curves correspond to points where waves intersect under phases which are neither perfectly coincident nor perfectly opposite, and where consequently they only partially efface each other. Hence the light gradually diminishes in these spaces between the bright and the dark curves. The difference between the distances of these intermediate points from the foci F and F′ exceeds a complete number of half undulations by a quantity which is less than half an undulation.

1232. *How the phenomena of interference are affected by the different refrangibilities of different homogeneous lights.* — In what has been here stated, it has been assumed that the light proceeding from the points F and F′ is homogeneous light. Now there are, as has been shown, various species of homogeneous light, differing from each other in refrangibility and colour; and it is necessary to explain in what respects each variety of refrangibility and colour affects the phenomena of the bright and dark curves just explained. We find accordingly, that by causing pencils of homogeneous light of different colours and refrangibilities to intersect as above described, the bright and dark curves formed by their interference retain the character of the hyperbola, and that, although their general disposition on either side of the central line *o* O is the same, they are at different distances from each other; that is to say, the distance of the first bright curve 2 2 from the central line *o* O, as well as the distance of any two corresponding curves from each other, are different for different species of homogeneous light. In general, the more refrangible the light is, the nearer are the bright curves to each other. Thus the distance between one bright curve and another for violet or blue colour is less than the distance between the corresponding bright lines for red or orange colour.

1233. *The lengths of the undulations of the different homogeneous lights computed from the phenomena of interference.* — By an exact measurement of the dark and bright hyperbolic curves produced by each species of homogeneous light, aided by their known geometrical properties, Fresnel was enabled to deduce from these curves the lengths of the undulations of the ether which correspond to each species of homogeneous light. The following are the results of his observations and calculations:—

olour of homogeneous Light.	Length of Wave in ten-millionth Parts of an Inch.	Number of Undulations to an Inch.
violet	160	62,500
let	167	59,880
rdering on dark blue	173	57,803
e	177	56,497
e bordering on light blue	180	55,555
ae	187	53,422
e bordering on green	194	51,546
	205	48,780
rdering on yellow	209	47,847
	217	46,083
ordering on orange	225	44,444
	230	43,480
ordering on red	235	42,559
	244	40,983
red	254	39,370

Effects of the interference of compound solar light.—Since nces between the bright and dark curves are different for ies of homogeneous light, it follows, that if the light which from F and F′ be white solar light which is composed of all rs of the spectrum, we shall have all the systems of bright curves which would be separately produced by each of the it parts of the solar lights superposed, and a mixture of rill consequently ensue which will produce rows of fringes, rs of which will be determined by the prismatic tints which ius mingled together.

plete analysis of the combination of colour which would pro e fringes in the case of solar light would be extremely com Some idea, however, may be formed of the manner in e combination of colours is produced from *fig.* 396., in which ve breadths and distances of the light and dark curves produced by the three homogeneous lights, red, green, and violet, are represented. The series of red fringes with their alternate dark spaces are represented by R R, the series of green stripes are represented by G G, and that of violet stripes by V V. If these

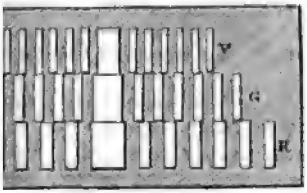

Fig. 396.

ered, instead of being placed as in the figure one above the be superposed, the effects which would be produced by a

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light proceeding from the two foci F and F′ composed of these three colours may be inferred.

1235. *Inflection or diffraction of light.* — If the rays of light diverging from a focus F, *fig.* 397., be incident upon an opaque object A B, all those rays of the pencil which are included within the angle A F B will be intercepted, so that a screen held at C D will receive none of those rays.

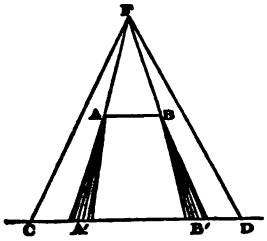

Fig. 397.

If the lines F A and F B be continued to A′ and B′, they will include upon the screen those spaces which would have been illuminated by the rays proceeding from F, which are intercepted by the opaque body A B. All the rays of the pencil included in the angles A F C and B F D will proceed uninterruptedly, and will fall upon the screen. If these rays underwent no change of direction, they would illuminate those portions of the screen included between C and A′ and D and B′. There would thus be an exact and well-defined shadow of the object A B formed upon the screen at A′ B′, and the remainder of the screen would be illuminated in the same manner as it would have been if the opaque body A B had not been present.

It is found, however, by experiment, that no such exact and well-defined shadow of the opaque object would be formed upon the screen.

The outline of the space which would limit an exact and geometrical shadow of A B being determined, it is found that within this space light will enter, and that outside this space the illumination is not the same as it would have been if the object A B had not been interposed.

From this it is inferred that the rays of light which pass the edge A B of the opaque object do not proceed in the same straight lines A A′ and B B′, in which they would have proceeded if the opaque object were not present. In a word, the appearance of the edge of the shadow is not a well-defined line separating the illuminated from the dark part of the screen, but a line of gradually decreasing brilliancy from the illuminated part of the screen to that in which the shadow becomes decided.

This effect produced by the edges of an opaque body upon the light passing in contact with them, by which the rays are bent out of their course either inwards or outwards, is called *inflection* or *diffraction.*

This phenomenon is a consequence of the general property of undulation explained in the Theory of Undulation (see Handbook of Sound, 810, 811, and 812). When the system of waves propagated round F as a centre encounters the obstacles A B, subsidiary systems of undulation will be formed round A and B respectively as centres,

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and will be propagated from those points independently of and simultaneously with the original system of waves whose centre is F, and which will also proceed towards C A′ and D B′. In a certain space round the lines A A′ and B B′, along which the rays grazing the edge of the opaque body would have proceeded, the two systems of undulation will intersect each other and produce the phenomena of interference.

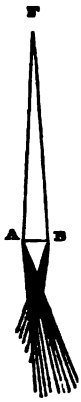

Fig. 398.

1236. *Combined effects of inflection and interference.*— If the opaque body A B be very small, and the distance of the focus F from it be considerable, the two pencils formed by inflection, of which A and B are the foci, will intersect each other as represented in *fig.* 398., and in this case all the phenomena of interference already described in 1231. will ensue. Thus, if the light be homogeneous, a bright line of light will be formed under the centre of the opaque object A B, outside which will be dark lines, and then bright and dark lines alternately. If the arrangment of these lines be examined, they will be found to be hyperbolic, as exhibited in *fig.* 395., and to vary in their relative distance with the quality of the light which radiates from the focus F. If the light radiating from such focus be compound solar light, then a series of coloured fringes will be formed, as already explained.

1237. *Examples of the effects of inflection and interference.*—The variety of optical phenomena produced by light passing the edges of small opaque objects, or small openings made in opaque plates, is infinite. The principles, however, on which all these appearances are explained, are the same.

The following experiments form examples of the variety of which these phenomena are susceptible.

I. If a small sphere formed of any opaque substance be suspended in a dark room, and a pencil of homogenous light be allowed to fall upon it, so that its shadow may be received upon a screen, it will be found that a bright spot will appear in the middle of the shadow, outside which will be a dark circle, beyond which there will be a bright circle, and beyond that a dark circle, and so on, the circles corresponding successively to the interference of the rays, by which their brilliancy is either doubled or extinguished, and the colour of the bright circles corresponding to that of the light.

If the light which falls on the sphere in this case be compound solar light, the central spot on the screen will be white, and will be surrounded by a series of coloured fringes, produced by the superposition of the coloured rings which would be produced separately by each compound of the solar light.

II. If a fine wire or sewing-needle be held close to one eye, the other being closed, and be looked at so as to be projected upon the light of a window, or a white screen, several needles will be seen.

III. If the eye be directed in a dark room to a narrow slit in the window-shutter by which light is admitted, several slits will be seen separated by dark bands.

IV. If a piece of card having a narrow incision made in it, be held between the eye and a candle, a series of slits will be seen parallel to each other, exhibiting the colours of the spectrum. The same appearance may be produced with increased effect by looking through the slit at the sun-light admitted through an opening in the window-shutter.

1288. *Phenomena of interference of light reflected and refracted by thin transparent laminæ.*—It has been already shown that when light passes from any transparent medium to another of different density, a part of it is reflected from their common surface, and a part only transmitted. Thus, when light passing through air is incident upon the surface of glass, a certain part of it is reflected from such surface, but the greater part enters it. When that portion which penetrates the glass arrives at the second surface, which separates the glass from the air, on the other side a like effect ensues, a portion of the light is reflected from the second surface, the greater part, however, penetrating it, and passing into the air. There are, therefore, two systems of reflected rays, one reflected from the first surface of the glass, and the other by the second surface.

The first system of reflected rays is thrown back immediately into the air; the second system is thrown back into the glass, and must pass through the first surface of the glass before it returns into the air.

If the two surfaces which thus successively reflect a portion of the light which passes through the transparent medium be very close together, and if they be not precisely parallel, the reflected rays will intersect each other, and produce the phenomena of interference.

1289. *Iridescence of fish-scales, soap-bubbles, mother-of-pearl, feathers, &c. explained.*—Hence arise the curious and beautiful appearances of iridescence which are observable whenever transparent substances are exhibited in sufficiently thin plates or laminæ, the prismatic colours observable in the scales of fishes, in spirit of wine spread in thin films on dark surfaces, in oil thinly diffused over the surface of water, and the thin laminæ of crystals and soap-bubbles, and bubbles of glass blown to extreme tenuity, in the laminæ of mother-of-pearl, and in the wings of insects and feathers of birds.

1240. *Newton's experimental illustration of the physical laws of such phenomena.*—In these and similar cases, the forms and thinness of the various laminæ being irregular, the iridescence affords no indications of general laws. Newton, however, by a series of beautiful experiments, reduced these phenomena to a form in which he was

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enabled to determine their laws, and by which they have since been shown in a rigorous manner to be consequences of the principle of interference.

Newton placed a flat plate of glass, D E, *fig.* 399., of uniform thinness, upon a convex lens A C B, of a very slight degree of convexity,

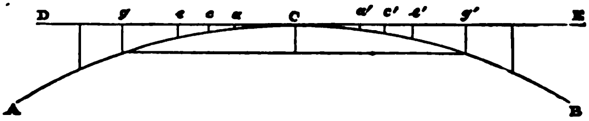

Fig. 399.

so that the surface of the two glasses should be separated by exceedingly minute spaces, even at considerable distances from the point of contact C. By this expedient a plano-concave lens of air of extreme thinness was formed by the two glasses.

Let us now suppose homogeneous light to fall upon the surface D E. The appearance will be that of a dark spot in the centre C, surrounded by a bright ring, outside which is a dark ring, followed by a bright ring, and so on, a series of bright and dark rings being formed round the central black spot.

If homogeneous light of different colours be successively thrown upon the glass, a system of rings, such as here described, will in each case be produced; but their diameters will be different, the rings being closer together for the more refrangible than for the less refrangible lights. If compound solar light be allowed to fall upon the glass, a series of rings will be formed of colours which would be produced by the superposition of all the systems of rings which are separately formed by the various homogeneous lights which form the compound solar light.

1241. *Newton's coloured rings explained by interference.*—These phenomena, which were explained by Newton upon an hypothesis called by him the *theory of fits*, of easy reflection and transmission, are easily explicable upon the principle of interference.

When homogeneous light falls upon the glass, a portion of it is reflected from the under surface of the plate D E, which separates the glass from the thin plano-concave lens of air. Another portion is reflected after passing through the air from the convex surface of the glass. These two systems of rays being reflected from surfaces not precisely parallel, intersect each other, and alternately destroy or increase each other's effect, according as the waves of light meet under the same or different phases. The dark rings comprehend the intersections under different phases, and the light rings the intersections under the same phases.

The thinness of the lens of air at the successive dark and bright rings respectively determine the difference between the lengths of the intersecting rays measured from their origin to either point of intersection, and thus show where the point of intersection comprehends the waves meeting under the same or under different phases. The measurement of this thickness, accordingly, at the bright and dark rings, is found to be in entire accordance with the calculations already made, and explained in the table of the length of the waves of homogeneous light of different colours.

CHAP. XVIII.

DOUBLE REFRACTION.

1242. *Transparent media resolved into two classes.*—Transparent substances consist of two classes, which present optical phenomena depending on certain physical properties inherent in the constitution of each class of media respectively.

The phenomena, both optical and physical, suggest in the first class the supposition that they consist of molecules which are uniform in their form and reciprocal effects, so that the forces which they exercise one upon the other are the same in every direction. To this class belong every species of æriform fluid, all liquids, and certain transparent solids, such as glass, when properly annealed.

1243. *Single refracting media.*—In all these substances the constituent molecules appear to be so arranged, that we might conceive them to be spherules of matter, from the centres of which forces emanate which are equal in every direction.

1244. *Double refracting media.*—The second class of substances, which includes crystallized minerals, generally exhibits phenomena which lead to the supposition that their constituent molecules are not spherules, or, at least, that they do not exercise like forces in all directions round their centres. The phenomenon of crystallization, explained in the Handbook of Mechanics, sections 60–66., itself suggests this supposition; for when a substance passes from the liquid to the solid state, and undergoes the process of crystallization, the particles affect a particular arrangement with reference to one another, so as to present themselves towards each other in certain directions, as if they had sides which mutually attracted or repelled each other.

1245. *Effects of an uncrystallized medium on light.*—To render more clearly intelligible the effects produced by crystallized substances on light transmitted through them, we shall first briefly recapitulate the effects produced on rays of light by an ordinary transparent unrystallized medium, such as air, water, or glass.

Let us suppose such a substance reduced to the form of a sphere, h, if it be gas or liquid, may be done by enclosing it in a thin

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globe of glass; and if it be a solid, it may be reduced to the spherical form in the lathe.

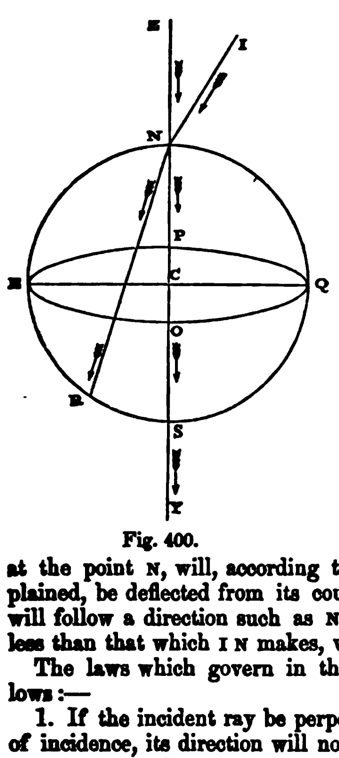

Fig. 400.

Let E N Q S, *fig*. 400., represent a section of this transparent sphere, and E P Q O another section at right angles to it.

Let Z N and I N represent two rays of homogeneous light incident at N, one in a direction which, being continued, would pass through the centre O of the sphere, and the other, I N, in a direction oblique to the former.

If the sphere be composed of non-crystallized transparent matter, the ray Z N will pass through it, pursuing the original direction, and consequently, after passing the centre C, will emerge from the lowest point S in the direction S Y, so that its course shall be in no wise changed by the transparent medium through which it has passed; but the ray I N, which falls obliquely at the point N, will, according to the law of refraction already explained, be deflected from its course towards the diameter N C S, and will follow a direction such as N R, which makes an angle with N S less than that which I N makes, with N Z.

The laws which govern in this case the refracted ray are as follows:—

1. If the incident ray be perpendicular to the surface at the point of incidence, its direction will not be changed in passing through the transparent medium.

2. If the incident ray form an angle, such as I N Z, with the perpendicular N Z at the point of incidence, then the refracted ray N R will form an angle with the same perpendicular N Z, or with its production N S, the plane of which will coincide with the plane of the angle of incidence Z N I.

3. If the angle of incidence I N Z be varied, the angle of refraction R N S will be also varied, but in such a manner that the ratio of the sine of the angle of incidence I N Z to that of the angle of refraction R N S shall always be the same, so long as the transparent medium into which the ray passes is the same.

4. If while the incident rays Z N and I N preserve their position, the sphere be turned round its centre C, so as to bring successively every part of its surface to coincide with the point of incidence N, the

[redacted]

refracted ray N R will still maintain the same direction and position, and the ray Z N will still pass through the centre of the sphere C, no matter what position may be given to the sphere, so long as the position of its centre C remain unchanged.

Thus the direction and position of the incident rays I N and Z N, and of the refracted rays N R and N S, will remain fixed, although the transparent sphere which they penetrate may be changed in an infinite variety of ways, so as to bring all its points in succession to coincide with the point of incidence N of the rays.

Such are the phenomena which are produced when the rays I N and Z N are incident upon a sphere composed of uncrystallised transparent substance. The same phenomena will always prevail in the case even of certain crystallized substances; but in the case of other crystallised media, different and far more complicated phenomena are developed, which we shall now proceed to explain.

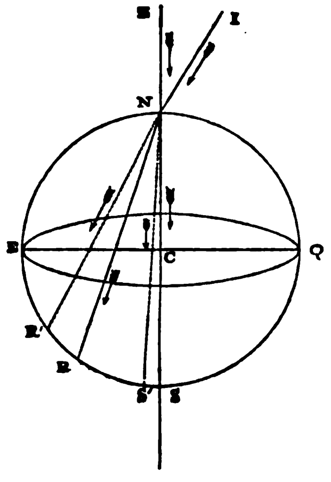

Fig. 401.

1246. *Effects of certain media on light.*—Let a sphere be formed of one of the class of crystals of which Iceland spar or the crystallized carbonate of lime is a specimen, and let this sphere be submitted to the same experiments as have been described in the former case. When the rays I N and Z N, *fig.* 401., penetrate the sphere at N, they will each of them be resolved into two rays, one of which, in the figure, is indicated by the uniform line, and the other by the dotted line. The rays indicated by the uniform lines N S and N R, will conform to the laws of refraction which prevail in uncrystallized media; that is to say, the ray N S will pass through the centre of the sphere C, preserving the direction of the incident ray Z N, which strikes the surface of the sphere at N in a perpendicular direction, and the ray N R will be in the plane of the angle of incidence I N Z. Also, if the ray I N be made to fall at N, so as to form any other angle of incidence, the ray N R will vary its inclination to the perpendicular N S, in conformity with the law of refraction, which establishes a constant ratio between the sines of the angles of incidence and refraction.

But none of these characters are found to attend the other rays N S' and N R', into which the original incident rays are resolved by the crystal.

The ray N S', although proceeding from the ray Z N, which is incident perpendicularly at the point N, does not penetrate the medium in the same direction, but makes a certain angle S′ N S with the perpendicular. Thus, in the case of this ray there is an acute angle of refraction corresponding to perpendicular incidence. In the case of the ray N R′ it is found that it deviates on the one side or the other of the plane of the angle of incidence I N Z, and thus this ray violates that general law of common refraction which declares that the plane of the angle of refraction coincides with the plane of the angle of incidence.

If the angle formed by the incident ray I N with the perpendicular Z N be varied, the angle which the refracted ray N R′ makes with the perpendicular N S will be also varied, but not according to the law of sines which prevails in the case of ordinary refraction.

1247. *The ordinary and extraordinary rays.* — Thus it appears that in such crystallized media the incident ray is resolved into two rays, one of which conforms to the laws of common refraction, and the other violates them, and is regulated by other and different conditions. The two rays into which the incident ray is thus resolved are called the *ordinary* and *extraordinary* rays; that which conforms to the laws of common refraction being called the *ordinary*, and that which violates them the *extraordinary* ray.

If the sphere be now supposed to be moved, as before, round its centre C, so as to bring successively all the points of its surface to coincide with the point of incidence N, it will be found that the ordinary rays N S and N R will preserve their direction and position fixed in all positions which the sphere shall assume; but that the direction and position of the extraordinary rays N S′ and N R′ will vary with every change of position of the sphere. They will sometimes approach to, and sometimes recede from the ordinary rays; and they will sometimes deviate on one side, and sometimes on the other, of the plane of the angle of incidence; but in all cases there will be a maximum deviation from the ordinary ray, which will not be exceeded.

1248. *The axis of double refraction.* — By varying the position of the sphere so as to bring the various points of its surface to coincide with the point of incidence N, a point will be found upon it at which the extraordinary ray N S′ will coincide with the ordinary ray N S. As this point approaches the point N, the angle S′ N S under the ordinary and extraordinary ray will be observed continually to diminish; an effect which will indicate the change of position necessary to bring the desired point to coincide with the point of incidence N.

This point of the sphere then possesses a distinctive character, in virtue of which the incident ray Z N is not, as at all other points, resolved into two rays, but passes through the sphere in the direction N O S, exactly as it would pass through a sphere composed of an uncrystallised substance.

[redacted]

The diameter of the sphere which possesses this property is called its *optical axis*, or the *axis of double refraction*, being the only line in the sphere along which a ray of ordinary light can pass without being decomposed into two.

1249. *Laws of double refraction.* — Having thus determined this optical axis of the sphere, let us next examine the conditions which affect a ray of light, such as I N, which falls obliquely at the extremity of such optical axis.

Let N C S, *fig.* 402., be the optical axis of the sphere. The ray Z N will then, as has just been explained, pass through the centre C to the point S, without double refraction, as it would through an ordinary medium. The ray I N, which falls obliquely at N, will, however, be doubly refracted, and will be resolved into the ordinary ray N R, and the extraordinary ray N R′. But this extraordinary ray N R′ will, in this case, conform to one of the laws of ordinary refraction, for it will invariably lie in the plane of the angle of incidence I N Z; and so long as the angle of incidence shall not be varied, the direction of this extraordinary ray will remain the same. This may be proved by causing the sphere to revolve round the axis N S. While it so revolves, the extraordinary ray N R′ will remain fixed in its direction, being always in the plane of the angle of incidence, and forming always the same angle of refraction with the axis N S.

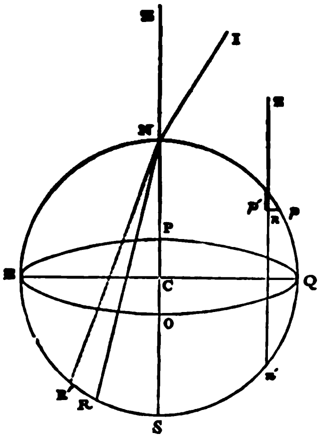

Fig. 402.

If the incident ray I N be varied in its inclination, so as to form, as before, a greater angle with Z N, the extraordinary ray N R′ will also vary its inclination to the axis N S and to the ordinary ray N R. But, although it will remain during such variation always in the plane of the angle of incidence, it will not conform to the invariable ratio of sines which constitutes the law of ordinary refraction.

If we suppose the incident ray I N gradually to approach Z N, so that the angle of incidence continually diminishes, then the two rays N R and N R′ will at the same time approach the axis N S and each other; and when the incident ray coincides with Z N, the ordinary and extraordinary rays N R and N R′ will coalesce with the axis N S.

As, on the other hand, the inclination of the ray I N to Z N is gradually increased, the ordinary and extraordinary rays N R and N R′ will also gradually recede from the axis N S, so that their angles of refraction will continually increase, and they will also recede from each other.

1250. *Positive and negative crystals.* — In the case represented in the figure, the angle of refraction of the extraordinary ray N R′ is greater than that of the ordinary ray N R, so that the latter is more deflected by the refraction of the crystal than the former. This, however, is not always the case.

In some crystals the angle of refraction of the extraordinary rays is less than that of the ordinary ray, and, consequently, the former is more deflected towards the perpendicular than the latter.

Crystals are accordingly resolved into two classes, based upon this distinction; those in which the extraordinary ray is less deflected than the ordinary ray being called *negative crystals,* and those in which it is more deflected *positive crystals.*

It is evident that in the former case the index of ordinary refraction is greater, and in the latter less than the index of extraordinary refraction.

It must be observed, that while the incident ray varies its obliquity to Z N, increasing gradually from 0 to 90°, and while the index of ordinary refraction throughout this variation remains constant, the index of extraordinary refraction varies with every change of obliquity. In the case of positive crystals this index increases, in the case of negative crystals it diminishes, with the angle of incidence; while, in all, it is equal to the index of ordinary refraction when the ray of I N coincides with Z N. It increases and becomes a maximum when I N is at right angles to Z N in positive crystals, it diminishes and becomes a minimum when I N is at right angles to Z N in negative crystals.

1251. *All lines parallel to the axis of double refraction are themselves axes of double refraction.* — It is easy to show that all lines passing through the crystal which are parallel to the line N S possess also the property which characterizes such axis; that is to say, a ray which is incident perpendicularly in the direction of such lines will penetrate the crystal without double refraction. This we may prove by cutting a portion of the crystal in a direction perpendicular to the line N S.

Thus, at the point *p*, let a surface *p p′* be formed, which shall be perpendicular to N S. Then a ray *z n*, falling perpendicularly on such surface *p p′* will penetrate the crystal in the direction *n n′* without double refraction.

1252. *Axis of double refraction coincides with crystallographic axis.*—Thus it appears that the lines passing through the crystal parallel to N S are axes of double refraction as well as the line N S. On comparing the direction of the line N S with the direction of the planes of cleavage of the crystal, it is found that this line has a direction which is symmetrical with respect to all these planes, and that it is

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in fact the direction of the crystallographic axis; that is to say, a line the direction of which bears the same relation to all the faces of the crystal.

1253. *Case of Iceland spar.*—Thus in the case of Iceland spar, the primitive form of whose molecules is that of such a rhomboid as is represented in *fig.* 403., the crystallographic axis is the diagonal A X joining the obtuse angles of the rhomb. The rhomb itself is a solid bounded by six equal and similar parallelograms, whose obtuse angles B A C and C D B are each 101° 55′, and whose acute angles A B C and A C D are accordingly each 78° 5′. The inclination of the faces of the rhomb, which meet at A, to each other is 105° 5′, consequently the inclination of those which meet at B is 74° 55′. The crystallographic axis A X is equally inclined, not only to the three faces of the rhomb, which meet at A and X respectively, but also to its three edges. The angles which this axis makes with the three edges of the rhomb forming the angle A are equal to each other, their common magnitude being 66° 44′ 46″.

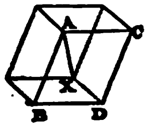

Fig. 403.

It is evident from this measurement, that the line A X is symmetrically placed with respect to all the elements which determine the primitive form of the crystal, and we thus find accordingly a distinct relation established between the optical and mineralogical characters of this substance, so that whenever the direction of its crystallographic axis is required to be ascertained, it can be done without any mechanical experiment or measurement, by merely determining that direction in which a ray of light incident perpendicularly on a surface of the crystal will pass through it without double refraction.

What has been here stated with regard to Iceland spar will, *mutatis mutandis*, be applicable to a numerous class of crystallized substances, which are distinguished by the denomination of crystals having a single axis of double refraction.

In all such crystals the crystallographic axis coincides with the optical axis.

1254. *General description of the phenomena of double refraction in uni-axial crystals.*—In attempting to explain the complicated phenomena of double refraction and other effects related to them, much convenience and clearness will be obtained by the adoption of a nomenclature indicating the position of the axis of double refraction in certain sections of the crystal analogous to the well-known circles used in geography and astronomy for expressing the relative position of points on the earth and in the heavens. We shall therefore call the extremities of the axis N and S the *poles* of the crystal, and a section of the crystal E P Q O, *fig.* 402., intersecting this axis at right angles the *equator*. We shall also call all sections of the crystal made by planes passing through the axis *meridians*.

These terms being understood, it will follow that whenever the plane of the angle of incidence coincides with the plane of a meridian, the angles of refraction, both of the extraordinary and ordinary rays, will be in the plane of the same meridian; but the ratio of the sine of the angle of incidence to the sine of the angle of extraordinary refraction will not in this case be constant.

If the plane of the angle of incidence intersect the crystal at right angles to the optical axis N S, and be consequently parallel to the line coincident with the plane of the equator, the angle of extraordinary refraction will have its plane coincident with that of the angle of incidence, thus fulfilling one of the laws of ordinary refraction, as is the case when the plane of the angle of incidence coincides with the plane of a meridian. But in this case the second law of refraction, which establishes a constant ratio between the sines of the angles of incidence and refraction, is also fulfilled by the extraordinary ray, so that when the angle of incidence coincides with, or is parallel to, the plane of the equator, the extraordinary refraction fulfils all the conditions of ordinary refraction, although the extraordinary ray does not coincide with the ordinary ray; the constant index of refraction of the one being greater or less than the constant index of refraction of the other, according as the crystal is positive or negative.

There are therefore two systems of planes which intersect crystals, one system having the axis of the crystal as their common line of intersection, and the other having directions parallel to each other and perpendicular to this axis. In the former, one of the laws of ordinary refraction is fulfilled, and in the latter both of them. In the former, the plane of the angle of extraordinary refraction coincides with the plane of the angle of incidence, but the ratio of the sines is not constant; in the latter, the planes also coincide, and the ratio of the sines is constant, but not the same as that of the ordinary ray.

1255. *Table of uni-axial crystals.* — The following is a table, according to Sir David Brewster, of the crystals which have a single axis of double refraction, arranged under their respective primitive forms; the sign + being prefixed to those which have a positive axis of double refraction, and — to those which have a negative axis of double refraction.

Rhomb with obtuse summit.

— Carbonate of lime (Iceland spar).
— Carbonate of lime and iron.
— Carbonate of lime and magnesia.
— Phosphato-arseniate of lead.
— Carbonate of zinc.
— Nitrate of soda.
— Phosphate of lead.
— Ruby silver.
— Levyne.
— Tourmaline.
— Rubellite.
— Alum stone.
+ Dioptase.
+ Quartz.

Rhomb with acute summit.

— Corundum.
— Sapphire.
— Ruby.
— Cinnabar.
— Arseniate of copper.

[redacted]

Regular hexahedral prism.

— Emerald.
— Beryl.
— Phosphate of lime (apatite).
— Nepheline.
— Arseniate of lead.
+ Hydrate of magnesia.

Octohedron with a square base.

+ Zircon.
+ Oxide of tin.
+ Tungstate of lime.
— Mellite.
— Molybdate of lead.
— Octohedrite.
— Prussiate of potassa.
— Cyanuret of mercury.

Right prism with a square base.

— Idocrase.
— Wernerite.
— Paranthine.
— Meionite.
— Somervillite.
— Edingtonite.
— Arseniate of potassa.
— Subphosphate of potassa.
— Phosphate of ammonia and magnesia.
— Sulphate of nickel and copper.
— Hydrate of strontia.
+ Apophyllite of uton.
+ Oxahverite.
+ Superacetate of copper and lime.
+ Titanite.
+ Ice (certain crystals).

M. Pouillet, "Élémens de Physique," tome ii., Paris, 1847, gives also the following: —

— Hydrochlorate of lime.
— Hydrochlorate of strontia.
+ Mica de kariat.
+ Oxide of iron.
+ Tungstate of zinc.

+ Stannite.
+ Boracite.
+ Sulphate of potassa and iron.
+ Hydrosulphate of lime.
+ Red silver.

1256. *Crystals having two axes of double refraction.* — There is another class of crystals which present optical phenomena still more complicated. Let us suppose, as before, one of these formed into a sphere, and let its various points, as before, be brought to coincide with the point of incidence N of two rays, one of which, Z N, *fig.* 402., is directed to the centre of the sphere, and the other I N forming any angle with the latter. By bringing the various points of the spherical surface to coincide with the point N, it will be found that two points, and two only, upon it possess the property of transmitting the ray Z N, which falls perpendicularly upon the surface, through the object without double refraction. The diameters passing through these two points have each of them the character of an axis of double refraction; and the crystals characterized by this property are accordingly called crystals with two axes of double refraction.

In this class of crystals it is found that neither of the rays into which the incident ray is resolved conforms to the laws of ordinary refraction; that both deviate from the plane of the angle of incidence, and that neither of them fulfils the second law, which determines the constant ratio between the sines of incidence and refraction.

Both rays, therefore, are extraordinary rays.

There are, however, two planes in which the angle of incidence may be placed, in one of which one of the two rays and in the other

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the other will conform to both the laws of ordinary refraction, so that in these planes one or other of the two extraordinary rays becomes an ordinary ray. The position of these planes is determined by the following conditions.

Let N S and N′ S′, *fig.* 404., be the two axes of double refraction. Let P P′ be a line which divides into equal parts the angle N C N′ formed by these two angles, and let Q Q′ be a line which divides into equal parts the other angle N′ C S formed by the same axis.

If a plane pass through C perpendicular to P C, any ray incident upon the crystal in that plane will be resolved into two rays, one of which will conform to the laws of ordinary refraction; and if a plane be drawn perpendicular to the line Q Q′, any ray incident upon the crystal in that plane will be resolved into two, one of which will also conform to the laws of ordinary refraction, and the ray which thus becomes an ordinary ray in the one plane will be different from that which becomes an ordinary ray in the other plane.

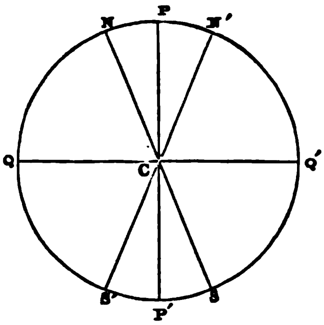

Fig. 404.

1257. *Table of bi-axial crystals.* — The following list of crystals having two axes of double refraction, with the magnitude of the angle included between such axes, is given by M. Pouillet in the work already cited.

TABLE OF CRYSTALS WITH TWO AXES.

Names of Substances.	Angles of Axis.	
	°	′
Sulphate of nickel (certain samples)	8	0
Sulpho-carbonate of lead	"	"
Carbonate of strontia	6	56
Carbonate of baryta	"	"
Nitrate of potassa	5	20
Mica (certain samples)	6	0
Talc	7	24
Pearl	11	28
Hydrate of baryta	13	18
Mica (certain samples)	14	0
Arragonite	18	18
Prussiate of potassa	19	24
Mica (certain samples)	25	0
Cymophane	27	51
Anhydrite	28	7
Borax	28	42

Names of Substances.	Angles of Axes.	
	°	′
Mica (several samples examined by M. Biot)	30	0
	31	0
	32	0
	34	0
	37	0
Apophyllite	35	8
Sulphate of magnesia	37	24
Sulphate of baryta	37	40
Spermaceti (about)	37	42
Borax (native)	38	48
Nitrate of zinc	40	0
Stilbite	41	42
Sulphate of nickel	42	4
Carbonate of ammonia	43	24
Sulphate of zinc	44	28
Anhydrite (examined by M. Biot)	44	21
Mica	45	0
Lepidolite	45	0
Benzoate of ammonia	45	8
Sulphate of soda and magnesia	46	49
Sulphate of ammonia	49	42
Brazilian topaz	49 to 50	0
Sugar	50	0
Sulphate of strontia	50	0
Sulpho-hydrochlorate of magnesia and iron	51	16
Sulphate of magnesia and ammonia	51	22
Phosphate of soda	55	20
Comptonite	56	6
Sulphate of lime	60	0
Oxynitrate of silver	62	16
Iolite	62	50
Feldspar	63	0
Aberdeen topa	65	0
Sulphate of potassa	67	0
Carbonate of soda	70	1
Acetate of lead	70	25
Citric acid	70	29
Tartrate of potassa and soda	80	0
Carbonate of potassa	80	30
Cyanite	81	48
Chlorate of potassa	82	0
Epidote	84	19
Hydrochlorate of copper	84	30
Peridot	87	56
Succinic acid	90	0
Sulphate of iron	90	0

1258. *Images formed by double refracting crystals.* – If a visible object be placed behind a double refracting crystal, the pencil of rays proceeding from each point in it will be resolved into two pencils, and will emerge from the crystal as if they had proceeded from two different objects in directions corresponding to the respective directions of the two pencils.

An eye, therefore, placed before the crystal, so as to receive these emerging pencils will see two different images of the object, corresponding to the two systems of pencils. If the crystal be one having a single axis of double refraction, then one of these images will be that produced by the pencils consisting of ordinary rays, and the other will be that produced by pencils consisting of extraordinary rays.

1259. *Ordinary and extraordinary image.*—The one is called the ordinary, the other the extraordinary image.

Thus, if P, *fig.* 405., be such an object, and A B C D be a double refracting crystal, such as Iceland spar, the pencils which proceed from P and are incident upon the surface B C will be divided into two systems of pencils, the axis of the ordinary system passing perpendicularly through the crystal in the direction I O, and emerging on the other side in the same direction, so as to meet the eye at Y. The extraordinary pencils will follow the direction I E through the crystal, and will emerge parallel to the ordinary pencil in the direction E Y′, so as to reach the eye at Y′. An eye placed therefore at any point, in looking towards the crystal, will perceive two images of the point P in juxtaposition in the direction of the rays Y′ E and Y O.

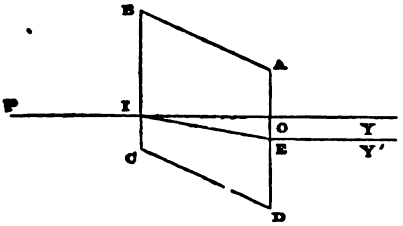

Fig. 405.

1260. *The separation of the images dependent on the thickness of the crystal.*—It is evident that the thicker the crystal is, the more widely separated will be these two images.

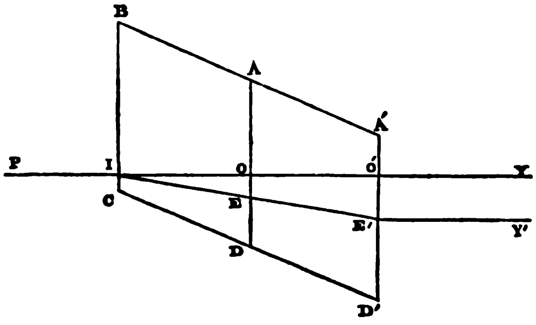

Fig. 406.

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A crystal of Iceland spar three inches thick will be sufficient to produce a distinct separation of the two images of a spherical object having a diameter of one-third of an inch.

If while the object and the eye remain fixed, the crystal be turned round the line P Y, joining the eye and the object as an axis, the extraordinary image will appear to revolve round the ordinary image, showing that in this case the extraordinary pencil I E revolves round the ordinary pencil I O, so as to move in the surface of a cone. This effect is in conformity with what has been already explained.

If, after passing through a crystal A B C D, *fig.* 406., the rays be received by another crystal A A′ D′ D, whose sides and axes have a position similar to those of the first, the two crystals being in contact at the surface A D, the ordinary and extraordinary rays will pass through the second crystal, following the same direction as those which they followed in the first crystal, the lines O O′ and E E′ being the continuation of the lines I O and I E.

1261. ***Case in which two similar crystals neutralize each other.***— If the two crystals in this case have the same thickness, then the effect will be that the rays E′ Y′ and O′ Y emerging from the second will be separated by a space twice as great as that by which they were separated in passing through the first crystal.

If the second crystal, instead of having been placed upon the first crystal so that its corresponding sides shall have the same direction, be placed upon it so that they shall have contrary directions, as represented in *fig.* 407., then the second crystal will have the effect of causing the reunion of the two pencils separated by the first crystal,

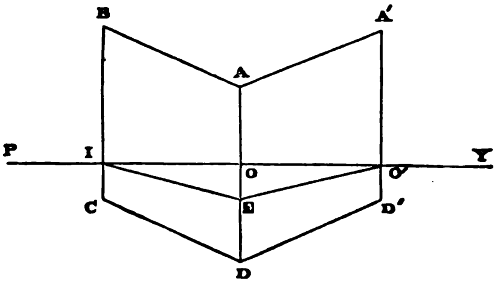

Fig. 407.

and the ordinary and extraordinary rays will accordingly emerge from the same point O′ of the second crystal in the same direction, so that an eye placed at Y will see but one image of the object P. In this case the ordinary ray follows the direction P I O O′ Y, and the extraordinary ray follows the direction P I E O′ Y. Thus, the separation of

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the rays takes place only in passing through the crystals, the reunion being established at the point of emergence O′ from the second crystal.

1262. *Cases in which four images are formed by the combination of two similar crystals.* — If we suppose the second crystal, A A′ D′ D, *fig.* 406., to be turned round the line P I O Y as an axis, the moment it moves from the position represented in *fig.* 406., the ordinary and extraordinary rays I O and I E incident upon it from the first crystal will be each doubly refracted, so as to be resolved into four rays, and thus an eye placed at Y would see four images of the point P. As the second crystal is gradually turned round, these four images assume a series of different positions with relation to each other, and also have different degrees of brilliancy. After the crystal has made one half a revolution and assumed the position represented in *fig.* 407., all these four images unite in one. In the position intermediate between these two, that is to say, when the second crystal has made a quarter of a revolution round the line P I O Y, then the four images will be reduced to two, which, however, will have a different position relative to the line A D from that which the images produced in the position represented in *fig.* 406. have.

1263. *Their successive positions.* — The successive positions assumed by the four images during the half revolution of the second crystal between the position represented in *fig.* 406., and that represented in *fig.* 407., are given in *fig.* 408., where B represents the position of the images corresponding to *fig.* 406., and K to *fig.* 407.; F represents their position when the second crystal has made one-fourth of a revolution; C, D, and E represent three successive positions of the images in three equally distant stages of the first quarter of a revolution; and G, H, and I represent their respective positions in three equally distant stages of the second quarter of a revolution. The relative brilliancies of the images are indicated by the shading of the dots, the dark dots being understood to represent greater brilliancy than the shaded ones.

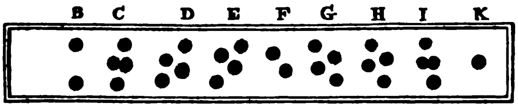

Fig. 408.

1264. *Position of axes different for different coloured lights in bi-axial crystals.* — In uni-axial crystals the axis has the same position, whatever be the colour of the light, but in bi-axial crystals the position of the axes is different for different coloured lights. Sir John Herschel found that in tartrate of potassa and soda (Rochelle salts), their inclination for violet light was 56°, and for red light 76°. In

[redacted]

other crystals, such as nitre, their inclination for violet was greater than for red, but in all cases the axes for all coloured light in the same crystal are in the same plane. Sir David Brewster found that glauberite had two axes, for red light, inclined at an angle of 50°, and only one for violet light. The same eminent philosopher found that in the case of analcine there were several planes along which there was no double refraction, however various the angle of incidence might be, so that that substance might be considered as having an infinite number of axes of double refraction.

1265. *Doubly refracting structure produced by artificial processes.* — The property of double refraction may in some cases be imparted by artificial processes to substances which do not naturally possess it. If a cylinder of glass be brought to a red heat, and held upon a plate of metal until it becomes cold, it will acquire the doubly refracting property, the axis of the cylinder being a single positive axis of double refraction. This axis differs, however, from the positive axis of crystallisation, because in this case it is a single line, while in the crystal the lines parallel to it are equally axes of double refraction. Sir David Brewster says, that if instead of heating the cylinder it had been immersed in a vessel of boiling water, it would have acquired the same doubly refracting virtue when the heat had reached its axis, but that the property would not be permanent, disappearing when the cylinder should become uniformly heated. Also if the cylinder were uniformly heated in boiling oil, or at a fire so as not to soften the glass, and had been placed in a cold fluid, it would acquire a temporary doubly refracting virtue when the cooling had reached the axis; but in this case the axis would be a negative one, instead of a positive, as in the former case.

According to him some other analogous structures may be produced by pressure, and by the induration of soft solids, such as animal jellies, isinglass, &c.

If the cylinder in the preceding explanations is not a regular one, but have its section perpendicular to the axis, every where an ellipse in place of a circle, it will have two axes of double refraction.

In like manner, if we use rectangular plates of glass instead of cylinders, as in the preceding experiment, we shall have plates with two planes of double refraction, a positive structure being on one side of each plane, and a negative one on the other.

If we use perfect spheres there will be axes of double refraction along every diameter, and consequently an infinite number of them.

The crystalline lenses of almost all animals, whether they are lenses, spheres, or spheroids, have one or more axes of double refraction.

[redacted]

CHAP. XIX.

POLARIZATION OF LIGHT.

1266. *Characteristic property of polarized light.* — When a ray of light, whether natural or artificial, has been submitted, under peculiar conditions, to reflection or refraction, it is then in a state in which it acquires new properties, and is denominated polarized light; and the process by which this modification in the ray is effected is called polarization.

To render the properties by which polarized is distinguished from unpolarized or common light clearly intelligible, let us imagine a ray of light admitted into a dark room through a hole in the window-shutter, so as to pass in a horizontal direction. Supposing such a ray to be cylindrical, let its section, made by a vertical plane, be represented by the circle A C B D, *fig.* 409.

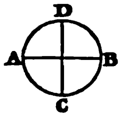

Fig. 409.

This ray, if it were common or unpolarized light, would be reflected or refracted in exactly the same manner, and according to the common laws of reflection and refraction already explained, on whatever side of it, and at whatever angle with it, the reflecting or refracting surface might be presented.

If, however, the ray be polarized, the effects will be different.

Let a plate of glass be blackened on one side so that when used as a reflector no light will be reflected from its posterior surface. Such a plate will therefore reflect light only from one surface, which will be its anterior surface. This precaution is necessary in the cases now to be examined, in order to prevent the effects which would ensue from the combination of the rays, which would otherwise be reflected from both the anterior and posterior surfaces of the glass.

Let such a plate, so prepared, be presented to the polarized ray at an angle of incidence of 54° 35′, so that the plate shall make with the ray an angle of 35° 25′; and let it be turned round the ray, so as to be presented on every side of it, still forming, however, the same angle with it. During this process, it will be observed that there is a certain direction of the plane of the angle of incidence at which no reflection will take place; the ray will be absorbed or extinguished, so to speak, by the reflecting surface. The plane of incidence will have this direction in two opposite positions of the reflector.

Let the line D C, *fig.* 409., represent this position of the plane of incidence: then D and C will be the two opposite sides of the ray, at which the reflector being presented will cause the extinction of the light. Now as the reflector is carried round from either of these positions respectively, so that the plane of the angle of incidence shall turn round the axis of the ray, reflection will begin to take place, and will increase in intensity until the plane of the angle of incidence take a position, such as A B, at right angles to D C, when the intensity of the reflection will be a maximum. After passing this position, the intensity of the reflection will again diminish, and will continue to decrease until the plane of the angle of incidence shall again coincide with the diameter D C.

It is evident, therefore, that different sides of such a ray have different properties. Thus, the sides A and B have a susceptibility of being reflected, of which the sides D and C are deprived; and the susceptibility of reflection diminishes gradually in going round the ray from either A or B towards C or D, when it altogether ceases.

A plane passing through the axis of the ray and coinciding with the diameter A B is called the plane of polarization.

It is evident, therefore, from what has been explained, that when the reflector is so presented to the ray that the plane of the angle of incidence shall coincide with the plane of polarization, reflection will take place with the greatest intensity, and that when the plane of the angle of incidence is at right angles to the plane of polarization, no reflection takes place, and the ray is extinguished.

1267. *Angle of polarization.* — If, instead of glass, any other reflecting surface be used, like effects would be produced; only that the angle at which it would be necessary to present the reflecting surface to the ray would be different, each species of reflector having its own particular angle.

This angle is, for reasons which will be hereafter explained, called the angle of polarization.

1268. *Polariscopes.* — Instruments called *Polariscopes* adapted for the experimental illustration of the phenomena of polarization, have been constructed in various forms.

One of the most convenient for the purposes of elementary explanation consists of several detached pieces, which are represented in *fig.* 410. A B is a brass tube like that of a telescope, along the axis of which the polarized pencil to be submitted to examination is transmitted. C is a short tube capable of being inserted, after the manner of telescopic tubes, in the main tube at A. This tube C carries a plane reflector D, of the blackened glass already described, which is capable of being turned on pivots, and is supplied with a double scale and index, by which the angle it makes with the axis of the tube can be regulated at pleasure. By turning the tube C round its axis, the plane of the reflector D may be presented successively on every side of the axis of the main tube.

A diaphragm is fixed in the tube at *d*, having a circular hole in its centre, to limit the magnitude of the transmitted pencil. The pieces E, F, G, and H, are severally capable of being inserted in the ends of the tube, and of being turned round in the same manner as already described with relation to the piece C inserted at the end A.

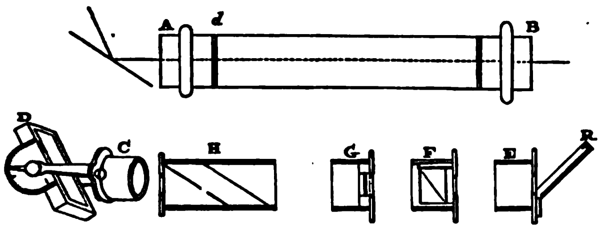

Fig. 410.

The short tube E carries a plane reflector R, similar to that already described, which is capable of being adjusted at any desired angle with the axis of the tube. The tube F contains a doubly refracting prism, the tube G contains a thin disk of tourmaline with parallel faces, so cut that the optic axis is parallel to these faces. In fine, the tube H contains a bundle of plates of glass, with parallel surfaces placed in contact with each other, and inclined obliquely to the axis of the tube.

All these pieces being severally inserted in the tube A B can be turned round its axis, so that the reflector R, or the prism, or the tourmaline G, or the included plates H, may be severally presented in succession on all sides of the ray transmitted along the axis of the tube A B.

1269. *Polarization by reflection.* — Let the tube C, *fig.* 410., carrying the reflector D, be inserted in the main tube A, and let a plate of blackened glass be inserted in the frame D, as already described. Let the apparatus be so adjusted that when a ray of light falling upon the plate D at an angle of incidence equal to 54° 35′ is reflected, the reflected ray will pass along the axis of the tube A B. Such a ray will be polarized, and the plane of its polarization will coincide with the plane of the angle of incidence upon the plate D.

To prove this, let the tube E carrying the reflector R be inserted in the end B of the main tube, and let the reflector R be adjusted so that the ray which passes along the axis of the tube shall fall upon it at the same angle of incidence, 54° 35′, as represented in *fig.* 411.

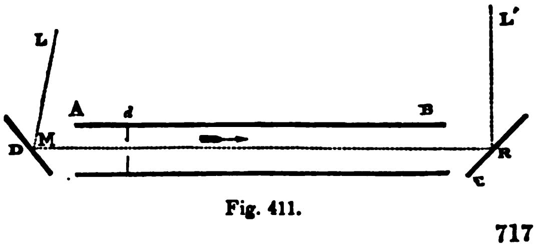

Fig. 411.

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If the tube E be so placed that the plane of the angle of incidence upon the reflector R shall coincide with the plane of the angle of incidence on the reflector D, then the ray coming along the axis of the tube will be reflected from R with the greatest possible intensity. If the tube E be then turned round within the tube B, so as to present the reflector R successively on different sides of the ray which passes along the axis of the tube, it will be found that when the reflector R assumes such a position that the plane of the angle of incidence upon it is at right angles to the plane of the angle of incidence upon the reflector D, no reflection will take place, and the ray will be extinguished.

It follows, therefore, from this, first, that the ray passing along the axis of the tube is polarized; and, secondly, that its plane of polarization coincides with the plane of the angle of incidence of the original ray upon the reflector D.

If, instead of a blackened glass, any other reflecting surface were placed in the frame D, the same effects would ensue; but the angle of incidence upon such surface which would produce polarization, would be different for different surfaces.

1270. *Method of determining the polarizing angle for different reflecting surfaces.* — It was discovered by Sir David Brewster by observation, and afterwards confirmed by theory, that the polarizing angle for any reflecting surface is that angle of incidence which, being added to the corresponding angle of refraction, supposing the ray to enter the medium, would make up the sum of 90°. Thus, if ABCD, *fig.* 412., be a transparent medium bounded by parallel surfaces AB and CD, and if PI be a ray of light incident upon it at such an angle of incidence PIF that the angle of refraction RIF′ corresponding to it shall, when added to FIP, make 90°, then the angle PIF will be the polarizing angle, and a ray incident at such angle and reflected from I in the direction IP′ will be polarized.

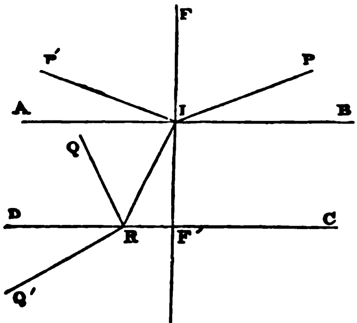

Fig. 412.

It is easy to show that in this case the directions of the reflected ray IP′ and the refracted ray IR are at right angles; for we have

$$\text{FIP} + \text{PIB} = 90^\circ.$$

And since PIB is equal to P′IA, we shall have

$$\text{FIP} + \text{P}'\text{IA} = 90^\circ.$$

But since F I P + R I F′ = 90°, it follows that

F′ I A = R I F′.

If to both of these we add the angle A I R, we shall have the angle P′ I R equal to the angle A I F′; but since A I F′ is 90°, the angle P′ I R will be also 90°.

The angle of polarization is therefore determined by the condition that the reflected ray I P′ shall be at right angles to the direction it would have pursued, had it been reflected instead of refracted at I.

It is easy to show that when the ray I R emerges from the lower surface in the direction R Q′, parallel to P I, it will be at right angles to the direction it would have taken, if, instead of passing through the surface at R, it were reflected from it in the direction R Q; for since R Q′ and R D are respectively parallel to P I and B I, the angle D R Q′ is equal to the angle P I B, or, what is the same, the angle F′ I A, or, in fine, to the angle R I F′.

But the angle I R F′ is equal to the angle Q R D, therefore the angles R I F′ and I R F′ taken together, are equal to the angle Q R Q′; and since the former are equal to 90°, Q R Q′ is a right angle. Hence it follows that the ray I R also falls upon the surface D C at R at the angle of polarization, since its directions reflected and refracted are at right angles.

It follows from what precedes, that the polarizing angle corresponding to any surface separating two media is that angle whose trigonometrical tangent is equal to the index of refraction; for since the angle R I F′ is the complement of the angle F I P, the sine of F I P divided by the sine of R I F′ will be equal to the tangent of the angle F I P.

Thus, whenever the index of refraction for any medium is known, the polarizing angle for the surface of such medium can be determined; and whenever the polarizing angle can be found by observation, the index of refraction may be inferred.

Since the indices of refraction for the different component parts of solar light are different, it follows that the polarizing angle for each species of homogeneous light will also be different.

1271. *Table showing the polarizing angle of certain media.*—Sir David Brewster gives the following table, showing the polarizing angles corresponding to the mean and extreme rays for the undermentioned transparent media:—

[redacted]

		Index of Refraction.	Maximum polarising Angle.	Difference between the greatest and least polarising Angles.
			° ′	° ′
Water	Red rays	1·330	53 4	15
	Mean rays	1·336	53 11	
	Violet rays	1·342	53 19	
Plate glass	Red rays	1·515	56 34	21
	Mean rays	1·525	56 45	
	Violet rays	1·535	56 55	
Oil of cassia	Red rays	1·597	57 57	1 24
	Mean rays	1·642	58 40	
	Violet rays	1·687	59 21	

1272. *Effects of reflection on polarized light.*—If a ray of polarized light be incident upon any plane reflecting surface, the position of the plane of its polarization will in general be changed after reflection, and will be turned more or less towards the plane of the angle of incidence. If the angle at which the ray is incident be equal to the polarizing angle, then the plane of polarization, whatever may be its position in the incident ray, will coincide with the common plane of incidence and reflection in the reflected ray, so that the effect of reflection will be to turn this plane round the axis of the ray through the angle formed by it with the plane of incidence.

If, however, the angle at which the ray is incident be not equal to the polarizing angle, then the plane of polarization will not be turned entirely round to coincide with the plane of the angle of incidence, but will be turned towards that plane, so that the angle formed by the plane of polarization of the reflected ray with the plane of incidence will be less than the angle formed by the plane of the angle of polarization of the incident ray with the same plane.

The angle through which the plane of polarization is thus turned will depend upon the relation which the angle of incidence bears to the polarizing angle.

If the ray be incident perpendicularly upon the surface, no change will take place in the position of the plane of polarization, that of the reflected ray coinciding with that of the incident ray. If the angle of incidence be very small, then the plane of polarization of the reflected ray will be slightly turned towards the plane of incidence, and it will be more and more turned towards it as the angle of incidence approaches to equality with the polarizing angle. When they are equal, the plane of polarization will coincide with the plane of the angle of incidence. When the angle of incidence exceeds the angle of polarization, the plane of polarization of the reflected ray will be turned from the plane of the angle of incidence, and on the other side of it; and it will continue to be turned from it more and more as the angle

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of incidence is increased, until it becomes a right angle. All these phenomena can be illustrated experimentally by means of the polariscopic apparatus already described, the plane of polarization being always capable of being determined by the means already explained.

1273. *Effects of ordinary refraction on polarized light.*—When a ray of polarized light enters any transparent medium, the plane of its polarization is changed after refraction, and is turned from the plane of the angle of incidence more or less, according as the angle of incidence differs more or less from the polarizing angle. The effect, therefore, of refraction on the plane of polarization is contrary to that produced by reflection. The more nearly the angle of incidence approaches to equality with the polarizing angle, the more nearly will the plane of polarization in the refracted ray be turned to a direction at right angles to the plane of incidence; and if the angle of incidence be absolutely equal to the polarizing angle, then the plane of polarization of the refracted ray will be at right angles to the plane of incidence, whatever may have been its position in the incident ray.

It follows, therefore, that if the plane of polarization of the incident ray be at right angles to the plane of incidence, it will suffer no change by refraction; but the further it departs from this direction the greater will be the change produced upon it by refraction.

1274. *Composition of unpolarized light.*—It was first suggested by Sir D. Brewster, and since confirmed by theory, that a ray of ordinary or unpolarized light consists of two rays polarized in planes at right angles to each other, the absolute direction of these planes being arbitrary. When such a ray is perfectly polarized, these planes of polarization are made to coincide, either or both being turned round the axis of the ray.

Polarized rays, may, however, also be obtained from a ray of natural light, either by resolving the ray into the two pencils of which it consists, and exhibiting them separately polarized in planes at right angles to each other, or by extinguishing one of the two rays, and not the other.

1275. *Polarization by double refraction.*—A doubly refracting crystal supplies the means of obtaining polarized rays by the first method.

When a ray of common light is incident upon such a crystal in a plane passing through its axis, it will be divided, as has been already explained, into two rays, the ordinary and extraordinary, both of which will be found to be polarized if examined by the test already explained. The plane of polarization of the ordinary ray will coincide with the plane of the angle of incidence, and the plane of polarization of the extraordinary ray will be at right angles to it. Thus the doubly refracting crystal resolves the ray of common light into its two component polarized rays, exactly as a common prism resolves a ray of solar light into its component rays of different refrangibility.

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1276. *Partial polarization.* — As a ray of light is completely polarized when the two planes of polarization naturally at right angles are brought to absolute coincidence, and as it is completely unpolarized when these planes are at right angles, it is partially polarized when they are in any intermediate position; and it approaches more and more to the state of complete polarization as the obliquity of the two planes of polarization increases. Thus when they form an angle of 45° the ray may be considered as half polarized.

It was long contended that a pencil partially polarized consisted of rays completely polarized mixed with rays completely unpolarized in various proportions, according to the degree of partial polarization of the pencil; but Sir David Brewster suggested, what has been since confirmed by theory, that partial polarization must be otherwise understood, and that a pencil partially polarized contains in it no ray, either perfectly polarized or perfectly unpolarized, but consists of rays, each of which is imperfectly polarized, as just explained.

1277. *Polarization by successive refractions.* — It has been already shown that a ray of polarized light when it enters a transparent medium, and is refracted by it, has its plane of polarization turned from the plane of the angle of incidence through an angle greater or less in magnitude according to the relation which the angle of incidence bears to the polarizing angle. Now, since a ray of natural light consists of two rays of light polarized in planes at right angles to each other, such a ray when it enters a refracting medium will have both planes of polarization of its component rays turned towards a right angle with the plane of the angle of incidence.

If such a ray then be successively refracted by a series of media bounded by parallel planes, the planes of polarization of its component rays will undergo a series of changes of direction, each having a tendency to turn them into a direction at right angles to the common plane of incidence and emergence.

Sir David Brewster found that the light of a wax candle placed at the distance of ten or twelve feet from a series of parallel plates of ground glass was polarized at angles of incidence which depended on the number of plates as exhibited in the following table:—

No. of Plates of Crown Glass.	Observed Angle at which the Pencil is polarized.	
	°	′
8	79	11
12	74	0
16	69	4
21	63	21
24	60	8
27	57	10
31	53	28
35	50	5
41	45	35
47	41	41

[redacted]

He inferred from these experiments that if we divide the number 41·84 by any number of crown glass plates, we shall obtain the tangent of the angle at which a pencil of light may be polarized by this number. He also inferred that the power of polarizing the refracted light increased with the angle of incidence between 0, or a minimum, at a perpendicular incidence, and the greatest possible, or a maximum, as the incidence approached 90°.

The apparatus represented at H, *fig.* 410., is adapted for the experimental demonstration of this. In the tube H is placed a series of five or more plates of glass, resting with their surfaces one upon the other, and capable of being adjusted in the tube, so as to form any desired angle with its axis.

If this piece H be inserted in the end A of the tube, and if the plates of glass be applied at the proper angle, it will be found that the light, after passing through them, is nearly polarized, and that its plane of polarization is perpendicular to the common plane of the angles of incidence and refraction. In this case, the more brilliant the pencil of light transmitted through the plates, the more numerous the plates must be in order to effect complete polarization.

Strictly speaking, no number of plates can bring the planes of polarization to absolute coincidence; but they may be said to approach so near to it, that the pencil will be to all appearance completely polarised with lights of ordinary intensity

A pencil thus polarized by refraction will exhibit the same properties when submitted to reflection, or when incident upon a plate of tourmaline, as have been already described with respect to light polarized by reflection.

1278. *Effect of tourmaline on polarized light.*—Let a plate of tourmaline be cut with surfaces parallel to each other and to its optic axis. Such a plate being fixed in the piece G, *fig.* 410., may be inserted in the end of the tube B, so as to receive the polarized ray transmitted along the axis of the tube perpendicular to its surface. When thus arranged, the tube G being turned within the tube B, so as to bring the optic axis of the tourmaline to coincide with the plane of polarization of the ray, the ray will be totally intercepted. If the tube be then turned, so that the axis of the tourmaline shall form an increasing angle with the plane of polarization, light will begin to be transmitted, and the intensity of the light so transmitted will gradually increase, until the axis of the tourmaline is at right angles to the plane of polarization, when its intensity will be a maximum. After it passes that, the tube G being slowly turned, the intensity will again diminish until the axis of the tourmaline again coincides with the plane of polarization, when the light will be completely intercepted. The tourmaline supplies, therefore, a test of polarization, and a means of ascertaining the position of the plane of

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polarization more convenient still than that which has been already explained by means of the reflecting surface R.

1279. *Polarization by absorption.* — Sir David Brewster showed that agate and some other crystals had the effect of intercepting one of the two polarised rays which constitute common light and transmitting the other, and suggested this as a means of obtaining polarised light. Thus, if a ray of common light be transmitted through a plate of agate, one of the oppositely polarised beams will be converted into nebulous light in one position of the crystal, and the other in another position, so that one of the polarized beams will in each case be transmitted. The same effect may be produced by Iceland spar, Aragonite, or artificial salts, prepared in a peculiar manner, so as to produce a dispersion of one of the two polarised rays forming common light.

If common light be transmitted through a thin plate of tourmaline, one of the polarized rays which constitute it will in like manner be absorbed by the tourmaline, and the other transmitted; and when the tourmaline is applied in a position at right angles to this, the ray which was before transmitted is absorbed, and *vice versâ*.

1280. *Polarization by irregular reflection.* — When a pencil of light is directed obliquely on any imperfectly polished surface so as to be irregularly reflected from it, the rays thus reflected will be partially polarized, as may be ascertained by looking at the reflecting surface through the plate of tourmaline G, *fig.* 410. On turning round the plate of tourmaline, it will be found that the brightness of the surface will vary according to the direction of the axis of the tourmaline, the positions of the axis which render its brilliancy greatest and least being at right angles to each other. That the polarization in this case is imperfect is demonstrated by the fact that the tourmaline in no position produces a complete extinction of the light.

Since light is more or less polarized by successive refractions and by successive reflections, whether regular or irregular, it follows that light is almost never found without being more or less polarized. Thus the light of day proceeding from the solar rays reflected and refracted by the atmosphere and the clouds must always be more or less polarized, — an effect of which may be verified by examining this light by one or other of the tests of polarization, but more especially by the tourmaline already described.

1281. *The interference of polarized pencils.* — If two pencils of light have their planes of polarization parallel, they will exhibit the same phenomena of interference as have been already described for ordinary light. The production of bright and dark fringes, when the pencils are homogeneous, and the production of coloured fringes, when the pencils consist of compound light, will occur as in the case of unpolarized light

But if the two pencils be polarized in planes at right angles to each other, none of the phenomena of interference will be exhibited. No matter under what circumstances the rays shall intersect, it can never happen that either ray will extinguish the other, or that the phenomena of dark and light or coloured fringes are produced.

When two pencils are polarized in planes forming with each other an oblique angle, they will produce fringes, but of inferior brilliancy to those exhibited when their planes of polarization are parallel.

If two pencils are first polarized in planes at right angles to each other, and afterwards have their planes of polarization rendered parallel, which may always be accomplished either by refraction or reflection, they will not recover the property of forming fringes of interference, of which they were deprived by rectangular polarization.

But if a pencil of common light be first completely polarized, and then be divided into two pencils polarized in rectangular planes, these two pencils, if their planes of polarization be again rendered parallel, will acquire the property of interference, and will exhibit fringes.

All these phenomena admit of verification by the polariscopic apparatus already described.

1282. *Compound solar light cannot be completely polarized by reflection, but may be nearly so.* — Since the polarizing angle varies with the index of refraction, and since white solar light is a compound of rays having different indices of refraction, it follows that a pencil of solar light can never be completely polarized by a reflecting surface, for the angle which would polarize completely one of its constituents would be different from the angle which would polarize completely another. But since the difference between the polarizing angles for the extreme rays in the case of glass is only 21′, and in the case of water still less, it follows that if the polarizing reflector be adjusted at the polarizing angle of the rays of mean refrangibility, the rays of extreme refrangibility will fall upon it at an angle differing very little from their polarizing angle, and, consequently, although they will not be completely, they will still be very nearly polarized.

1283. *Absence of complete polarization rendered evident by tourmaline.* — Nevertheless, the absence of complete polarization in this case is rendered extremely evident by the the test of the plate of tourmaline already described.

If the reflector D, *fig.* 410., be adjusted to the polarizing angle of the rays of mean refrangibility, and the plate of tourmaline G be applied to the end B of the tube, the rays corresponding to the middle of the spectrum only will be completely intercepted when the axis of the tourmaline is brought into the plane of polarization. A portion of the extreme rays at both ends of the spectrum will be transmitted through the tourmaline, and will be perceivable as bright purple light proceeding from the mixture of the red and violet rays which are transmitted. If the plate D be then adjusted to the polarizing angle of the violet rays, the red rays will be transmitted in considerable quantity, and the yellow less, so that the light transmitted will be a

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reddish-orange; and if, on the other hand, the polarizing plate D be adapted to the polarizing angle of the red rays, the light transmitted will be a bluish-green. If the polarizing plate D be composed of any highly dispersive substance, such as cassia, diamond, chromate of lead, realgar, or specular iron, the colour of the unpolarised light transmitted from the tourmaline will be found to be extremely bright and beautiful.

1284. *Effect of a doubly refracting crystal on polarized light.*— Let us suppose a pencil of polarised light R P, *fig.* 413., to be incident perpendicularly upon a plate A B, cut from a doubly refracting crystal, in such a manner that its surfaces are parallel to each other and to the optic axis of the crystal. The pencil R P, in passing through this plate, will be doubly refracted, the ordinary pencil proceeding in the direction P O O of the original pencil R P, and the extraordinary pencil taking another direction P E through the crystal, and emerging in the direction E E, parallel to that of the incident ray R P. These two pencils will be polarised in rectangular planes, the plane of polarization of the ordinary pencil O O coinciding with the optical axis of the crystal, and the plane of polarization of the extraordinary pencil E E being perpendicular to it.

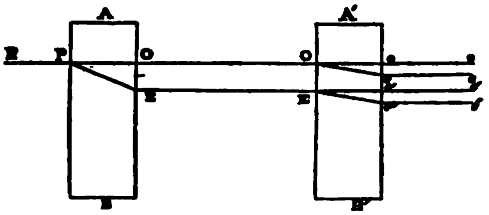

Fig. 413.

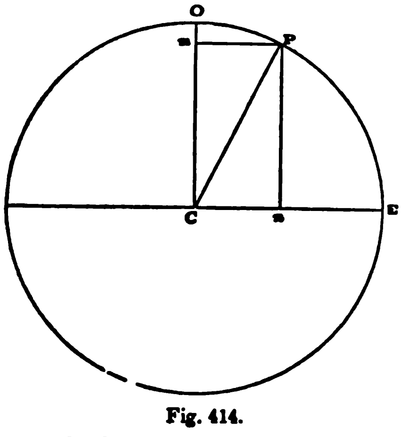

Fig. 414.

To render this more clear, let the circle, *fig.* 414., represent a section of the incident ray R P, and let C P be the direction of the plane of primitive polarization of the ray R P. Let C O be parallel to the optic axis of the crystal A B, and C E be perpendicular to it. It follows, therefore, that C O, *fig.* 414., will be the direction of the plane of polarization of the ordinary pencil O O, *fig.* 413., and C E will be the direction of the plane of polarization of the extraordinary pencil E E, *fig.* 413.

It follows from the principles of the undulatory theory (and this consequence is confirmed by observation) that the proportion in which the light of the original pencil R P is shared by the ordinary and extraordinary pencils O O and E E will be expressed by the squares of the cosines of the angles which the plane of primitive polarization C P, *fig.* 414., makes with the planes of polarization of the two pencils O O and E E, *fig.* 413., respectively.

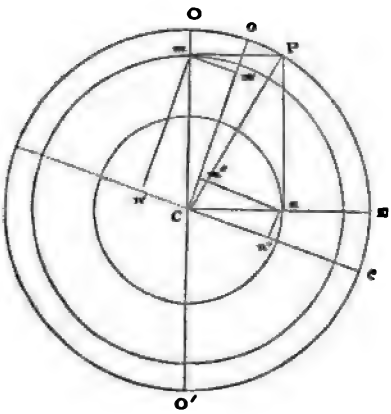

Fig. 415.

If, therefore, the number of rays in the original pencil R P be expressed by the square of the radius, *fig.* 415., the number of rays in the ordinary pencil O O will be expressed by the square of C *m*, and the number of rays in the extraordinary pencil E E will be expressed by the square of C *n*. The changes incident to the relative intensities of the ordinary and extraordinary pencils produced by the plate A B, may then be easily inferred from the diagram, *fig.* 414.

If the plane of polarization of the original ray R P coincide with the axis of the crystal A B, then C P, *fig.* 414., will coincide with C O, and the number of rays in the pencil O O, *fig.* 413., will be expressed by the square of the radius C O, while the pencil E E will vanish; for, in this case, C *m* will become equal to C O, and C *n* will vanish.

As the plane of primitive polarization C P makes an increasing angle with C O, C *m*, whose square represents the number of rays in the pencil O O, will decrease, and C *n*, whose square represents the number of rays in the pencil E E, will increase. The one pencil, therefore, will diminish, and the other increase in intensity. When the plane of primitive polarization C P makes an angle of 45° with the axis C O of the crystal, the line C P will bisect the angle O C E, and and C *m* will become equal to C *n*. In this position, therefore, the ordinary and extraordinary pencils O O and E E, *fig.* 413., will become equally intense, or contain the same number of rays.

When the plane of primitive polarization C P makes with the axis C O of the crystal A B a greater angle than 45°, C *m* becomes less than C *n*, and consequently the ordinary pencil O O, *fig.* 413., contains less

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rays than the extraordinary pencil E E; and as the angle included between C P and C O increases, the extraordinary pencil will become relatively more intense, and the ordinary pencil less so, until the plane of primitive polarisation C P makes a right angle with the axis C O of the crystal; in which case C P will coincide with C E, C N will become equal to C E, and C M will vanish.

Thus the ordinary pencil O O, *fig.* 413., will disappear, and all the rays of the incident pencil R P will pass into the emergent extraordinary pencil E E. A like succession of changes of intensity will take place if we suppose the axis of primitive polarization C P to revolve through another quadrant; the rays of the extraordinary pencil gradually passing into the ordinary one, and the extraordinary one vanishing, and the ordinary pencil acquiring the same intensity as the incident pencil, when the plane of polarization again coincides with the direction of the axis of the crystal.

It thus appears, that in a complete revolution of the plane of primitive polarization, or, what is the same, if that plane be fixed, in a complete revolution of the plate A B in its own plane, there will be two positions, 180° asunder, in which all the rays of the primitive pencil will pass into the ordinary pencil, and consequently, in which the primitive pencil will undergo no change either in its intensity or its polarization. Therefore, there will be two positions at right angles to these in which the primitive pencil again undergoes no change in intensity, but in which it is converted into the extraordinary pencil E E, its plane of polarization being turned through 90°, and receiving a direction at right angles to that of the plane of primitive polarization. In the intermediate positions between these four directions, the relative intensities of the ordinary and extraordinary pencils undergo constant change; that of the ordinary pencil being greater or less than that of the extraordinary pencil, according as the plane of primitive polarization makes a less or greater angle than 45° with the axis of the crystal A B, and the intensities of the two pencils are equal in the four positions in which the axis of primitive polarization is inclined at 45° to the axis of the crystal.

1285. *Effects produced by a second doubly refracting crystal.* — If we now suppose the ordinary and extraordinary pencils O O and E E, *fig.* 413., to be incident perpendicularly upon another doubly refracting plate A′ B′, cut with surfaces parallel to each other and to its optic axis, as before, they will each be again doubly refracted. The ordinary pencil O O will be divided into another ordinary pencil *o o* and an extraordinary pencil *e e*, while the extraordinary pencil E E will also be doubly refracted and resolved into two, an ordinary pencil *o′ o′*, and an extraordinary pencil *e′ e′*, all these four pencils emerging parallel to the primitive pencil R P.

To determine the proportion in which the rays of the original pencil R P are distributed among these four pencils, let C O, *fig.* 415.,

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represent, as before, the direction of the optical axis of the plate A B and therefore the plane of polarization of the ordinary pencil O O; and consequently C E, perpendicular to C O, will represent the plane of polarization of the extraordinary pencil E E. Let C *o* represent the direction of the optical axis of the plate A′ B′, and let C *e* be a line perpendicular to it.

According to what has been already explained, the planes of polarization of the ordinary pencils *o o* and *o′ o′* will coincide with C *o*, the optical axis of the plate A′ B′, while the planes of polarization of the extraordinary pencils *e e* and *e′ e′* will coincide with the line C *e* perpendicularly to C *o*.

If the square of the radius C P, *fig.* 415., express the number of rays in the original pencil R P, the square of C *m*, as already explained, will express the number of rays in the pencil O O, and the square of C *n* the number of rays in the pencil E E

To obtain expressions for the intensities of the pencils into which these latter are resolved by the second crystal A′ B′, let circles be described with C as a centre, and C *m* and C *n* respectively as radii. From *m* draw *m n′* perpendicular to C *e*, and *m m′* perpendicular to C *o*. Since, then, the square of C *m* expresses the number of rays in the pencil O O, the square of C *m′* will express the number of rays in the pencil *o o*, and the square of C *n′* will express the number of rays in the pencil *e e*.

In like manner, if from *n* we draw *n m″* and *n n″* at right angles respectively to C *o* and C *e*, the number of rays in the pencil *o′ o′* will be expressed by the square of C *m″*, and the number of rays in the pencil *e′ e′* will be expressed by the square of C *n″*. We shall therefore have the following analysis of the intensities of the emergent pencils of the ordinary and extraordinary rays produced by the first plate A B, and of the four pencils, ordinary and extraordinary, produced by the second plate A′ B′.

Intensity of original pencil R P is expressed by ………………… C P^2.
" ordinary pencil O O " ………………… C m^2.
" extraordinary pencil E E " ………………… C n^2.
" ordinary pencil *o o* " ………………… C m'^2.
" extraordinary pencil *e e* " ………………… C n'^2.
" ordinary pencil *o′ o′* " ………………… C m''^2.
" extraordinary pencil *e′ e′* " ………………… C n''^2.

If we suppose the plate A′ B′ to be turned round its centre, so as to make its optical axis C *o*, *fig.* 415., revolve, making varying angles with the planes of polarization of the rays O O and E E, a succession of changes will take place in the two pairs of ordinary and extraordinary pencils emerging from the plate A′ B′, in all respects analogous to those which have been already described as having taken place in the pencils O O and E E emerging from the first plate A B.

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This change can be easily inferred from *fig.* 415., where C *e* represents the direction of the optical axis of the crystal A′ B′, and C O and C E the planes of polarisation of the pencils O O and E E.

Thus, if we suppose the crystal A B turned into such a position that its optical axis C *o* shall coincide with C O, then C *m*′ will become equal to C *m*, and C *n*′ will vanish; therefore the pencil *o o* will contain all the rays of the incident pencil O O, and will have the same plane of polarisation, while the pencil *e e* will vanish. At the same time that this takes place, C *e* will coincide with C E, and consequently C *n*″ will become equal to C *n*, and C *m*″ will vanish. Therefore the pencil *e*′ *e*′ will contain all the rays of the incident pencil E E. Thus it appears that in this case the second plate A′ B′ will make no change whatever, either on the intensities or the planes of polarisation of the two rays O O and E E that emerge from the first crystal A B. If the axis of the second crystal C *o* be turned round so as to make a gradually increasing angle with the axis C O of A B, then the lines C *n*′ and C *m*″ will gradually increase, and the lines C *m*′ and C *n*″ will gradually diminish. Therefore the intensities of the ordinary pencil *o o* will gradually diminish, and that of the extraordinary pencil *e e* will gradually increase; and, at the same time, the intensity of the extraordinary pencil *e*′ *e*′ will gradually diminish, and that of the ordinary pencil *o*′ *o*′ will gradually increase.

When the axis C *o* of the crystal A′ B′ makes an angle of 45° with the axis C O of the crystal A B, then the four pencils will have equal intensities, for in such case C *o* will bisect the angle O C E, and the line C *e* will bisect the angle O′ C E; and in this case it is evident that all the four lines C *m*′, C *n*′, C *m*″, and C *n*″ will be equal; and since their squares express the intensities of the four pencils, these intensities will be equal. When the angle formed by the axis C *o* of the plate A′ B′, still increasing, forms an angle greater than 45° with the axis C O of the plate A B, then the line C *n*′ becomes greater than C *m*′, and consequently the pencil *e e* becomes more intense than the pencil *o o*. At the same time, the line C *n*″ will become less than C *m*″, and consequently the pencil *e*′ *e*′ will become less intense than the pencil *o*′ *o*′. These inequalities between the respective pencils will gradually increase with the gradually increasing angle formed by the axis of the plate A′ B′ with the axis of the plate A B, until these axes form a right angle with each other, in which case the pencils *o o* and *e*′ *e*′ wil vanish, and the pencil *e e* will contain all the rays of the pencil O O and the pencil *o*′ *o*′ will contain all the rays of the pencil E E. Thus when the axis of the crystal A′ B′ is applied at right angles to the axis of the crystal A B, no change is made in the intensities of the two pencils incident upon this second crystal; but the planes of polarisation are respectively moved through a right angle, the ordinary pencil being converted into an extraordinary one, and the extraordi-

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nary pencil being converted into an ordinary pencil. It is clear that the same succession of changes will take place throughout each successive quadrant through which the optical axes of the plates are turned.

CHAP. XX.

CHROMATIC PHENOMENA OF POLARIZED LIGHT.

1286. *Chromatic phenomena explicable by undulatory hypothesis.* — The splendid prismatic colours arranged in the form of concentric rings, intersected by dark and bright rectangular crosses, and occasionally by hyperbolic curves, are among the most remarkable and beautiful phenomena developed by modern experimental researches in optics. No triumph of theory can be more complete than the solution of these complicated appearances afforded by the undulatory hypothesis.

Any description, however, of these multitudinous and various appearances, much more any exposition of the mathematical solution of them supplied by the undulatory theory of light, would be incompatible with the objects and the necessary limits of this volume. While, however, we cannot enter into these details, we must not, on the other hand, pass over in absolute silence such phenomena.

1287. *Effect produced by the transmission of polarized light through thin doubly refracting plates.* — To convey some idea of the principles on which these phenomena are based, let us suppose the plates A B and A′ B′ to be so thin that the separation of the pencils into which the primitive pencil R P is resolved will be inconsiderable In such case, although the changes described in the last chapter will still be made in their planes of polarization, the pencils will more or less overlay each other, so that the rays composing one will fall within the limits of and be mixed with, the rays of the other.

It might therefore be inferred that the intensity or brilliancy of the pencils formed by each combination would be found by adding together the measures of their separate intensities. Thus, the two pencils $o\,o$ and $o'\,o'$, whose separate intensities are expressed by $\mathrm{C}\,m'^2$, and $\mathrm{C}\,m''^2$, would have thire combined intensity expressed by

$$\mathrm{C}\,m'^2 + \mathrm{C}\,m''^2.$$

But it must be considered that polarized light is subject to interference when its planes of polarization are parallel, which they are in the two cases here supposed, the planes of polarization of the pencils *e e* and *e′ e′* being both parallel to the axis of the crystal A′ B′, and the planes of polarization of the pencils *e e* and *e′ e′* being both perpendicular to it. If, therefore, the other conditions of interference be fulfilled, it will follow that the rays of these two pairs of pencils would alternately extinguish one another, or produce a brilliancy equal to the sum of their intensities, according to the phases under which the luminous undulations meet.

But it is easy to show, that, provided one or both of the crystals A B and A′ B′ have a certain degree of thinness, the rays of the two pencils would fulfil the conditions which determine interference.

To prove this, it must be considered that the indices of ordinary and extraordinary refraction are different; therefore the velocities of the undulations in passing through the crystals will be different, if one be ordinarily and the other extraordinarily refracted; and if this difference be such as to produce by the undulation of the emergent pencils that relation which determines interference, that phenomenon must ensue. Now, on considering the refraction which the pencils *o o* and *o′ o′* have suffered, it will appear that the former has undergone ordinary refraction by both crystals, while the latter has suffered extraordinary refraction, by the crystal A B, and ordinary refraction by the crystal A′ B′. Their velocities, therefore, through the crystal A B will be different; and if the thinness of the crystal be such that the undulations of the original rays are so related as to fulfil the conditions of interference, interference will ensue.

The same observations will be applicable to the pencils *e e* and *e′ e′*, the latter of which has suffered extraordinary refraction by both crystals, and the former ordinary refraction by A B, and extraordinary refraction by A′ B′.

1288. *Coloured rings and crosses explained.* — If, therefore, the plates be reduced to such a degree of thinness as to produce the phenomena of interference, a series of bright and dark rings will be produced; but as such rings will depend on the indices of refraction, and as these indices differ for each species of homogeneous light, it will follow that a different system of rings would be produced by each species of homogeneous light, of which the primitive pencil R P might be composed; and if such pencil be composed of compound solar light, then the resulting appearances are those which will be produced by the superposition of all the systems of rings which would be separately produced by each species of homogeneous light. The effect of the optical axes of the crystals, and of the revolution of either of them round its centre in its own plane, will be to produce dark or bright rectangular crosses corresponding to the planes of polarization of the emergent pencils, these crosses intersecting the systems of coloured rings.

We have here adopted for illustration, for the sake of simplicity, the case of crystals having a single axis of double refraction. The appearances produced by crystals with two axes are analogous to these, though somewhat more complicated.

In these, two systems of rings, which sometimes assume the form of the curves called lemniscates, which have the form of the figure of 8, are produced, and the cross is often converted into hyperbolic curves, which in certain positions assume the form of a cross, the hyperbola passing into its asymptos.

To give a complete analysis of these complicated and beautiful chromatic phenomena would be impossible within the necessary limits of this volume; enough, however, has been explained of the principles of polarization to render their general theory intelligible; and we shall therefore now confine ourselves to a general description of some of the most interesting of the phenomena produced by transmitting polarized light through doubly refracting media.

1289. *Method of observing and analyzing these phenomena.*— *Apparatus of Noremberg.*—The polariscopic apparatus of Noremberg, represented in *fig.* 416., supplies convenient means of observing and analyzing the chromatic phenomena of polarized light.

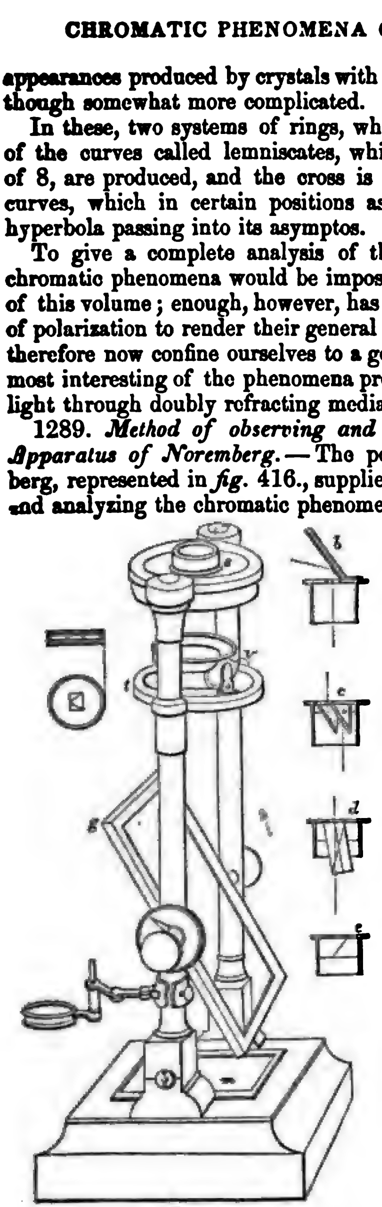

Fig. 416.

The polarizing apparatus is mounted in the lower part of the instrument, and consists of the frame *g* containing the polarizing plate, the horizontal reflector *m*, and other accessories. By means of these a pencil of light polarized in any required plane can be transmitted vertically upwards, so as to pass through the centre of the rings *v* and *s*.

The rings *v* and *s* are graduated, and a tube is inserted in each of them, having an index which plays on the divided scale as the tube is turned round its centre within the ring. Plane reflectors inclined at variable angles, plates of doubly refracting crystals, doubly refracting prisms, bundles of parallel plates of glass and other polariscopic tests, are set in short tubes capable of being fixed in one or other of the rings *v* and *s*. So the polarized pencil transmitted upwards along the axis of the

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apparatus may first be made to pass through the plate inserted in *v*, and may then be examined by an inclined reflector or tourmaline plate, a doubly refracting prism, or by any other polariscopic test which may be fixed in *s*. The position of the indices which move on the divided circles of *v* and *s* will indicate the position and changes of position of the planes of polarization.

1290. *Effect of rock crystal.* — Let a plate of rock crystal, with surfaces cut parallel to its optic axis, the thickness of which does not exceed the 50th of an inch, be placed on the ring *v*; and let a doubly refracting prism, with a single axis of double refraction, be placed in *s*.

Let us first suppose that the axis of this prism coincides with the plane of polarization of the pencil incident on the plate *v*, and let the axis of this plate be first placed in the plane of polarization. In that case the incident ray will pass through both crystals without change, and an eye placed above the prism at *s* will see only the ordinary image of the object from which the pencil issues. If the axis of *v* be turned at right angles to the plane of polarization, a single image only will be seen; but in this case it will be the extraordinary image, and the plane of its polarization will be perpendicular to the plane of primitive polarization. The images will in both cases be white.

In all intermediate positions of the axis of the plate *v*, two images will be seen, which will partly overlay each other, as represented in *fig.* 417. Those parts which are not superposed will have colours exactly complementary, and the superposed parts on which these colours are combined will be white.

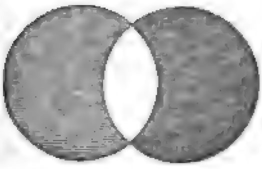

Fig. 417.

As the plate *v* is turned round its centre through 90°, from the position in which its axis coincides with the plane of primitive polarization to the position in which it is at right angles to that plane, the two images pass through a series of tints of colour (always, however, complementary), and through various degrees of relative brightness, their most vivid colours being exhibited when the axis is at 45° with the plane of primitive polarization.

The same changes take place in each successive quadrant through which the axis of *v* revolves.

If the axis of the prism *s* be placed at right angles with the plane of primitive polarization, a like succession of appearances will be exhibited, the ordinary and extraordinary images, however, interchanging places.

If the axis of the prism *s* be placed at any oblique angle with the plane of primitive polarization, a like succession of effects will be observed; but, in this case, the single images will be exhibited when the axis of the prism *s* coincides with, and is at right angles with that of

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the plate *v*; and the double coloured images appear in the intermediate positions, the images having the greatest splendour when the two axes intersect at an angle of 45°.

There is therefore, in all cases, a single image in four positions in each revolution, these four positions being at right angles to each other; and intermediate between these, there are four other positions, also at right angles to each other, at which the complementary images attain their greatest brightness.

Plates of rock crystal more than the 50th of an inch in thickness produce like effects, but with less brilliant colours. In general, the colours vary with the thickness of the plate, the more brilliant tints being produced by the thinnest plates.

Different crystals exhibit striking differences in these chromatic phenomena. Thus Biot found that carbonate of lime cut parallel to the axis, required to be eighteen times thinner than rock crystal to produce the same tint. This circumstance renders it difficult to observe these phenomena with carbonate of lime.

1291. *Effect of Iceland spar inclosed between two plates of tourmaline.* — Let a plate of Iceland spar less than an inch thick be cut with parallel surfaces at right angles to its optic axis. If this be placed between two plates of tourmaline cut parallel to their axes, a series of beautiful chromatic phenomena will be observed by looking through it at the clouds. If the axes of the tourmalines are placed at right angles, the crystal will exhibit a system of concentric rings of the most vivid colours, intersected by a dark cross, as represented in *fig.* 418.

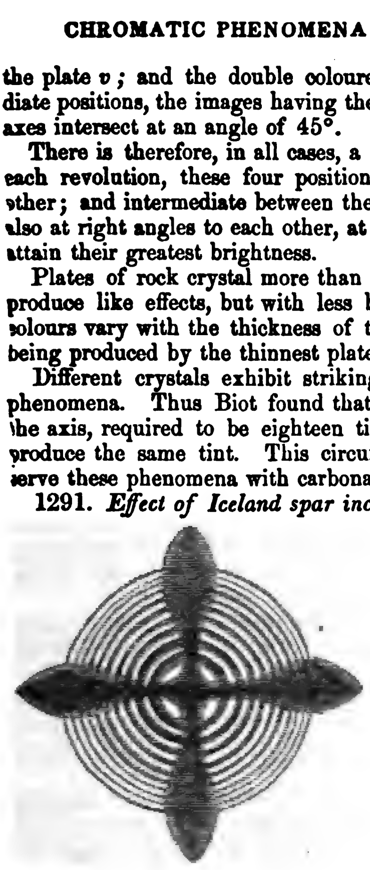

Fig. 418.

If the axis of one of the tourmalines be turned gradually round, making a decreasing angle with the axis of the other, the tints of the rings will undergo a series of changes, and the dark cross will show a space in the midst of each of its arms faintly luminous, as represented in *fig.* 419. These changes will proceed until the axis of the one tourmaline becomes parallel to the other, when the cross will become white, and all the tints of the rings will become complementary to those which they had in the first position, as represented in *fig.* 420.

If, instead of presenting the crystal to the white light of the heavens, a pencil of homogeneous light be transmitted through it, the rings, instead of showing various tints will be alternately dark

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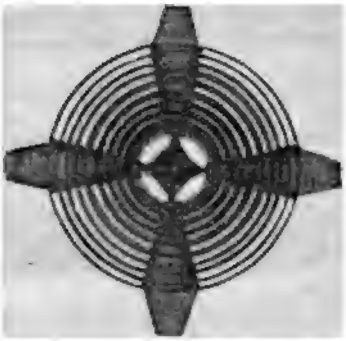

Fig. 419.

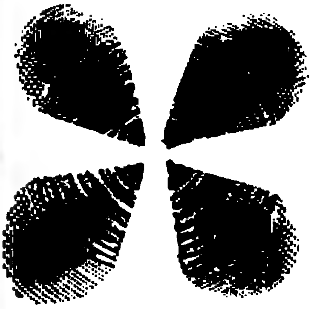

Fig. 420.

and of the colour of the homogeneous light; and the cross, in like manner, will be either dark or of the colour of the same light. The diameters of the successive rings will be different for each coloured light, being greater for the more refrangible colours; and the diameters of rings for the same colour will increase as the thickness of the crystal is diminished.

It is evident that the system of rays produced by white light results from the superposition of the several systems produced separately by the homogeneous coloured lights.

The white cross produced by white light, when the axes of the tourmalines are parallel, is in like manner produced by the superposition of all the coloured crosses produced by the homogeneous lights severally.

1292. *Effects produced by other uni-axial crystals.*—Phenomena analogous to these are produced by all crystals having a single axis of double refraction, such as rock crystal, tourmaline, zircon, nitrate of soda, mica, hyposulphate of lime, apophyllite, &c. In some cases, however, the effects are modified by conditions peculiar to the species of crystal under examination. Thus, in the case of rock crystal, the cross disappears, in consequence of the effect of circular polarization, which we shall presently notice. In other crystals there appear to be different optic axes for lights of different refrangibilities, which produce modifications in the appearance of the rings and crosses.

Of all crystals, the most convenient for the exhibition of these phenomena is Iceland spar.

1293. *Effect of bi-axial crystals; nitrate of potassa.*—If a plate of nitrate of potassa (a crystal having two axes), with parallel surfaces cut at right angles to its optic axis, be placed in like manner between two plates of tourmaline cut parallel to their axes, a series of chromatic appearances will be observed, which are represented in *figs.* 421., 422., and 423.

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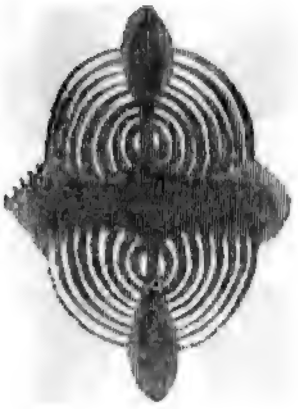

Fig. 421.

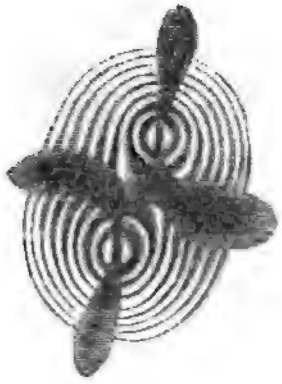

Fig. 422.

If the axes of the tourmalines are placed at right angles, the crystal itself being properly placed between them, a dark cross, *fig.* 421., will be seen intersecting a double system of coloured rings, the common centres of which correspond to the position of the two axes of the intermediate crystal.

If the crystal be turned gradually round its centre between the tourmaline plates without deranging the position of the latter, the cross will gradually assume the form of two hyperbolic curves, and the rings will change their position and tints as represented in *fig.* 422. When the crystal has been turned through half a quadrant, the appearance will be that represented in *fig.* 423., and after which it will assume a form like that of *fig.* 422., but more inclined to the horizontal position; and, in fine, when the crystal has been turned through a quadrant, the appearance will be that represented in *fig.* 421., the vertical arms of the cross, and the line joining the centres of the systems of concentric rings, being, however, horizontal.

1294. *Effect of the carbonate of lead.* — The carbonate of lead, another crystal with two axes, gives appearances analogous to those of nitrate of potassa. These are represented in *fig.* 424.

1295. *Coloured bands produced by an acute prism of rock crystal.* —If a piece of rock crystal be cut in the form of a prism, with a very acute angle, one surface forming the angle being parallel to the optic axis, and the other therefore slightly inclined to it, a pencil of polarized light transmitted through it will exhibit to the naked eye a series of alternated red and green fringes, provided the eye is placed at some distance from the crystal, and the thickness through which the light passes does not exceed the 50th of an inch. These coloured bands

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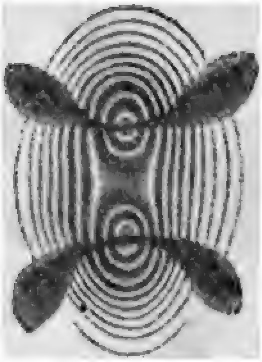

Fig. 423.

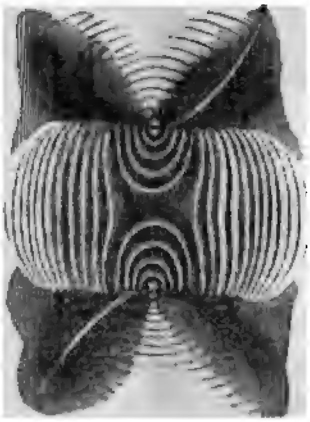

Fig. 424.

are more vivid when viewed through a plate of tourmaline, and it is easy to observe that they attain their greatest brightness when the axis of the prism is inclined at 45° to the plane of primitive polarization.

1296. *Polarizing structure artificially produced in glass and other media.* — A doubly refracting and polarizing structure may be produced in glass and other transparent bodies by molecular changes in their structure consequent on sudden changes of temperature, and sometimes by mere mechanical pressure.

If a circular plate of glass, about an inch in diameter and half an inch thick, be exposed to a high temperature by contact with a heated body which is a good conductor, so that its temperature near the edges shall be higher than at the centre; or if, on the contrary, it be raised to a higher temperature at the centre than near the edges, it will exhibit the phenomena of rectangular crosses and coloured rings, like those produced by doubly refracting crystals.

If, in this case, the plate be oval, it will exhibit appearances indicating two axes of double refraction.

When a plate is reduced to a uniform temperature, these appearances cease.

These phenomena are susceptible of infinite variation, according to the shape of the plate, which may be square, oblong, or of any other form. The disposition and form of the fringes and rings will vary with the form of the plate.

A permanent doubly refracting and polarizing structure may be imparted to glass by raising it to a high temperature, and then cooling it rapidly, by placing it in contact with the cold surfaces of metals.

The metallic surfaces, in this case, may be formed into an infinite variety of fancy patterns, which will have the effect of producing corresponding optical effects of great beauty.

CHAP. XXI.

CIRCULAR POLARIZATION.

1297. *Cases in which the change of the plane of polarization varies with the thickness of the crystal.* — In all the cases noticed in the preceding chapter in which a ray of polarized light passes through a plate of doubly refracting crystal, the change produced upon its plane of polarization is quite independent of the thickness of the crystal; this change depending solely upon the relative position of the plane of primitive polarization and the axis of the crystal.

We have now, however, to notice another class of polarizing influences, in which the change produced in the plane of polarization of the ray transmitted will vary with the thickness of the crystal.

If a plate of rock crystal be cut with parallel surfaces perpendicular to its optical axis, a ray of polarized light transmitted through it will have its plane of polarization changed, and turned through a certain angle. If the thickness of the plate be doubled or halved, then the angle through which the plane of polarization of the ray is turned will also be doubled or halved. In a word, the angle through which the plane of polarization would revolve when the ray passes through the crystal, will, for the same crystal, be proportional to the thickness of the plate.

The direction in which the plane of polarization is thus made to turn is different in different specimens. Thus, two different plates of rock crystal, having the same thickness, will turn the plane of polarization, one to the left and the other to the right.

1298. *The angle through which that plane is turned varies with the refrangibility of the light.* — The angle through which the plane of polarization is turned depends also upon the refrangibility of the light transmitted through the crystal, the angle increasing with the refrangibility. Thus, if a polarized ray of red light be transmitted through such a plate, the angle through which its plane of polarization will be turned will be less than that through which the plane of polarization of an orange ray would be turned, and this latter less than that through which the plane of polarization of a yellow, green or any other more refrangible ray would be turned.

It follows from this, that if a polarized pencil of white light be incident upon such a plate, the emergent pencil will have different planes of polarization for light of each degree of refrangibility.

1299. *The plane may make a complete revolution if the thickness of the crystal be sufficient.*—It follows from these phenomena, that in its progress through the thickness of such a crystal the plane of polarization of a ray of homogeneous light is gradually turned round its centre, so that a thickness may be assigned which will cause this plane to make a complete revolution, so that the emergent ray will in this case appear as if it had suffered no change, although in reality, in its progress through the crystal, the plane of its polarization had in succession formed all angles with its original direction from 0° to 360°.

This effect on the plane of polarization may be illustrated by the motion of the thread of a screw in penetrating any substance.

To understand this distinction, as Sir John Herschel has observed, it is only necessary to take a common corkscrew, and holding it with the head towards him, let the observer turn it in the usual manner, as if to penetrate a cork. The head will then turn the same way as the plane of polarization of a ray in its progress from the spectator through a right-handed crystal may be conceived to do. If the thread of the corkscrew be reversed, as in a left-handed screw, then the motion of the head, as the instrument advances, would represent that of the plane of polarization in a left-handed crystal.

1300. *Angles through which the plane is turned by a crystal of quartz.*—The angles through which the plane of polarization of each of the component rays of the spectrum is made to turn by a plate of quartz cut perpendicular to the axis of the twenty-fifth of an inch thick is given in the following table:—

Homogeneous Ray.	Arcs of Rotation.	
	°	′
Extreme red	17	30
Mean red	19	00
Limit of red and orange	20	29
Mean orange	21	24
Limit of orange and yellow	22	19
Mean yellow	24	00
Limit of yellow and green	25	40
Mean green	27	51
Limit of green and blue	30	03
Mean blue	32	19
Limit of blue and indigo	34	34
Mean indigo	36	07
Limit of indigo and violet	37	41
Mean violet	40	53
Extreme violet	44	05

1301. *Effect of amethyst.*—Sir David Brewster says, that in examining the phenomena of circular polarization produced in amethyst, he found that it possessed the same power in the same specimen of

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turning the plane of polarization, both from left to right and from right to left, and that it actually consisted of alternate strata of right and left-handed quartz, whose planes were parallel to the axis of double refraction. These strata are not united together like the parts of certain composite crystals, whose dissimilar faces are brought into mechanical contact; for the right and left-handed strata destroy each other at the middle line between each stratum, and each stratum has its maximum polarizing force in its middle line, the force diminishing gradually to the line of junction.

1302. *Circular polarization in liquids and gases.*—Rock crystal is the only solid substance in which circular polarization has been observed. This phenomenon, however, has been discovered in several fluids. Thus, right-handed circular polarization exists in turpentine, essence of laurel (?), gum arabic and inuline; and left-handed polarization is observed in essence of citron (?), syrup of sugar, alcoholic solution of camphor, dextrine, and tartaric acid.

1303. *Magnetic circular polarization.*—It has lately been shown by Dr. Faraday, that several transparent solids and liquids acquire the property of circular polarization, when they have been submitted in a certain manner to magnetic and electric action.

These bodies appear to acquire a photogyric virtue, or a property by which they are enabled to cause the planes of undulation of the liquid which traverse them to revolve.

Thus, a link of connection is indicated between two physical influences which seemed hitherto distinct and independent; the force which produces the undulation of the luminous ether, and those of the electric and magnetic fluids.

PRACTICAL QUESTIONS FOR THE STUDENT.

1. What must be the length of a plane mirror, in order that an observer may see his whole length therein, the mirror being placed parallel to the observer?

2. The radius of a concave reflector is 3 inches, and the distance of the focus of incident rays from the vertex 9 inches: what is the position of the focus of reflected rays? (959.)

NOTE.—The formula (A), given in 959., may be reduced to a more convenient form for use.

Thus, by transposing the term $\frac{1}{f}$, we have

$$\frac{1}{f'}=\frac{2}{r}-\frac{1}{f}=\frac{2f-r}{rf}.$$

Hence

$$f'=\frac{rf}{2f-r};$$

which gives us the following Rule for obtaining the position of the focus of reflected rays:—*Multiply the radius of the mirror by the distance of the focus of incident rays from the vertex, and divide the product by twice that distance minus the radius.*

By a proper attention to the signs of r and f, the formula and rule may be applied to all cases of reflection from spherical mirrors, whether concave or convex, and whether the rays be diverging or converging.

For concave mirrors, r is positive; for convex, negative. For diverging rays, f is positive; for converging, negative.

We will discuss briefly the various cases which may occur. For *concave* mirrors, there are three cases.

1st. *The rays may be diverging and the focus beyond the principal focus.* In this case $2f$ being greater than r, it will be seen from the formula that f' will always be positive; that is, the reflected rays will converge to a *real focus* in front of the mirror.

For the question proposed above, we have

$$f' = \frac{3 \times 9}{18 - 3} = 1\tfrac{4}{5} \text{ inches.}$$

2d. *The rays may be diverging and the focus between the principal focus and the vertex.* In this case $2f$ being less than r, f' will be negative, that is, the reflected rays will diverge from an *imaginary* focus behind the mirror.

3d. *The rays may be converging.* In this case, f is negative, and the formula becomes

$$f' = \frac{r \times -f}{-2f - r} = \frac{rf}{2f + r};$$

which is always positive. Hence such rays are always brought to a real focus.

For *convex* mirrors, there are also three cases.

1st. *The rays may be diverging.* Here, f is positive and r negative; and the formula becomes

$$f' = -\frac{rf}{2f + r};$$

which makes f' always negative. Hence, in this case, the reflected rays always diverge from an imaginary focus behind the mirror.

2d. *The rays may converge, and their focus be between the principal focus and the vertex.* In this case, r and f are both negative; $2f$ being less than r Hence the formula is

$$f' = \frac{-r \times -f}{-2f + r} = \frac{rf}{-2f + r};$$

which is positive so long as $2f$ is less than r. Consequently, in this case, the reflected rays converge to a real focus in front of the mirror.

3d. *The rays may converge, and their focus be beyond the principal focus.* In this case, $2f$ being greater than r, the value of f', in the preceding formula, will be negative. Hence the reflected rays will diverge.

It will be perceived from this discussion, that rays, incident upon a concave mirror, are always reflected converging, unless their focus be between the principal focus and the vertex.

On the contrary, rays, incident on a convex mirror, are always reflected diverging, unless their focus be between the principal focus and the vertex.

3. A candle is placed 16 feet from the vertex of a convex mirror whose radius is 2 inches: what is the position of the focus of reflected rays?

4. The focus of converging rays incident upon a convex mirror is 2 inches behind the vertex, the radius of the mirror being 5 inches; the vertex of a concave mirror having the same radius is placed at a distance of 8 feet from the vertex of the first mirror: determine the position of the focus of the rays reflected from the second mirror.

5. Show that, in all cases of reflection from spherical surfaces, the conjugate foci lie on the same side of the principal focus; that they move in opposite directions; and that they meet at the centre of the reflector.

6. A plane mirror, moveable about an axis in its own plane parallel to the axis of the earth, revolves from east to west with half the sun's appa-

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